PEARSON

PHYSICS

WESTERN AUSTRALIA

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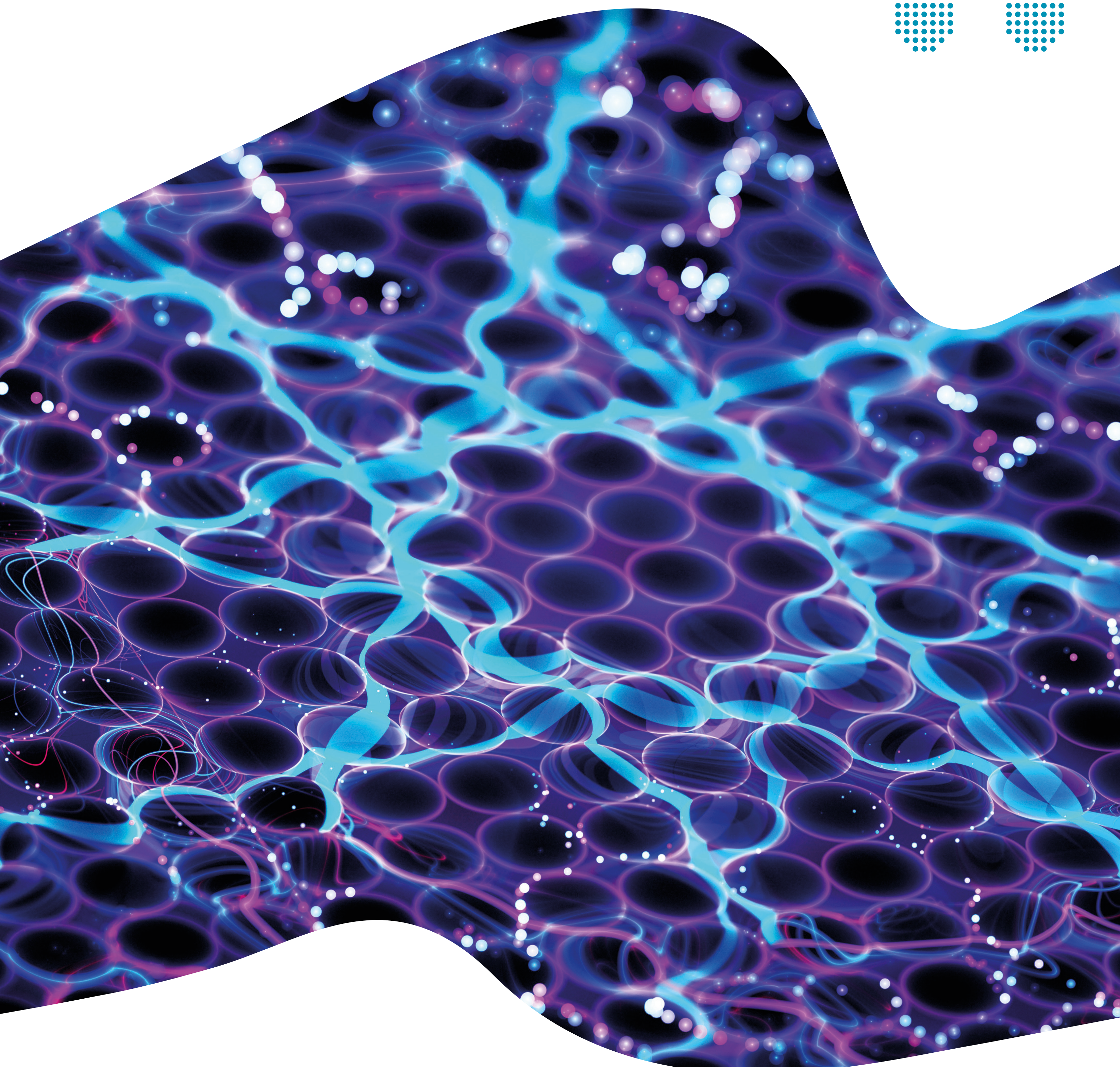


2ND EDITION

CHAPTER 02

Scalars and vectors

Scalars and vectors are mathematical representations of quantities that are used in physics. An understanding of scalars and vectors is essential to learning concepts involving forces and motion.

By the end of this chapter, you will be able to distinguish between scalar and vector quantities. You will be able to use arrows to represent vectors and then add and subtract vectors in one and two dimensions. You will also be able to resolve vectors into their perpendicular components.

Science Understanding

Motion and forces

- distinguish between vector and scalar quantities
- addition and subtraction of vectors in one and two dimensions
- resolution of vectors into components and manipulation (addition and subtraction) of components

2.1 Scalars and vectors

You come into contact with many physical quantities in the natural world every day. Time, mass and distance are all physical quantities. Each of these physical quantities has units to measure them; for example, seconds, kilograms and metres.

Some measurements only make sense if there is also a direction included. For example, a GPS system tells you when to turn and in which direction. Without both of these two instructions, the information is incomplete.

All physical quantities can be divided into two broad groups based on what information you need for the quantity to make logical sense. These groups are called **scalars** and **vectors**. Both of these types of measures will be investigated throughout this section.

SCALARS

There are a number of properties in nature that can be measured or determined, and described using only a number and a unit. For example, if the time taken for a student to travel to school is measured, you need the **magnitude** (size) and the **units** in order to understand the journey. It may take 90 minutes or one and a half hours, the number is important and the units are also important.

Quantities that require magnitude and units are called scalar quantities. Scalars do not need a direction to make logical sense.

Examples of scalar quantities are:

- time
- distance
- volume
- speed
- temperature.

VECTORS

A vector is a physical quantity that requires magnitude, direction and units to make logical sense.

Some physical quantities require magnitude, direction, and units to make sense of them. For example, to describe a force applied on an object, you would need to state the magnitude, the direction of the force, and the units to fully convey the information to a classmate. Such quantities are known as vectors.

Examples of vectors include:

- position
- displacement
- velocity
- acceleration
- force
- momentum.

These measures are discussed in more detail in the coming chapters.

VECTORS AS ARROWS

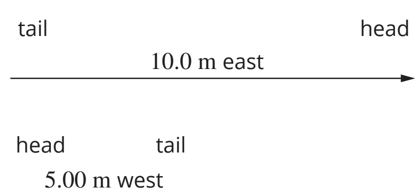


FIGURE 2.1.1 Two vector diagrams. As the top vector is twice as long as the bottom vector, it represents a measure twice the magnitude of the bottom vector. The arrowheads indicate that the vectors are in opposite directions.

A vector is a measurement that has a magnitude, a direction and a unit. A vector can be visually represented as an arrow.

Figure 2.1.1 shows two vector diagrams. In a **vector diagram,** the length of the arrow indicates the magnitude of the vector. The arrowhead shows the direction of the vector. The direction of the vector is always from its tail to its head.

A force is a push or a pull and the unit of measurement for force is the newton (N). If you push a book to the right, it will respond differently to if you push the book to the left. Therefore, a force is only described properly if a direction is included, and so force is considered to be a vector. Forces are described in more detail in Chapter 4. Force is an important concept to understand in physics, so many of the examples in this chapter refer to forces.

In most vector diagrams, the length of the arrow is drawn to scale so that it accurately represents the magnitude of the vector.

In the scaled vector diagram in Figure 2.1.2, a force $F = 4.00$ N to the left acting on the toy car is drawn as a 2.00 cm long arrow pointing to the left. In this example, a scale of 1 cm = 2 N force is used.

An exact scale for the magnitude is not always used. However, it is important that vectors are drawn relative to one another. For example, a vector of 50.0 m north should always be about half as long as a vector of 100.0 m north.

FIGURE 2.1.2 A force of 4.00 N to the left acts on a toy car.

Point of application of vectors

Vector diagrams may be presented slightly differently depending on what they are depicting. If the vector represents a force, the tail end of the arrow is drawn at the point where the force is applied to the object. If it is a displacement vector, attach the tail of the arrow to the position where the object starts. Friction vectors are drawn at the point where they act between an object and a surface.

Figure 2.1.3 shows a force applied by a foot to a ball (95.0 N east) and an opposing friction force (20.0 N west).

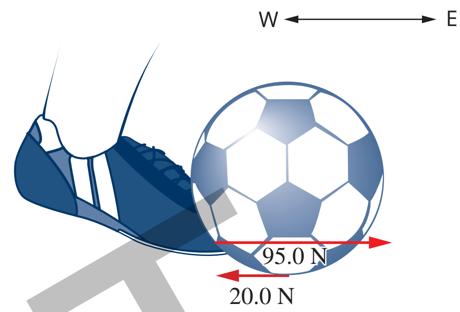


FIGURE 2.1.3 The force on the ball acts at the point of contact between the ball and the foot. The friction force acts between the ball and the ground. The kicking force, as indicated by the length of the arrow, is larger than the friction force.

DIRECTION CONVENTIONS

Vectors need a direction in order to make logical sense. However, for the description of a vector quantity to be useful, there needs to be a way of describing the direction so that everyone understands and agrees upon it.

Vectors in one dimension

For vector problems in one **dimension**, there are a number of **direction conventions** that can be used. For example:

- forwards or backwards
- up or down
- left or right.

You can also use more formal conventions such as:

- north or south
- east or west.

As you can see, for vectors in one dimension there are only two directions possible. The two directions must be in the same dimension or along the same line. The direction convention used should be presented graphically in all vector problems. This is shown in Figure 2.1.4. Arrows like these are placed near the vector diagram so that it is clear which convention is being used.

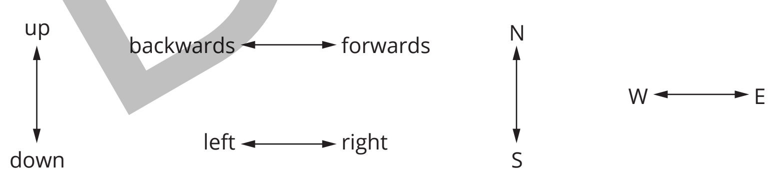


FIGURE 2.1.4 Some common one-dimensional direction conventions.

Sign convention

In calculations involving one-dimensional vectors, a sign convention can also be used to convert physical directions to the mathematical signs of positive and negative. For example, forwards can be positive and backwards can be negative, or right can be positive and left can be negative. A vector of 100 m up can be described as +100 m, provided the relationship between sign and direction conventions are clearly indicated in a legend or key. Some examples are provided in Figure 2.1.5.

The advantage of using a sign convention is that the signs of positive and negative can be entered into a calculator, while the words 'up' and 'right' cannot. This is useful when adding vectors together. This will be discussed in the next section.

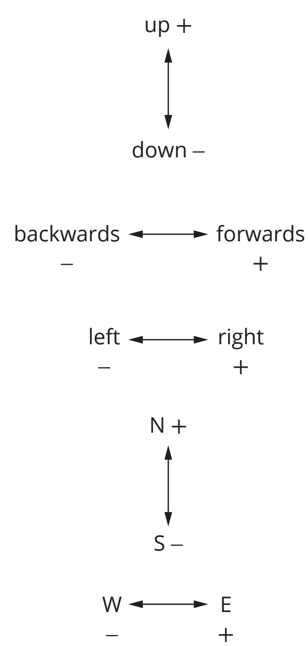
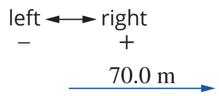


FIGURE 2.1.5 One-dimensional direction conventions can also be expressed as sign conventions.

Worked example 2.1.1

DESCRIBING VECTORS IN ONE DIMENSION



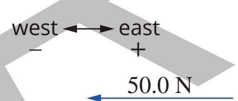
Describe the vector above using:

a the direction convention shown	
Thinking	**Working**
Identify the direction convention being used in the vector.	In this case, the vector is pointing to the right according to the direction convention.
Note the magnitude, unit and direction of the vector.	In this example, the vector is 70.0 m right.

b an appropriate sign convention.	
Thinking	**Working**
Convert the physical direction to the corresponding mathematical sign.	The physical direction of right is positive and left is negative. In this example, the arrow is pointing right, so the mathematical sign is +.
Represent the vector with a mathematical sign, magnitude and unit.	This vector is +70.0 m.

Worked example: Try yourself 2.1.1

DESCRIBING VECTORS IN ONE DIMENSION



Describe the vector above using:

a the direction convention shown

b an appropriate sign convention.

Vectors in two dimensions

When vectors are in one dimension, it is relatively simple to understand direction. However, some vectors will require a description in a two-dimensional plane. These planes could be:

- horizontal, which can be defined using north, south, east and west
- vertical, which can be defined in a number of ways including forwards, backwards, up, down, left and right.

The description of the direction of these vectors is more complicated. Therefore, a more detailed convention is needed for identifying the direction of a vector. There are a variety of conventions, but they all describe a direction as an angle from a known reference point.

Horizontal plane

For a horizontal, two-dimensional plane, the two common methods for describing the direction of a vector are:

- full circle (or true) bearing. A 'full circle bearing' describes north as zero degrees true. This is written as 0°T. In this convention, all directions are given as a clockwise angle from north. As an example, 95.0°T is 95.0° clockwise from north.
- quadrant bearing. An alternative method is to provide a 'quadrant bearing', where all angles are referenced from either north or south and are between 0° and 90.0° towards east or west. In this method, 30.0° T becomes N30.0°E, which can be read as 'from north 30.0° towards the east'.

Using these two conventions, north-west (NW) would be 315.0°T using a full circle bearing, or N 45.0°W using a quadrant bearing. Figure 2.1.6 demonstrates these two methods.

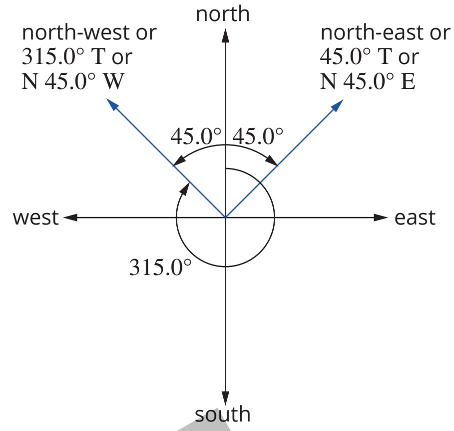


FIGURE 2.1.6 Two horizontal vector directions, viewed from above, using full circle bearings and quadrant bearings.

Vertical plane

For a vertical, two-dimensional plane the directions are referenced to vertical (upwards and downwards) or horizontal (left and right) and are between 0° and 90.0° up or down. For example, a vector direction can be described as '60.0° up from horizontal to the left'. The same vector direction could be described as '30.0° down from the vertical to the left'. The opposite direction to this vector would be '60.0° down from horizontal to the right'. This example is illustrated in Figure 2.1.7.

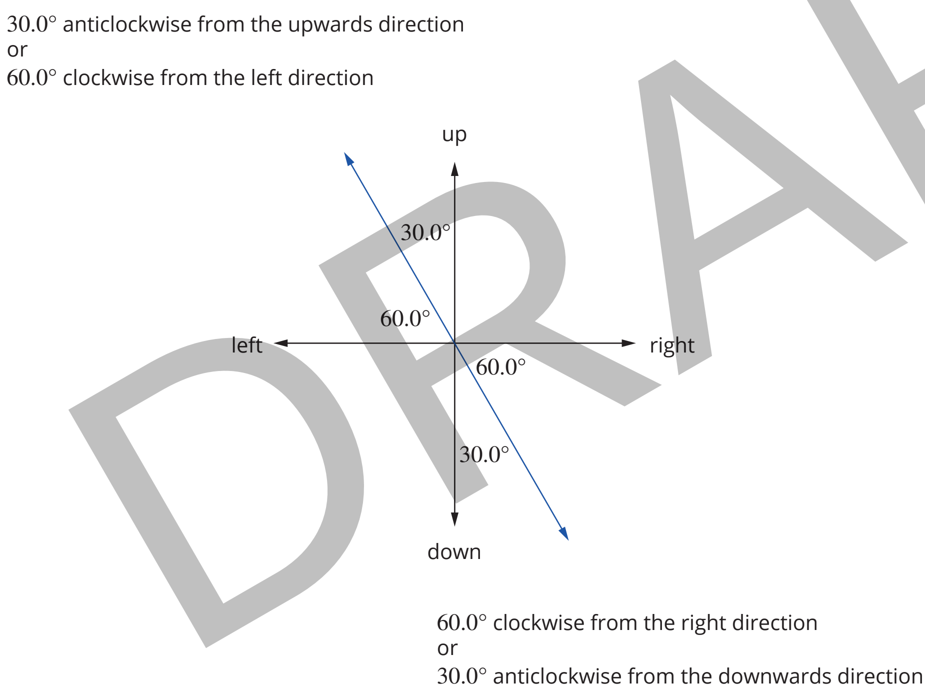


FIGURE 2.1.7 Two vectors in the vertical plane.

PHYSICSFILE

Orienteering

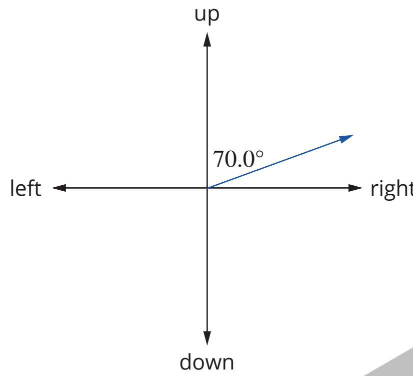
Orienteering is an adventure sport requiring participants to use a compass and map (Figure 2.1.8) to navigate from point to point. Finding your way from place to place involves determining angles measured from any of the cardinal points: north, south, east or west. The vectors between points are shown on the map but contestants need to determine the best way to get there, since the direct line might not be the fastest route. This sport is often timed and competitive and takes place in an unfamiliar, and sometimes challenging, environment. These courses can be followed individually or in teams. Not all orienteering courses have to be completed on foot, as some courses might be designed for mountain bikers or cross-country skiers depending on the environment. There are some permanent orienteering courses in popular spaces in WA such as Whiteman's Park and King's Park, which are free of charge and suitable for any age group. In addition, there are seriously competitive groups that meet on a regular basis.

FIGURE 2.1.8 A compass and map used in orienteering. The numbered points represent the order in which the course should be followed.

Worked example 2.1.2

DESCRIBING TWO-DIMENSIONAL VECTORS

Describe the direction of the following vector using an appropriate method.

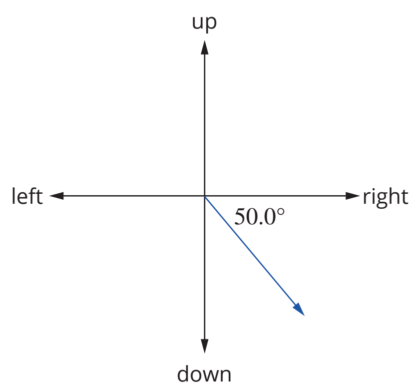


Thinking	Working
Choose the appropriate points to reference the direction of the vector. In this case using the vertical reference makes more sense, as the angle is given from the vertical.	The vector can be referenced to the vertical.
Determine the angle between the reference direction and the vector.	In this example, from vertical to the vector there is 70.0°.
Determine the direction of the vector from the reference direction.	From vertical, the vector is down to the right.
Describe the vector using the sequence: angle, up or down from the reference direction.	This vector is 70.0° down from vertical to the right.

Worked example: Try yourself 2.1.2

DESCRIBING TWO-DIMENSIONAL VECTORS

Describe the direction of the following vector using an appropriate method.



2.1 Review

SUMMARY

- Scalar quantities require a magnitude and a unit to make logical sense. No direction is required for scalar quantities.
- Distance, time, speed and mass are examples of scalar quantities.
- Vectors require magnitude, units and direction to make logical sense.
- Displacement, velocity, acceleration and force are examples of vectors.
- Arrows are used to represent vectors.
- The length of the arrow represents the magnitude of the vector.
- The direction the arrow is pointing indicates the direction of the vector.
- Vector arrows can be drawn to scale, or drawn with lengths that are relative to each other.
- Force vectors are drawn with their tails attached to the point of application of the force on the object.
- Displacement vectors are drawn from the start of the journey to the end of the journey.
- One-dimensional vectors use direction conventions and sign conventions to describe the direction of the vector. Examples include left and right, up and down, + and –.
- The direction of two-dimensional vectors in the horizontal plane can be described using a full circle bearing or a quadrant bearing. Vectors in the vertical plane can be described using angles measured up or down from the vertical or horizontal, to the right or to the left.

KEY QUESTIONS

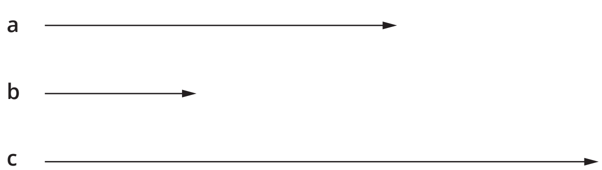
1 What information is required to fully describe a scalar measure?

2 What information is required to fully describe a vector measure?

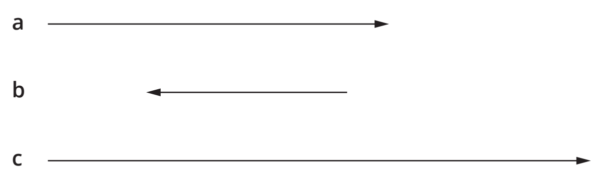
3 Classify each of the following as scalar or vector quantities.

a time
b force
c acceleration
d distance
e position
f displacement
g volume
h momentum
i speed
j velocity
k temperature

4 For the following, decide which of the vector magnitudes provided describes which vector diagram. Note: one of the vector magnitudes is not required.
5.40 N; 2.70 N; 9.00 N; 8.10 N



5 For the following, decide which of the vector magnitudes provided describes which vector diagram. Note: one of the vector magnitudes is not required.
10.80 N; –2.70 N; –5.40 N; 16.20 N

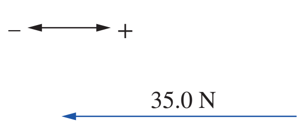


6 Give the opposing direction to each of the following one-dimensional descriptions.

a up
b north
c backwards
d down
e west
f negative

7 Why is it sometimes appropriate to rename direction conventions with a positive or negative sign—for example, + instead of north or – instead of left?

8 Describe the following vector using an appropriate convention.



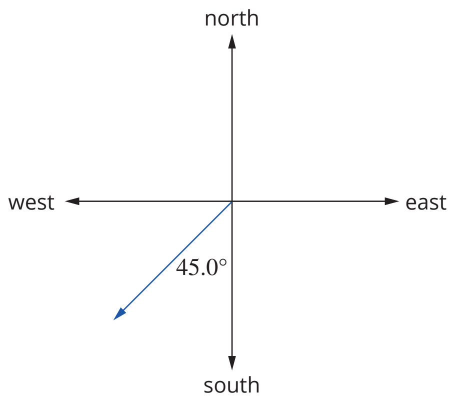
2.1 Review *continued*

9 Describe the following vectors using:

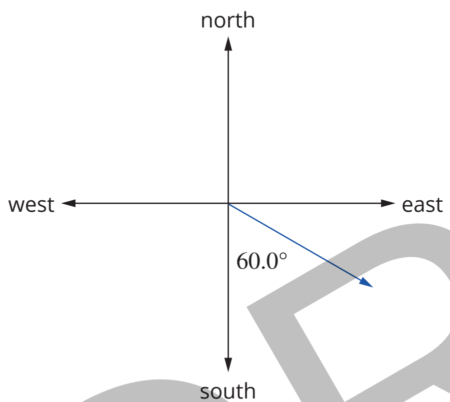
i full circle bearings

ii quadrant bearings.

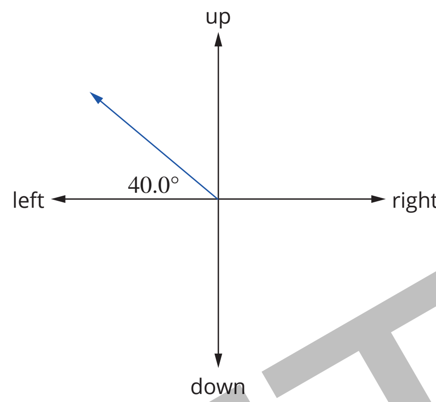
a



b



10 Describe the following vector using appropriate conventions.



2.2 Adding vectors in one and two dimensions

In real situations, more than one vector may act on an object. If this is the case, it is helpful to analyse the associated vector diagrams to find out the overall sum or combined effect of the vectors, known as the resultant vector.

When vectors are combined, it is called adding vectors. Adding vectors that are in one dimension means finding the one resultant vector that is the equivalent of any number of vectors in the same line. It is also possible to find the resultant of a number of vectors that are in two dimensions. The individual vectors can be in any direction, as long as they are all in the same plane.

Vectors can also be added in three dimensions, but this is beyond the realm of this course.

ADDING VECTORS IN ONE DIMENSION

When two or more vectors are in the same dimension, it means that the vectors are either pointing in the same direction or in the opposite direction to each other. They are **collinear** (in line with each other). For example, the displacements 10.0 m west, 15.0 m east and 25.0 m west are all in one dimension. They are all in the same or opposite direction to each other.

Graphical method of adding vectors

Vector diagrams, like those shown in Figure 2.2.1, are convenient for adding vectors. To combine vectors in one dimension, draw the first vector, then start the second vector with its tail at the head of the first vector. Continue adding arrows 'head to tail' until the last vector is drawn. The sum of the vectors, or the **resultant** vector, is drawn from the tail of the first vector to the head of the last vector.

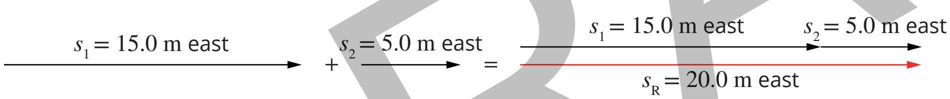


FIGURE 2.2.1 Adding vectors head to tail. This particular diagram represents the addition of 15.0 m east and 5.0 m east. The resultant vector, shown in red, is 20.0 m east.

In Figure 2.2.1, the two vectors s_1 (15.0 m east) and s_2 (5.0 m east) are drawn separately. The two vectors are then drawn with s_1 and s_2 connected head to tail. The resultant vector s_R is drawn from the tail of s_1 to the head of s_2. The magnitude (size) of the resultant vector can be deduced from the magnitudes of the separate vectors: 15.0 m + 5.0 m = 20.0 m.

Alternatively, vectors can be drawn to scale, for example: 1 m = 1 cm. The resultant vector is then directly measured from the scale diagram. The direction of the resultant vector is the same as the direction from the tail of the first vector to the head of the last vector.

Algebraic method of adding vectors

To add vectors in one dimension algebraically, a sign convention is used to represent the direction of the vectors (see Figure 2.2.2). When applying a sign convention, it is important to provide a key explaining the convention used.

The sign convention allows you to enter the signs and magnitudes of vectors into a calculator. The sign of the final magnitude gives the direction of the resultant vector.

Vectors are added head to tail. The resultant vector is drawn from the tail of the first vector to the head of the last vector.

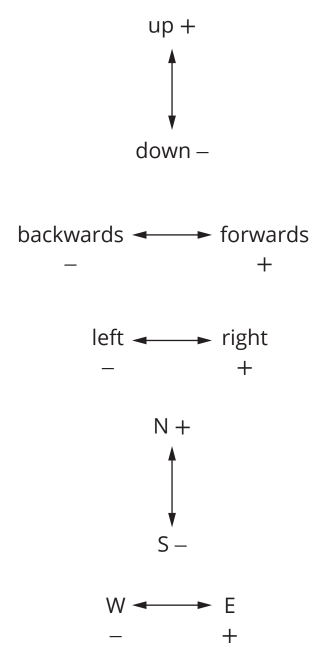


FIGURE 2.2.2 Common sign and direction conventions.

Worked example 2.2.1

ADDING VECTORS IN ONE DIMENSION USING ALGEBRA

Use the sign and direction conventions shown in Figure 2.2.2 on page xxx to determine the resultant vector of a student who walks 25.0 m west, 16.0 m east, 44.0 m west and then 12.0 m east.

Thinking	Working
Apply the sign conventions to change each of the directions to signs.	25.0 m west = –25.0 m 16.0 m east = +16.0 m 44.0 m west = –44.0 m 12.0 m east = +12.0 m
Add the magnitudes and their signs together.	Resultant vector = (–25.0) + (+16.0) + (–44.0) + (+12.0) = –41.0 m
Refer to the sign and direction conventions to determine the direction of the resultant vector.	Negative is west. ∴ Resultant vector = 41.0 m west

Worked example: Try yourself 2.2.1

ADDING VECTORS IN ONE DIMENSION USING ALGEBRA

Use the sign and direction conventions shown in Figure 2.2.2 on page xxx to determine the resultant force on a box that has the following forces acting on it: 16.0 N up, 22.0 N down, 4.0 N up and 17.0 N down.

ADDING VECTORS IN TWO DIMENSIONS

Adding vectors in two dimensions means that all of the vectors must be in the same plane (coplanar). The individual vectors can go in any direction within the plane, and can be separated by any angle. The examples in this section illustrate vectors in the horizontal plane, but the same strategies apply to adding vectors in the vertical plane.

The horizontal plane is the one that is looked down on from above. Examples include looking at a house plan or map placed on a desk. The direction convention that suits this plane best is the north, south, east and west convention, or the forwards, backwards, left and right convention. This is shown in Figure 2.2.3.

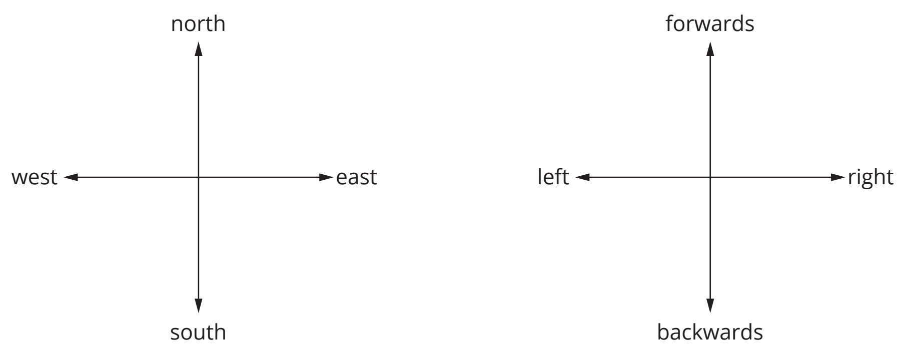


FIGURE 2.2.3 The direction conventions for the horizontal plane.

When two vectors are in the horizontal plane, the angles between them can be right-angled, acute or obtuse.

Graphical method of adding vectors

The magnitude and direction of a resultant vector can be determined by measuring an accurately drawn scaled vector diagram. There are two main ways to do this:

- head to tail method
- parallelogram method.

Head to tail method

To add vectors at right angles to each other using a graphical method, use an appropriate scale and then draw each vector head to tail. The resultant vector is the vector that starts at the tail of the first vector and ends at the head of the last vector. To determine the magnitude and direction of the resultant vector, measure the length of the resultant vector and compare it to the scale, then measure and describe the direction appropriately.

In Figure 2.2.4, two vectors, 30.0 m east and 20.0 m south, are added head to tail. The resultant vector, shown in red, is estimated to be about 36 m according to the scale provided. Using a protractor, the resultant vector is measured to be in the direction 34.0° south of east. This represents a direction of S 56.0° E when using quadrant bearings from south.

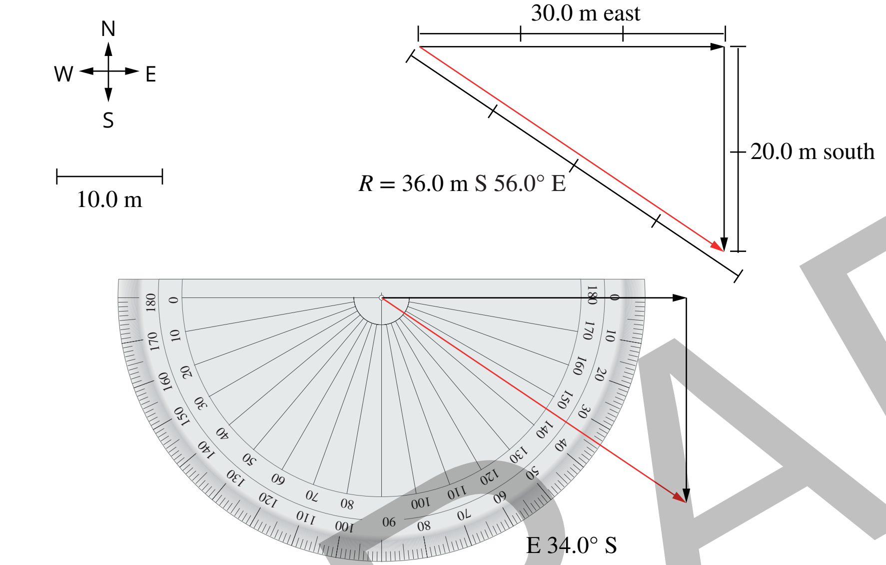


FIGURE 2.2.4 Adding two vectors at right angles, using the graphical method.

If the two vectors are at angles other than 90.0° to each other, the graphical method is ideal for finding the resultant vector. In Figure 2.2.5, the force vectors 15.0 N east and 10.0 N S 45° E are added head to tail. The magnitude of the resultant vector is estimated to be about 23 N. The direction of the resultant vector is measured by a protractor from east to be 18.0° towards the south, which should be written as E 18.0° S or S 72.0° E.

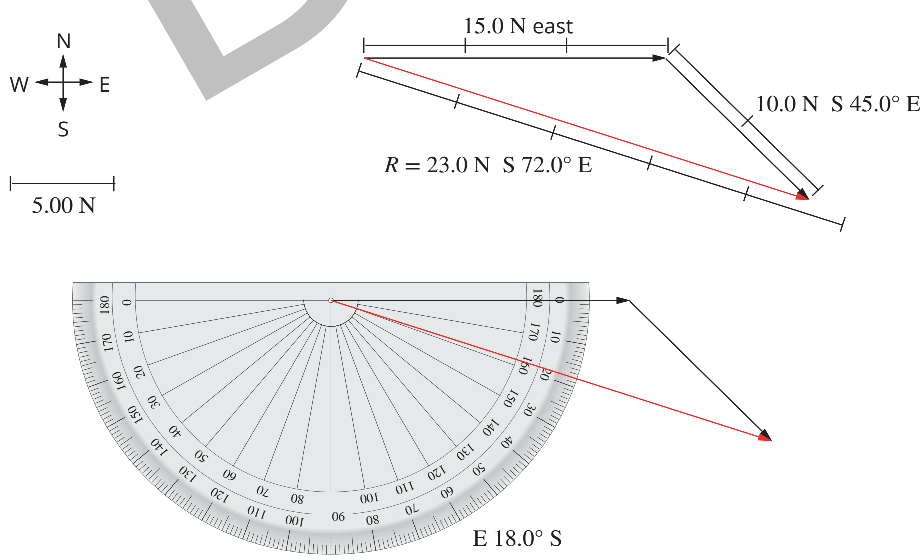


FIGURE 2.2.5 Adding two vectors not at right angles, using the graphical method.

Parallelogram method

An alternative method for determining a resultant vector is to construct a parallelogram of vectors. In this method, the two vectors to be added are drawn tail to tail. Next, a parallel line is drawn for each vector as shown in Figure 2.2.6. In this figure, the parallel lines have been drawn as dotted lines. The resultant vector is drawn from the tails of the two vectors to the intersection of the dotted parallel lines.

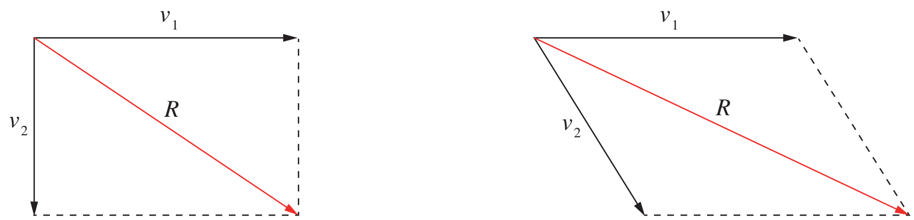
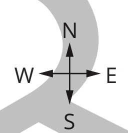


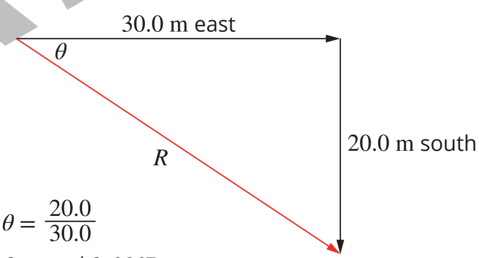
FIGURE 2.2.6 Parallelogram of vectors method for adding two vectors.

Geometric method of adding vectors

Graphical methods of adding vectors in two dimensions only give approximate results as they rely on comparing the magnitude of the resultant vector to a scale and measuring the direction with a protractor. A more accurate method to resolve vectors is to use Pythagoras' theorem and trigonometry. These techniques are referred to as geometric methods. Geometric methods can be used to calculate the magnitude of the resultant vector and its direction. However, Pythagoras' theorem and trigonometry can only be used for finding the resultant vector of two vectors that are at right angles to each other.

In Figure 2.2.7, two vectors, 30.0 m east and 20.0 m south, are added head to tail. The resultant vector, shown in red, is calculated using Pythagoras' theorem to be 36.1 m. The resultant vector is calculated to be in the direction S 56.3° E. This result is more accurate than the answer determined earlier in this section.





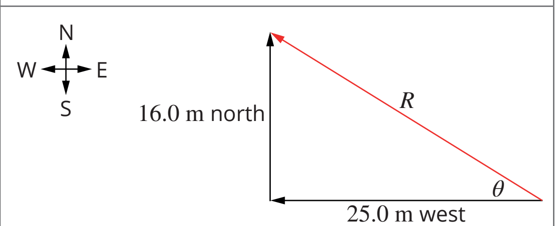
$$R^2 = 30.0^2 + 20.0^2$$
$$= 900 + 400$$
$$R = \sqrt{1300}$$
$$= 36.1 \text{ m}$$

$$\tan\theta = \frac{20.0}{30.0}$$
$$\theta = \tan^{-1} 0.6667$$
$$= 33.7°$$
E 33.7° S (or S 56.3° E)

FIGURE 2.2.7 Adding two vectors at right angles, using the geometric method.

Worked example 2.2.2

ADDING VECTORS IN TWO DIMENSIONS USING GEOMETRY

Determine the resultant vector that represents a child running 25.0 m west and then 16.0 m north. Refer to Figure 2.2.2 on page xxx for sign and direction conventions if required.	
Thinking	**Working**
Construct a vector diagram showing the vectors drawn head to tail. Draw the resultant vector from the tail of the first vector to the head of the last vector.	

PHYSICSFILE

Pythagoras' theorem and trigonometric ratios

The Year 11 ATAR Physics syllabus assumes students will be able to:

- use Pythagoras' theorem, similarity of triangles and the angle sum of a triangle
- solve simple sine, cosine and tangent relationships in a right-angle triangle.

Pythagoras' theorem is $a^2 + b^2 = c^2$ where c is the hypotenuse (the longest side) and a and b are the two shorter sides of a right-angled triangle. The hypotenuse is easily recognised as it is directly across from (opposite) the right angle of the triangle.

Most students learn the mnemonic SOHCAHTOA in their maths classes. It is often pronounced soh-cah-toa and provides a way to remember the trigonometric ratios:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the resultant vector.	$R^2 = 25.0^2 + 16.0^2$ $= 625 + 256$ $R = \sqrt{881}$ $= 29.7\text{ m}$
Using trigonometry, calculate the angle from the west vector to the resultant vector.	$\tan\theta = \frac{16.0}{25.0}$ $\theta = \tan^{-1} 0.640$ $= 32.6°$
Determine the direction of the vector relative to north or south.	$90.0° - 32.6° = 57.4°$ The direction is N 57.4° W
State the magnitude and direction of the resultant vector.	$R = 29.7\text{ m}$ N 57.4° W

Worked example: Try yourself 2.2.2

ADDING VECTORS IN TWO DIMENSIONS USING GEOMETRY

Determine the resultant force when forces of 5.00 N east and 3.00 N north both act on a tree at the same time. Refer to Figure 2.2.2 on page xxx for sign and direction conventions if required.

PHYSICS IN ACTION

Surveying

Surveyors use technology to measure, analyse and manage data about the shape of the land and the exact location of landmarks and buildings. They take many measurements, including angles and distances, and use them to calculate more advanced data such as vectors, bearings, co-ordinates, elevations and maps. Surveyors typically use theodolites (see Figures 2.2.8 and 2.2.9), GPS survey equipment, laser range finders and satellite images to map the land in three dimensions.

Surveyors are often the first professionals on a building site to ensure that the boundaries of the property are correct. They also ensure that the building is built in the correct location. Surveyors must liaise closely with architects both before and during a building project as they provide position and height data for walls and floors.

FIGURE 2.2.8 Surveying the land with a theodolite.

FIGURE 2.2.9 Surveying equipment being used on a building site.

2.2 Review

SUMMARY

- Combining vectors is known as adding vectors.
- One-dimensional vector addition refers to vectors in a line, while two-dimensional vector addition refers to vectors on a plane.
- Adding vectors in one dimension can be done graphically by using vector diagrams. After adding vectors head to tail, the resultant vector can be drawn from the tail of the first vector to the head of the last vector.
- Adding vectors in one dimension can be done algebraically by applying a sign convention. Vectors with direction become vectors with either positive or negative signs.
- Adding vectors in two dimensions can be estimated graphically with a scale and a protractor.
- An alternate method of adding vectors in two dimensions is to construct a parallelogram of vectors.
- Adding vectors in two dimensions can be done more accurately using Pythagoras' theorem and the trigonometric ratios of a right-angled triangle.

KEY QUESTIONS

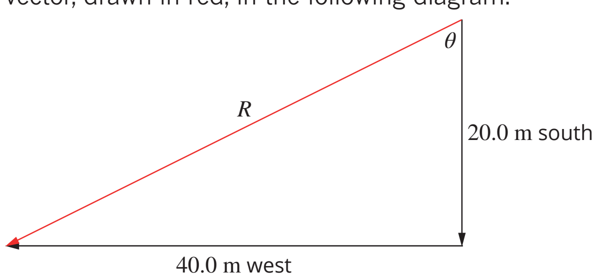
1 Every day, Monday to Friday, a man drives his car 3.00 km east to work and returns the same route home. He drives nowhere else.
  a What distance will he have driven in a week?
  b What is his displacement at the end of a week?

2 Add the following vectors to find the resultant vector: 3.00 m up, 2.00 m down and 3.00 m down.

3 Determine the resultant vector of a toy train that is made to move in these directions: 23.0 m forwards, 16.0 m backwards, 7.0 m forwards and 3.0 m backwards.

4 When adding vector B to vector A using the head to tail method, from what point, and to what point, is the resultant vector drawn?
  A from the head of A, to the tail of B
  B from the tail of B, to the head of A
  C from the head of B, to the tail of A
  D from the tail of A, to the head of B

5 Describe the magnitude and direction of the resultant vector, drawn in red, in the following diagram.



6 Forces of 2000 N north and 6000 N east act on an object. What is the resultant force?

7 Calculate the magnitude of the resultant vector when 30.0 m south and 40.0 m west are added.

8 Determine the resultant force acting on an object being pulled north by force 3000 N; south by 5000 N and east by 5000 N.

9 Determine the resultant vector of the following combination: 3350 N forwards, 6220 N backwards, 2235 N forwards and 634 N forwards.

2.3 Subtracting vectors in one and two dimensions

The previous section discussed combining or adding vectors. In physics, there are times when the difference between two vectors has to be determined. For example, a change in velocity is determined by the final velocity minus the initial velocity. In other words, you must subtract vectors. The best way to subtract one vector from another is to add the opposite vector.

SUBTRACTING VECTORS IN ONE DIMENSION

To find the difference between two vectors, you must subtract the initial vector from the final vector. To do this, work out which is the initial vector, then reverse its direction. These two vectors are then added: the final vector and the opposite of the initial vector.

This technique can be applied both graphically and algebraically.

Graphical method of subtracting vectors

Velocity is a quantity that gives an indication of how fast an object is moving in a certain direction. It is a vector because the direction is important when stating the velocity of an object. For example, the velocity of the tennis ball moving towards the racquet in Figure 2.3.2 is different from the velocity of the tennis ball as it leaves the racquet. The concept of velocity is covered in more detail in Chapter 3, but it is useful to use the example of velocity now when discussing the subtraction of vectors. The processes applied when subtracting velocity vectors work for all other vectors.

FIGURE 2.3.2 As velocity is a vector, direction is important. The tennis ball has a different velocity when it leaves the racquet from when it travelled towards the racquet.

To subtract velocity vectors in one dimension using a graphical method, determine which vector is the initial velocity and which is the final velocity. The final velocity is drawn first. The initial velocity is then drawn, but in the opposite direction to its original direction. The sum of these vectors, or the resultant vector, is drawn from the tail of the final velocity to the head of the reversed initial velocity. This resultant vector is the difference between the two velocities, or Δv.

In Figure 2.3.3, the two separate velocity vectors v_1 ($9.00\,\text{m}\,\text{s}^{-1}$ east) and v_2 ($3.00\,\text{m}\,\text{s}^{-1}$ east) are drawn separately. The initial velocity, v_1, is then drawn again in the opposite direction: $-v_1$ or $9.00\,\text{m}\,\text{s}^{-1}$ west.

FIGURE 2.3.3 Subtracting vectors using the graphical method: the initial vectors.

PHYSICSFILE

Double negatives

When a negative number is multiplied by another negative number, the result is a positive number. A double negative is also illustrated when a negative number is subtracted from another number. The effect is to add the two numbers together. For example, (5) – (–2) = 7.

It is important to differentiate between the terms subtract, minus, take away or difference between, and the term negative. The terms subtract, minus, take away or difference between are processes, like add, multiply and divide. You will find these processes grouped together on your calculator. The term negative is a property of a number that means that it is opposite to positive. There is a separate button on your calculator (usually shown as ±) for this property.

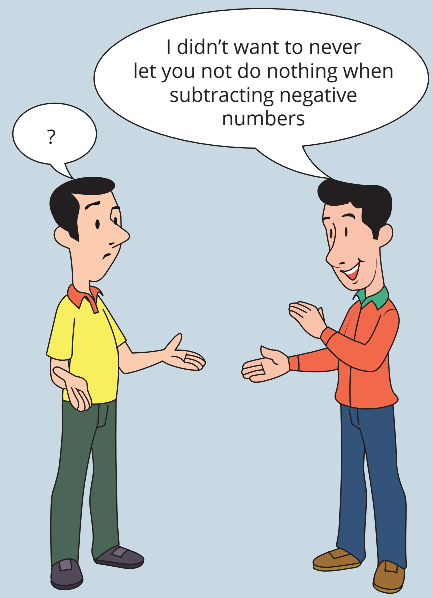


FIGURE 2.3.1 Double negatives can be confusing.

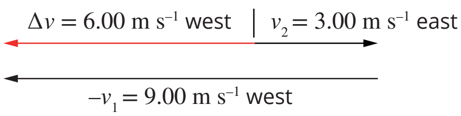


FIGURE 2.3.4 Subtracting vectors using the graphical method: the resultant vector.

> To find the difference between or change in vectors, subtract the initial vector from the final vector. Vectors are subtracted by adding the negative of a vector.

Figure 2.3.4 illustrates how the difference between the vectors is found. Firstly, the final velocity, v_2, is drawn. Then the opposite of the initial velocity, $-v_1$, is drawn head to tail. The resultant vector, Δv, is drawn from the tail of v_2 to the head of $-v_1$.

The magnitude of the resultant vector, Δv, can be calculated from the magnitudes of the two vectors. Alternatively, you could draw the vectors to scale and then measure the resultant vector against that scale—for example $1\,\text{m}\,\text{s}^{-1} = 1\,\text{cm}$.

The direction of the resultant vector, Δv, is the same as the direction from the tail of the final velocity, v_2, to the head of the opposite of the initial velocity, $-v_1$.

Algebraic method of subtracting vectors

To subtract velocity vectors in one dimension algebraically, a sign convention is used to represent the direction of the velocities. Some examples of one-dimensional directions include east and west, north and south, and up and down. These options are replaced by positive (+) or negative (–) signs when calculations are performed. To change the direction of the initial velocity, simply change the sign from positive to negative or from negative to positive.

The equation for finding the change in velocity is:

change in velocity = final velocity – initial velocity

$$\Delta v = v_2 - v_1$$

$$\Delta v = v_2 + (-v_1)$$

change in velocity = final velocity + the opposite of the initial velocity

The final velocity is added to the opposite of the initial velocity. Since the change in velocity is a vector, it will consist of a sign and a magnitude. The sign of the answer can be compared with the sign and direction convention (Figure 2.3.5) to determine the direction of the change in velocity.

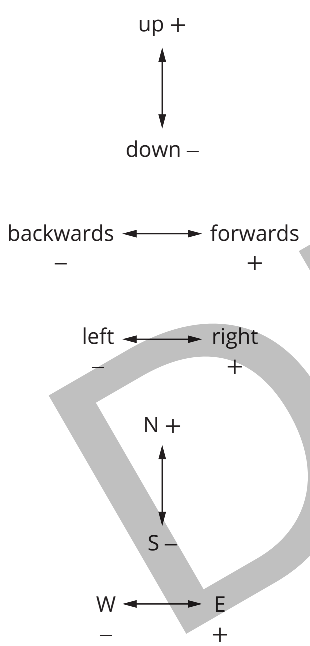


FIGURE 2.3.5 Sign and direction conventions.

Worked example 2.3.1

SUBTRACTING VECTORS IN ONE DIMENSION USING ALGEBRA

Use the sign and direction conventions shown in Figure 2.3.5 to determine the change in velocity of a plane as it changes from $255\,\text{m}\,\text{s}^{-1}$ west to $160\,\text{m}\,\text{s}^{-1}$ east.

Thinking	Working
Apply the sign and direction convention to change the directions to signs.	$v_1 = 255\,\text{m}\,\text{s}^{-1}$ west $= -255\,\text{m}\,\text{s}^{-1}$ $v_2 = 160\,\text{m}\,\text{s}^{-1}$ east $= +160\,\text{m}\,\text{s}^{-1}$
Reverse the direction of the initial velocity, v_1, by reversing the sign.	$-v_1 = 255\,\text{m}\,\text{s}^{-1}$ east $= +255\,\text{m}\,\text{s}^{-1}$
Use the formula for change in velocity to calculate the magnitude and the sign of Δv.	$\Delta v = v_2 + (-v_1)$ $= (+160) + (+255)$ $= +415\,\text{m}\,\text{s}^{-1}$
Refer to the sign and direction convention to determine the direction of the change in velocity.	Positive is east. $\therefore \Delta v = 415\,\text{m}\,\text{s}^{-1}$ east

Worked example: Try yourself 2.3.1

SUBTRACTING VECTORS IN ONE DIMENSION USING ALGEBRA

Use the sign and direction conventions shown in Figure 2.3.5 to determine the change in velocity of a rocket as it changes from $212\,\text{m}\,\text{s}^{-1}$ up to $2200\,\text{m}\,\text{s}^{-1}$ up.

SUBTRACTING VECTORS IN TWO DIMENSIONS

Changing velocity in two dimensions can occur when turning a corner. For example, walking at 3.00 m s^{-1} west, then turning to travel at 3.00 m s^{-1} north. Although the magnitude of the velocity is the same, the direction is different.

A change in velocity in two dimensions can be determined using either the graphical method or the geometric method described in the previous section. The initial velocity must always be reversed before it is added to the final velocity.

The two-dimensional direction conventions were introduced in the previous section and are shown here in Figure 2.3.6.

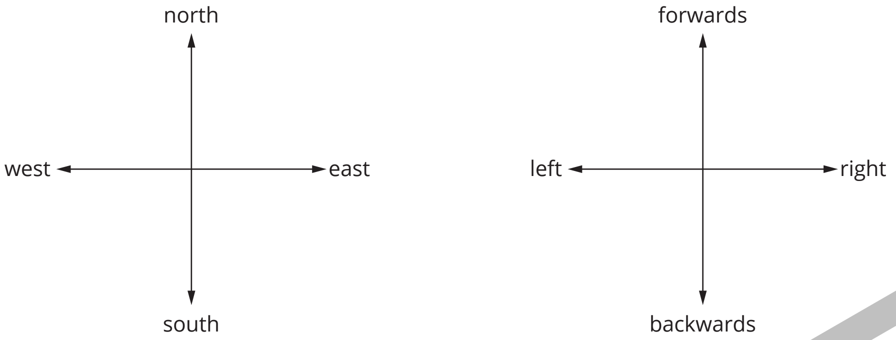


FIGURE 2.3.6 The direction conventions for the horizontal plane.

> When a change in a vector occurs, the magnitude and/or the direction of the vector can change.

Graphical method of subtracting vectors

To subtract vectors using a graphical method, choose a direction convention and a scale, and draw each vector.

Using velocity as an example, the steps to do this are as follows.

- Draw in the final velocity first.
- Draw the opposite of the initial velocity head to tail with the final velocity vector.
- Draw the resultant change in velocity vector, starting at the tail of the final velocity vector and ending at the head of the opposite of the initial velocity vector.
- Measure the length of the resultant vector and compare it to the scale to determine the magnitude of the change in velocity.
- Measure an appropriate angle to determine the direction of the resultant vector.

Figure 2.3.7 shows the velocity vectors for travelling 3.00 m s^{-1} west and then turning and travelling 3.00 m s^{-1} north. The opposite of the initial velocity is drawn as 3.00 m s^{-1} east.

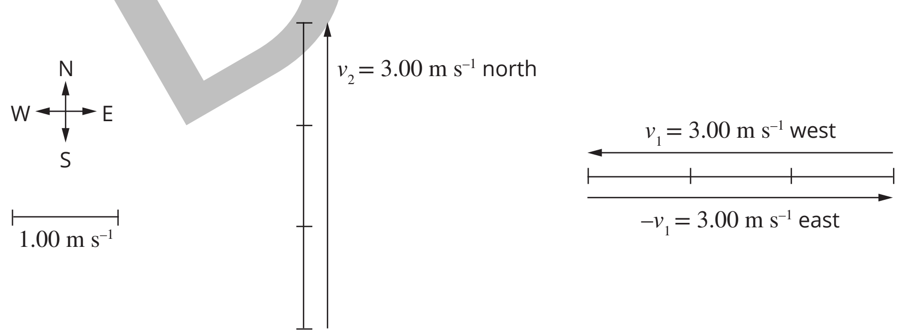


FIGURE 2.3.7 Subtracting two vectors at right angles, using the graphical method: the initial vectors.

To determine the change in velocity, the final velocity vector is drawn first, then from its head the opposite of the initial velocity is drawn. This is shown in Figure 2.3.8. The magnitude of the change in velocity (the resultant vector) is shown in red. It is estimated to be about 4.30 m s^{-1} according to the scale provided. Using a protractor, the resultant vector is measured to be in the direction N 45.0° E.

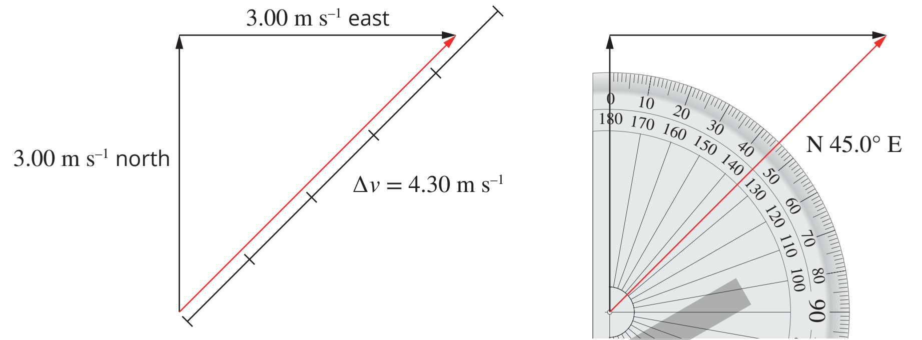


FIGURE 2.3.8 Subtracting two vectors at right angles, using the graphical method: the resultant vectors.

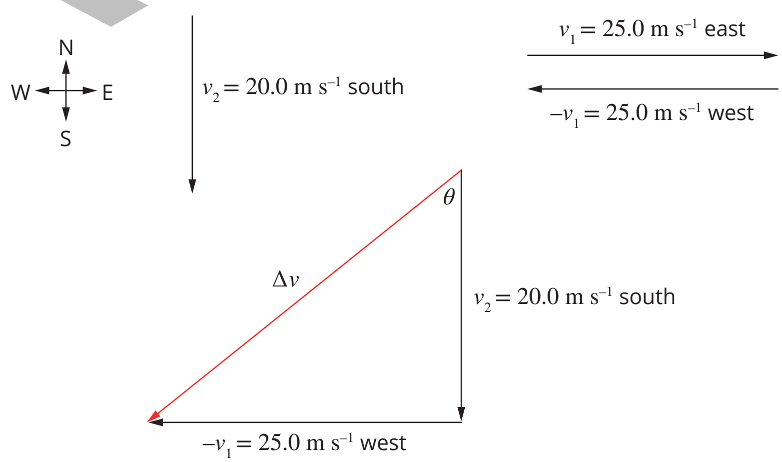
Geometric method of subtracting vectors

The graphical method of subtracting vectors in two dimensions only gives approximate results, as it relies on comparing the magnitude of the change in velocity vector to a scale and measuring its direction with a protractor.

As you saw earlier in the chapter when adding vectors, a more accurate method to subtract vectors is to use Pythagoras' theorem and trigonometry.

Figure 2.3.9 shows how to calculate the resultant velocity when changing from 25.0 m s^{-1} east to 20.0 m s^{-1} south. The initial velocity of 25.0 m s^{-1} east and the final velocity of 20.0 m s^{-1} south are drawn. Then the opposite of the initial velocity is drawn as 25.0 m s^{-1} west. The final velocity vector is drawn first, then from its head the opposite of the initial velocity is drawn. The resultant velocity vector, shown in red, is calculated to be 32.0 m s^{-1}. The resultant vector is calculated to be in the direction S 51.3° W.

The resultant vector is 32.0 m s^{-1} S 51.3° W.



$$R^2 = 25.0^2 + 20.0^2 \qquad \tan\theta = \frac{25.0}{20.0}$$

$$= 625 + 400 \qquad \theta = \tan^{-1} 1.25$$

$$R = \sqrt{1025} \qquad = 51.3°$$

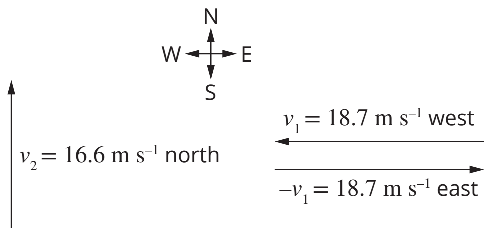
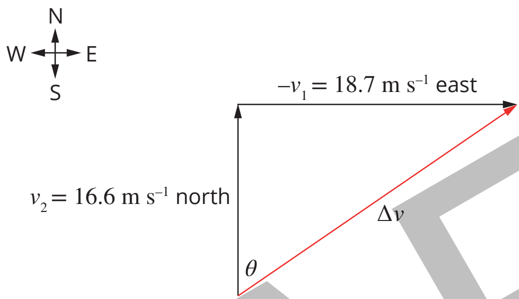
$$= 32.0\ \text{m s}^{-1} \qquad \text{S } 51.3°\ \text{W}$$

FIGURE 2.3.9 Subtracting two vectors at right angles, using the geometric method.

Worked example 2.3.2

SUBTRACTING VECTORS IN TWO DIMENSIONS USING GEOMETRY

Determine the change in velocity of Clare's scooter as she turns a corner if she approaches it at 18.7 m s^{-1} west and exits at 16.6 m s^{-1} north.

Thinking	Working
Draw the final velocity vector, v_2, and the initial velocity vector, v_1, separately. Then draw the initial velocity in the opposite direction.	
Construct a vector diagram drawing v_2 first and then from its head draw the opposite of v_1. The change of velocity vector is drawn from the tail of the final velocity to the head of the opposite of the initial velocity.	
As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the change in velocity.	$R^2 = 16.6^2 + 18.7^2$ $= 275.26 + 349.69$ $R = \sqrt{625.25}$ $= 25.0\ \text{m s}^{-1}$
Calculate the angle from the north vector to the change in velocity vector.	$\tan\theta = \frac{18.7}{16.6}$ $\theta = \tan^{-1} 1.16$ $= 48.4°$
State the magnitude and direction of the change in velocity.	$\Delta v = 25.0\ \text{m s}^{-1}$ N 48.4° E

Worked example: Try yourself 2.3.2

SUBTRACTING VECTORS IN TWO DIMENSIONS USING GEOMETRY

Determine the change in velocity of a ball as it bounces off a wall. The ball approaches at 7.00 m s^{-1} south and rebounds at 6.00 m s^{-1} east.

2.3 Review

SUMMARY

- To find the difference between, or change in vectors, subtract the initial vector from the final vector.
- Vectors are subtracted by adding the negative, or opposite, of a vector.
- Vector subtraction in one or two dimensions can be determined graphically using a scale and a protractor.
- Vector subtraction in one dimension can be determined algebraically by applying a sign and direction convention.
- Vector subtraction in two dimensions can be determined geometrically using Pythagoras' theorem and trigonometry.

KEY QUESTIONS

1 A car that was initially travelling at a velocity of 3.00 m s^{-1} west is later travelling at 5.00 m s^{-1} east. What is the difference between the two vectors?

2 Determine the change in velocity of a runner who changes from running on grass at 4.00 m s^{-1} to the right to running in sand at 2.00 m s^{-1} to the right.

3 A student throws a ball up into the air at 4.00 m s^{-1}. A short time later the ball is travelling back downwards to hit the ground at 3.00 m s^{-1}. Determine the change in velocity of the ball during this time.

4 Tom hits a tennis ball against a wall. If the ball travels towards the wall at 35.0 m s^{-1} north and rebounds at 32.5 m s^{-1} south, calculate the change in velocity of the ball.

5 Jamelia applies the brakes on her car and changes her velocity from 22.2 m s^{-1} forwards to 8.20 m s^{-1} forwards. Calculate the change in velocity of Jamelia's car.

6 A jet plane makes a turn after taking off, changing its velocity from 345 m s^{-1} south to 406 m s^{-1} west. Calculate the change in the velocity of the jet.

7 Yvette hits a golf ball that strikes a tree and changes its velocity from 42.0 m s^{-1} east to 42.0 m s^{-1} north. Calculate the change in the velocity of the golf ball.

8 A yacht tacks (changes course) during a race, changing its velocity from 7.05 m s^{-1} south to 5.25 m s^{-1} west. Calculate the change in the velocity of the yacht.

9 A cyclist travelling north at 40.0 km h^{-1} does a U-turn at the halfway point of a race and slows to 25.0 km h^{-1}. Determine:

a the change in speed

b the change in velocity.

10 A motorcyclist travelling south turns around a roundabout and exits at the third exit (now heading west) maintaining a speed of 30.0 km h^{-1}. Determine their change in velocity.

2.4 Vector components

Sections 2.2 and 2.3 explored how vectors can be combined to find a resultant vector. In physics, there are times when it is useful to break one vector up into two vectors that are at right angles to each other. For example, if a force vector is acting at an angle up from horizontal, as shown in Figure 2.4.1, this vector can be considered to consist of two independent components, one vertical and one horizontal.

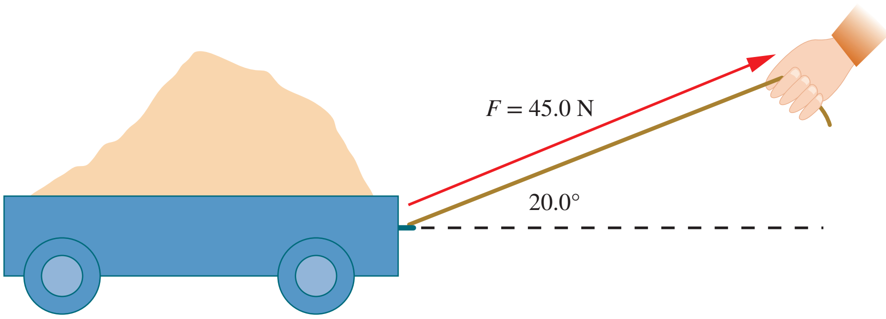


FIGURE 2.4.1 The pulling force acting on the cart has a component in the horizontal direction and a component in the vertical direction.

The components of a vector can be found using trigonometry.

FINDING PERPENDICULAR COMPONENTS OF A VECTOR

Vectors at an angle are more easily dealt with if they are broken up into perpendicular **components**, that is, two components that are at right angles to each other. These components, when added together, give the original vector. To find the components of a vector, a right-angled triangle is constructed with the original vector as the hypotenuse. This is shown in Figure 2.4.2. The hypotenuse is always the longest side of a right-angled triangle and is opposite the 90.0° angle. The other two sides of the triangle are each shorter than the hypotenuse and form the 90.0° angle with each other. These two sides are the perpendicular components of the original vector.

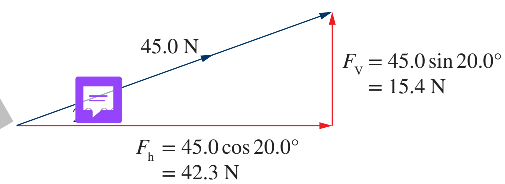


FIGURE 2.4.2 The perpendicular components (shown in red) of the original vector (shown in blue). The original vector is the hypotenuse of the triangle.

Geometric method of finding vector components

The geometric method of finding the perpendicular components of vectors is to construct a right-angled triangle using the original vector as the hypotenuse. This was illustrated in Figure 2.4.2. The magnitude and direction of the components are then determined using trigonometry. A good rule to remember is that no component of a vector can be larger than the vector itself. In a right-angled triangle, no side is longer than the hypotenuse. The original vector must be the hypotenuse and its components must be the other two sides of the triangle.

Figure 2.4.3 shows a force vector of 50.0 N (drawn in black) acting on a box in a direction 30.0° up from horizontal to the right. The horizontal and vertical components of this force must be found in order to complete further calculations.

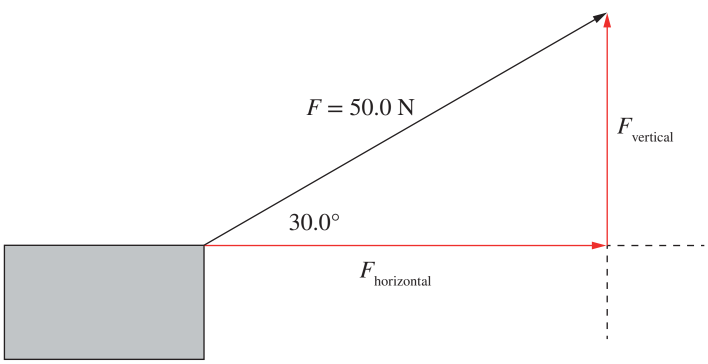


FIGURE 2.4.3 Finding the horizontal and vertical components of a force vector.

PHYSICSFILE

Projectile motion

When objects undergo projectile motion, such as when a ball leaves a tennis racquet, the vertical and horizontal motion are affected differently. So, problem solving begins with separating the initial velocity of the projectile into horizontal and vertical components. At all times during the flight of the ball, the force of gravity is pulling down on the ball, causing it to accelerate downwards until it hits something: another racquet, the net, or the ground. However, the ball will continue to move in the horizontal direction at the same constant velocity as there is no force in that direction, once the ball has left the racquet (figures 2.4.4 and 2.4.5).

FIGURE 2.4.4 A tennis ball in motion accelerates downwards while maintaining a constant horizontal velocity.

FIGURE 2.4.5 By judging the perfect vertical and horizontal components of the velocity required, a tennis player can hit the perfect shot.

The horizontal component vector is drawn from the tail of the 50.0 N vector towards the right, with its head directly below the head of the original 50.0 N vector. The vertical component vector is drawn from the head of the horizontal component to the head of the original 50.0 N vector.

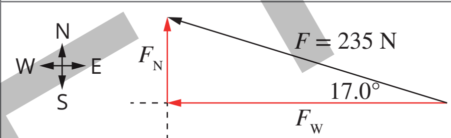
Using trigonometry, the horizontal component of the force is calculated to be 43.3 N horizontally to the right. The vertical component is calculated to be 25.0 N vertically upwards. The calculations are shown below:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotense}}$$
$$\text{adj} = \text{hyp}\cos\theta$$
$$F_h = (50.0)(\cos 30.0°)$$
$$= 43.3\text{ N horizontal to the right}$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotense}}$$
$$\text{opp} = \text{hyp}\sin\theta$$
$$F_v = (50.0)(\sin 30.0°)$$
$$= 25.0\text{ N vertically upwards}$$

Worked example 2.4.1

CALCULATING THE PERPENDICULAR COMPONENTS OF A FORCE

Use the direction conventions to determine the perpendicular components of a 235 N force acting on a bike at a direction of 17.0° north of west.	
Thinking	**Working**
Draw F_W from the tail of the 235 N force along the horizontal direction, then draw F_N from the horizontal vector to the head of the 235 N force.	
Calculate the west component of the force F_W using $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ adj = hyp $\cos\theta$ $F_W = (235)(\cos 17.0°)$ = 224.7 N west
Calculate the north component of the force F_N using $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ opp = hyp $\sin\theta$ $F_N = (235)(\sin 17.0°)$ = 68.7 N north

Worked example: Try yourself 2.4.1

CALCULATING THE PERPENDICULAR COMPONENTS OF A FORCE

Use the direction conventions to determine the perpendicular components of a 3540 N force acting on a trolley at a direction of 26.5° down from horizontal to the left.

2.4 Review

SUMMARY

- A vector can be resolved into two perpendicular component vectors.
- Perpendicular component vectors are at right angles to each other.
- Any component vectors must be smaller in magnitude than the original vector.
- The hypotenuse of a right-angled triangle is the longest side of the triangle and the other two sides are each smaller than the hypotenuse.
- A right-angled triangle vector diagram can be drawn with the original vector as the hypotenuse and the perpendicular components drawn from the tail of the original to the head of the original.
- The perpendicular components can be determined using trigonometry.

KEY QUESTIONS

1 Rayko applies a force of 462 N on the handle of a mower in a direction of 35.0° down from horizontal to the right.

a What is the downwards force applied?

b What is the horizontal right force applied?

2 A force of 25.9 N acts in the direction of S 40.0° E. Find the perpendicular components of the force.

3 A ferry is transporting students to Rottnest Island. At one point in the journey the ferry travels at 18.3 $m s^{-1}$ N 75.6° W. Calculate its velocity in the northerly direction and in the westerly direction at that time.

4 Zehn walks 47.0 m in the direction of S 66.3° E across a hockey field. Calculate the change in Zehn's position down the field and across the field during that time.

5 A cargo ship has two tugs attached to it by ropes. One of the tugs is pulling directly north, while the other tug is pulling directly west. The pulling forces of the tugboats combine to produce a total force of 235 000 N in a direction of N 62.5° W. Calculate the force that each tug boat applies to the cargo ship.

6 Resolve the following forces into their perpendicular components around the north–south line. In part d, use the horizontal and vertical directions.

a 108 N S 60.0° E

b 60.0 N north

c 312 N 165°T

d 3.00×10^5 N 30.0° upwards from horizontal to the right.

7 What are the horizontal and vertical components of a 348 N force that is applied along a rope used to drag an object across a yard at 60.0° up from horizontal to the left?

8 A ball is hit from a racquet with a velocity of 30.0 $m s^{-1}$ at 50.0° up from horizontal to the right. Calculate the horizontal and vertical components of the velocity.

9 A person walks 445 m in a south-east direction. How far south have they travelled in this time?

10 A sprinter starts a race by pushing against the starting block with a force of 486 N. If the block is positioned at 70.0° up from horizontal to the left, what horizontal force does the sprinter apply to the block?

Chapter review

02

KEY TERMS

collinear
component
dimension
direction conventions
magnitude
resultant
scalar
units
vector
vector diagram

1 Select the scalar quantities in the list below. (There may be more than one correct answer.)
A force
B time
C acceleration
D mass

2 Select the vector quantities in the list below. (There may be more than one correct answer.)
A displacement
B distance
C volume
D velocity

3 A basketballer applies a force with their hand to bounce the ball. Describe how a vector can be drawn to represent this situation.

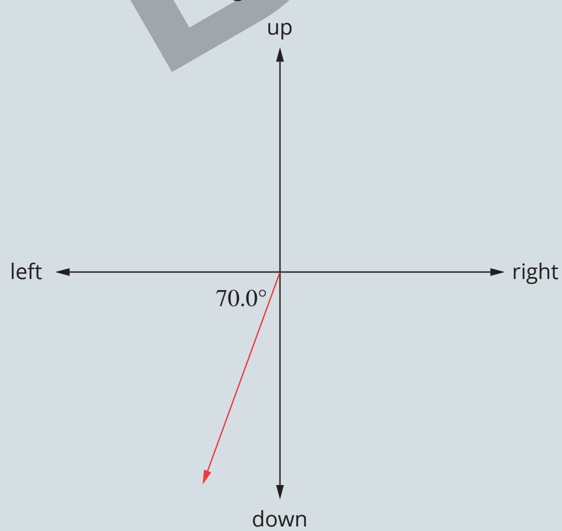
4 Vector arrow A is drawn twice the length of vector arrow B. What does this mean?

5 A car travels 15.0 m s^{-1} north and another travels 20.0 m s^{-1} south. Why is a sign convention often used to describe vectors like these?

6 When finding the change in velocity between an initial velocity of 34.0 m s^{-1} south and a final velocity of 12.5 m s^{-1} east, which two vectors need to be added together?

7 If the vector 20.0 N forwards is written as –20.0 N, how would you write a vector representing 80.0 N backwards?

8 Describe the following vector direction.



9 Add the following force vectors using a number line: 3.00 N left, 2.00 N right, 6.00 N right. Then draw and describe the resultant force vector.

10 Determine the resultant vector of the following combination: 45.0 m forwards, 70.5 m backwards, 34.5 m forwards, 30.0 m backwards.

11 Find the vector which results from the addition of 36.0 m south and 55.0 m west.

12 Add the following vectors: 481 N north and 655 N east. Give answers to three significant figures.

13 Determine the change in velocity of a bird that changes from flying 3.00 m s^{-1} to the right to flying 3.00 m s^{-1} to the left.

14 A car makes a turn, changing its velocity from 13.0 m s^{-1} south to 18.7 m s^{-1} west. Calculate the change in the velocity vector, Δv, of the car, to three significant figures.

15 Nina hits a cricket ball so that it changes its velocity from 38.8 m s^{-1} east to 55.5 m s^{-1} north. Calculate the change in the velocity vector, to three significant figures.

16 A force of 45.5 N acts in the direction of S 60.0° E. Find the eastern and southern components of this force. Give your answers to three significant figures.

17 Calculate the vertical velocity of a cannonball which is shot at 50.0° up from the horizontal ground to the right at a speed of 422 m s^{-1}.

18 Findlay pulls a heavy load with a force of 212 N at 60.0° up from horizontal to the right and Dougie pulls twice as hard with a different rope at 50.0° up from horizontal to the right. Determine the total horizontal force which is pulling the load along.

19 Lin shoots a basket 5.00 m away. It just manages to go down into the basket at 20.0° up from vertical to the right, with a vertical velocity component of 3.00 m s^{-1} down. Calculate its actual speed when it goes through the loop.

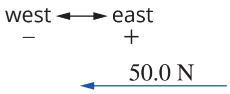
20 Aidan does a ski jump off a ramp and lands with a speed of 10.0 m s^{-1} at 45.0° down from horizontal to the right. Calculate their vertical and horizontal velocities when they land.

Chapter 2 Scalars and vectors

Section 2.1 Scalars and vectors

Worked example: Try yourself 2.1.1

DESCRIBING VECTORS IN ONE DIMENSION



Describe the vector using:

a the direction convention shown

Thinking	Working
Identify the direction convention being used in the vector.	In this case the vector is pointing to the west according to the direction convention.
Note the magnitude, unit and direction of the vector.	In this example the vector is 50.0 N west.

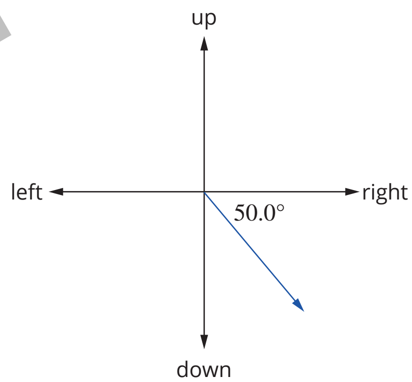
b an appropriate sign convention.

Thinking	Working
Convert the physical direction to the corresponding mathematical sign.	The direction west is negative.
Represent the vector with a mathematical sign, magnitude and unit.	This vector is –50.0 N.

Worked example: Try yourself 2.1.2

DESCRIBING TWO-DIMENSIONAL VECTORS

Describe the direction of the following vector using an appropriate method.



Thinking	Working
Choose the appropriate points to reference the direction of the vector. In this case using the horizontal reference makes more sense, as the angle is given from the horizontal.	The vector can be referenced to the horizontal.
Determine the angle between the reference direction and the vector.	From the right direction to the vector there is an angle of 50.0°.
Determine the direction of the vector from the reference direction.	From the right direction, the vector is down.
Describe the vector using the sequence: angle, up or down from the reference direction.	This vector is 50.0° down from horizontal to the right.

Section 2.1 Review

KEY QUESTIONS SOLUTIONS

1 Scalar measures require a magnitude (size) and units.

2 Vectors require a magnitude, units and a direction.

3

Scalar	Vector
time	force
distance	acceleration
volume	position
speed	displacement
temperature	momentum
	velocity

4 If the shortest arrow is 2.70 N, the middle length arrow is twice the length of the shortest (5.40 N) and the longest is three times the shortest (8.10 N). The 9.00 N magnitude is not required.

5 If the shortest arrow is –5.40 N, the middle length arrow is twice the length of the shortest (10.80 N) and the longest is three times the shortest (16.20 N). The –2.70 N magnitude is not required.

6 **a** down
b south
c forwards
d up
e east
f positive

7 Terms like north and left cannot be used in a calculation. + and – can be entered into calculators to do calculations with vectors.

8 The vector diagram shows –35.0 N.

9 **a** **i** 225.0° T
ii S 45.0° W
b **i** 120.0° T
ii S 60.0° E

10 40.0° up from horizontal to the left

Section 2.2 Adding vectors in one and two dimensions

Worked example: Try yourself 2.2.1

ADDING VECTORS IN ONE DIMENSION USING ALGEBRA

Use the sign and direction conventions shown in Figure 2.2.2 to determine the resultant force on a box that has the following forces acting on it: 16.0 N up, 22.0 N down, 4.0 N up and 17.0 N down.	
Thinking	**Working**
Apply the sign and direction conventions to change the directions to signs.	16.0 N up = +16.0 N 22.0 N down = –22.0 N 4.0 N up = +4.0 N 17.0 N down = –17.0 N
Add the magnitudes and their signs together.	resultant force = (+16.0) + (–22.0) + (+4.0) + (–17.0) = –19.0 N
Refer to the sign and direction convention to determine the direction of the resultant force vector.	Negative is down. ∴ resultant force = 19.0 N down

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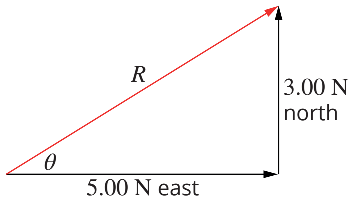
Worked example: Try yourself 2.2.2

ADDING VECTORS IN TWO DIMENSIONS USING GEOMETRY

Determine the resultant force when forces of 5.00 N east and 3.00 N north both act on a tree at the same time. Refer to Figure 2.2.2 on page xxx for sign and direction conventions if required.

Thinking	Working
Construct a vector diagram showing the vectors drawn head to tail. Draw the resultant vector from the tail of the first vector to the head of the last vector.	
As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the resultant vector.	$R^2 = 5.00^2 + 3.00^2$ $= 25.0 + 9.00$ $R = \sqrt{34.0}$ $= 5.83$ N
Using trigonometry, calculate the angle from the east vector to the resultant vector.	$\tan\theta = \frac{3.00}{5.00}$ $\theta = \tan^{-1} 0.600$ $= 31.0°$
Determine the direction of the vector relative to north or south.	$90.0° - 31.0° = 59.0°$ The direction is N 59.0° E.
State the magnitude and direction of the resultant vector.	$R = 5.83$ N, N 59.0° E

Section 2.2 Review

KEY QUESTIONS SOLUTIONS

1 **a** total weekly distance = 5 × 2 × 3.00 = 30.0 km
  b Since he returns home each day, his displacement is zero each day and each week.

2 Using sign conventions, resultant = (+3.00) + (–2.00) + (–3.00) = –2.00 The resultant vector is 2.00 m down.

3 Using sign conventions, resultant = (+23.0) + (–16.0) + (+7.0) + (–3.0) = +11.0. The resultant vector is 11.0 m forwards.

4 D. Adding vector B to vector A is equivalent to saying A + B. Therefore, draw vector A first, then draw vector B with its tail at the head of A. The resultant is drawn from the tail of the first vector (A) to the head of the last vector (B).

5 $R^2 = 40.0^2 + 20.0^2$

$= 1600 + 400$

$R = \sqrt{2000}$

$= 44.7$ m

$\tan\theta = \frac{40.0}{20.0}$

$\theta = \tan^{-1}$ r(2.00)

$= 63.4°$

$R = 44.7$ m, S 63.4° W

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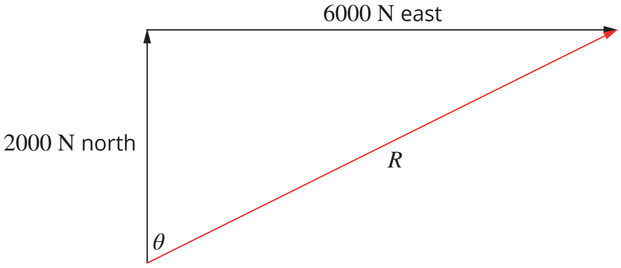
6



$R^2 = 2000^2 + 6000^2$

$= 4\,000\,000 + 36\,000\,000$

$R = \sqrt{40000000}$

$= 6325\text{ N}$

$\tan\theta = \frac{6000}{2000}$

$\theta = \tan^{-1} 3.00$

$= 71.6°$

$R = 6325\text{ N, N } 71.6°\text{ E}$

7 $R^2 = 40.0^2 + 30.0^2$

$= 1600 + 900$

$R = \sqrt{2500}$

$= 50.0\text{ m}$

8 First add 3000 N north and 5000 N south.

Resultant force is 2000 N south.

Then add 2000 N south to 5000 N east:

$F^2 = 2000^2 + 5000^2$

$= 29\,000\,000$

$F = 5385\text{ N}$

$\theta = \tan^{-1}\frac{5000}{2000} = 68.2°$

resultant force = 5385 N S 68.2° E

9 total forwards force = 3350 + 2235 + 634 = 6219 N

Apply a sign convention: forwards = +6219 N; backward = –6220 N

Add vectors = (+6219) + (–6220) = –1 N

resultant force = 1 N backwards

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Section 2.3 Subtracting vectors in on and two dimensions

Worked example: Try yourself 2.3.1

SUBTRACTING VECTORS IN ONE DIMENSION USING ALGEBRA

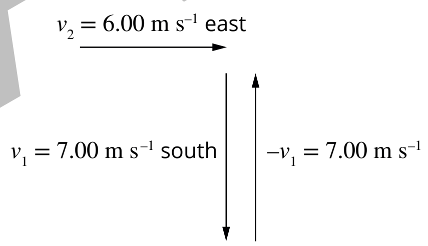
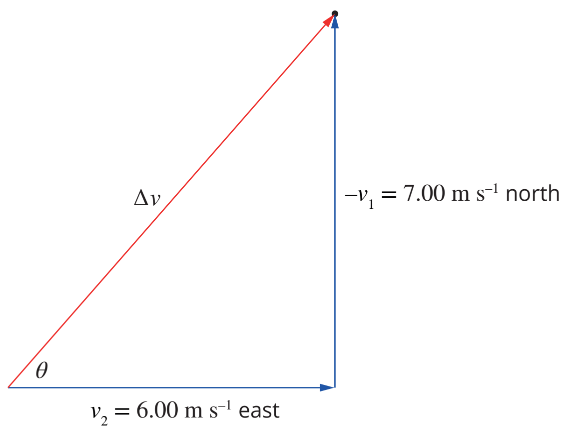
Use the sign and direction conventions shown in Figure 2.3.5 to determine the change In velocity of a rocket as it changes from 212 m s^{-1} up to 2200 m s^{-1} up.

Thinking	Working
Apply the sign and direction conventions to change the directions to signs.	v_1 = 212 m s^{-1} up = +212 m s^{-1} v_2 = 2200 m s^{-1} up = +2200 m s^{-1}
Reverse the direction of the initial velocity, v_1, by reversing the sign.	$-v_1$ = 212 m s^{-1} down = −212 m s^{-1}
Use the formula for change in velocity to calculate the magnitude and the sign of Δv.	$\Delta v = v_2 + (-v_1)$ = (+2200) + (−212) = +1988 m s^{-1}
Refer to the sign and direction convention to determine the direction of the change in velocity.	Positive is up. ∴ Δv = 1988 m s^{-1} up.

Worked example: Try yourself 2.3.2

SUBTRACTING VECTORS IN TWO DIMENSIONS USING GEOMETRY

Determine the change in velocity of a ball as it bounces off a wall. The ball approaches at 7.00 m s^{-1} south and rebounds at 6.00 m s^{-1} east.

Thinking	Working
Draw the final velocity vector, v_2, and the initial velocity vector, v_1, separately. Then draw the initial velocity in the opposite direction.	
Construct a vector diagram, drawing v_2 first, and then from its head draw the opposite of v_1. The change in velocity vector is drawn from the tail of the final velocity to the head of the opposite of the initial velocity.	
As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the change in velocity.	$R^2 = 7.00^2 + 6.00^2$ = 49.0 + 36.0 $R = \sqrt{85.0}$ = 9.22 m s^{-1}

Calculate the angle from the north vector to the change in velocity vector.	$\tan\theta = \dfrac{7.00}{6.00}$ $\theta = \tan^{-1} 1.17$ $= 49.40°$ Direction from north vector is 90.00 – 49.40 = 40.60°
State the magnitude and direction of the change in velocity.	$\Delta v = 9.22\,\text{m s}^{-1}$ N 40.6° E

Section 2.3 **Review**

KEY QUESTIONS SOLUTIONS

1 Change in velocity = final velocity – initial velocity
$= (5.00) + (+3.00)$
$= 8.00\,\text{m s}^{-1}$ east

2 Change in velocity = final velocity – initial velocity
$= (2.00) + (-4.00)$
$= 2.00\,\text{m s}^{-1}$ left

3 Change in velocity = final velocity – initial velocity
$= (-3.00) + (-4.00)$
$= 7.00\,\text{m s}^{-1}$ downwards

4 Change in velocity = final velocity – initial velocity
$= (-32.5) + (-35.0)$
$= 67.5\,\text{m s}^{-1}$ south

5 Change in velocity = final velocity – initial velocity
$= (8.20) + (-22.2)$
$= 14.0\,\text{m s}^{-1}$ backwards

6 $\Delta v^2 = (v_2)^2 + (-v_1)^2$
$= (406)^2 + (345)^2$
$\Delta v = \sqrt{1648.36 + 1190.25}$
$= \sqrt{2838.61}$
$= 533\,\text{m s}^{-1}$
$\tan\theta = \dfrac{345}{406}$
$\theta = \tan^{-1}\dfrac{345}{406}$
$= 40.4°$
Angle measured from the north = 90.0° – 40.4° = 49.6°
$\Delta v = 533\,\text{m s}^{-1}$ N 49.6° W

7 $\Delta v^2 = (v_2)^2 + (-v_1)^2$
$= (42.0)^2 + (42.0)^2$
$\Delta v = \sqrt{1764 + 1764}$
$= \sqrt{3528}$
$= 59.4\,\text{m s}^{-1}$
$\tan\theta = \dfrac{42.0}{42.0}$
$\theta = \tan^{-1}(1.000)$
$= 45.0°$
$\Delta v = 59.4\,\text{m s}^{-1}$ N 45.0° W

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8 $\Delta v^2 = (v_2)^2 + (-v_1)^2$

$= (5.25)^2 + (7.05)^2$

$\Delta v = \sqrt{27.56 + 49.70}$

$= \sqrt{77.27}$

$= 8.79\ \text{m s}^{-1}$

$\tan\theta = \frac{7.05}{5.25}$

$\theta = \tan^{-1}\frac{7.05}{5.25}$

$= 53.3°$

Angle measured from the north = 90° – 53.3° = 36.7°

$\Delta v = 8.79\ \text{m s}^{-1}$ N 36.7° W

9 **a** (40.0) – (25.0) = 15.0 km h^{-1}

b (25.0) – (– 40.0) = (25.0) + (40.0) = 65.0 km h^{-1} i.e. 65.0 km h^{-1} south

10 $\Delta v^2 = (v_2)^2 + (-v_1)^2$

$= (30.0)^2 + (30.0)^2$

$\Delta v = \sqrt{900 + 900}$

$= \sqrt{1800}$

$= 42.4\ \text{kmh}^{-1}$

$\tan\theta = \frac{30}{30}$

$\theta = \tan^{-1}$

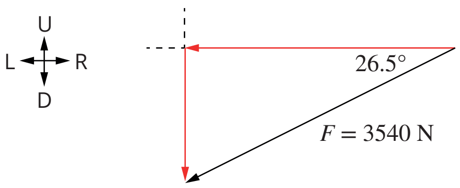
$= 45.0°$

$\Delta v = 42.4\ \text{kmh}^{-1}$ N 45.0° W

Section 2.4 Vector components

Worked example: Try yourself 2.4.1

CALCULATING THE PERPENDICULAR COMPONENTS OF A FORCE

Use the direction conventions to determine the perpendicular components of a 3540 N force acting on a trolley at a direction of 26.5° down from horizontal to the left.	
Thinking	**Working**
Draw F_L from the tail of the 3540 N force along the horizontal, then draw F_D from the horizontal line to the head of the 3540 N force.	
Calculate the left component of the force F_L using $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ adj = hyp cos θ $F_L = (3540)(\cos 26.5°)$ = 3168 N left
Calculate the downwards component of the force F_D using $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ opp = hyp sin θ $F_D = (3540)(\sin 26.5°)$ = 1580 N downwards

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Section 2.4 Review

KEY QUESTIONS SOLUTIONS

1 a $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\text{opp} = \text{hyp} \sin\theta$

$F_D = (462)(\sin 35.0°)$

$= 265\,\text{N downwards}$

b $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{adj} = \text{hyp} \cos\theta$

$F_R = (462)(\cos 35.0°)$

$= 378\,\text{N right}$

2 $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{adj} = \text{hyp} \cos\theta$

$F_S = (25.9)(\cos 40.0°)$

$= 19.8\,\text{N south}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\text{opp} = \text{hyp} \sin\theta$

$F_E = (25.9)(\sin 40.0°)$

$= 16.6\,\text{N east}$

Therefore, 19.8 N south and 16.6 N east

3 $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{adj} = \text{hyp} \cos\theta$

$v_N = (18.3)(\cos 75.6°)$

$= 4.55\,\text{m s}^{-1}\text{ north}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\text{opp} = \text{hyp} \sin\theta$

$v_W = (18.3)(\sin 75.6°)$

$= 17.7\,\text{m s}^{-1}\text{ west}$

Therefore, 4.55 ms^{-1} north and 17.7 ms^{-1} west

4 $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{adj} = \text{hyp} \cos\theta$

$s_S = (47.0)(\cos 66.3°)$

$= 18.9\,\text{m south}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\text{opp} = \text{hyp} \sin\theta$

$s_S = (47.0)(\sin 66.3°)$

$= 43.0\,\text{m east}$

Therefore, Zehn is 18.9 m south and 43 m east of his starting point.

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5 $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{adj} = \text{hyp}\cos\theta$

$F_N = (235\,000)(\cos 62.5°)$

$= 109\,000$ N north

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\text{opp} = \text{hyp}\sin\theta$

$F_W = (235\,000)(\sin 62.5°)$

$= 208\,000$ N west

6 **a** $F_S = (108)(\cos 60.0°) = 54.0$ N south

$F_E = (108)(\sin 60.0°) = 93.5$ N east

b $F_N = 60.0$ N north

c $F_S = (312)(\cos 20.0°) = 293$ N south

$F_E = (312)(\sin 20.0°) = 107$ N east

d $F_v = (3.00 \times 10^5)(\sin 30.0°) = 1.50 \times 10^5$ N up

$F_h = (3.00 \times 10^5)(\cos 30.0°) = 2.60 \times 10^5$ N horizontal

7 horizontal component $F_h = (348)(\cos 60.0°) = 174$ N

vertical component $F_v = (348)(\sin 600°) = 301$ N

8 vertical = $(30.0)(\sin 50.0°) = 23.0\,\text{m}\,\text{s}^{-1}$

horizontal = $(30.0)(\cos 50.0°) = 19.3\,\text{m}\,\text{s}^{-1}$

9 distance south = $(445)(\sin 450°) = 315$ m

10 horizontal component of force = $(486)(\cos 70.0°) = 166$ N

CHAPTER 2 REVIEW

1 B and D are both scalars. These do not require a magnitude and direction to be fully described.

2 A and D are vectors. These require a magnitude and direction to be fully described.

3 The vector must be drawn as an arrow with its tail at the point of contact between the hand and the ball. The arrow points in the direction of the 'push' of the hand.

4 Vector A is drawn twice the length of vector B, so it has twice the magnitude of B.

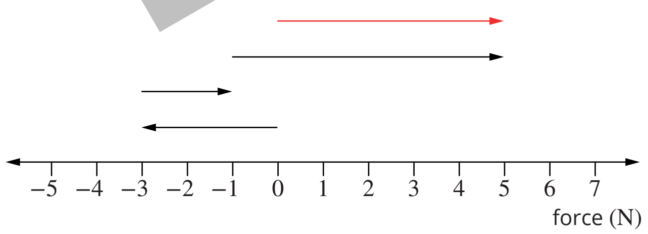
5 Signs are useful in mathematical calculations, as the words north and south cannot be used with a calculator.

6 $34.0\,\text{m}\,\text{s}^{-1}$ north and $12.5\,\text{m}\,\text{s}^{-1}$ east need to be added together. This is because the change in velocity is the final velocity plus the opposite of the initial velocity. The opposite of $34.0\,\text{m}\,\text{s}^{-1}$ south is $34.0\,\text{m}\,\text{s}^{-1}$ north.

7 + 80.0 N or just 80.0 N

8 70.0° down from horizontal to the left or 20.0° up from vertical to the left

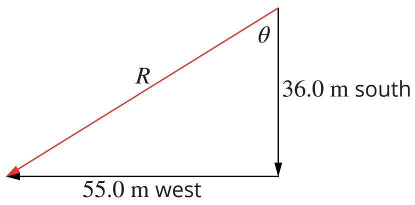
9



The resultant vector is 5.00 N right.

10 The vectors are (+45.0) + (–70.5) + (+34.5) + (–30.0). This equals –21.0. Backwards is negative, therefore the answer is 21.0 m backwards.

11



$R^2 = 36.0^2 + 55.0^2$

$= 1296 + 3025$

$R = \sqrt{4321}$

$= 65.7\,\text{m}$

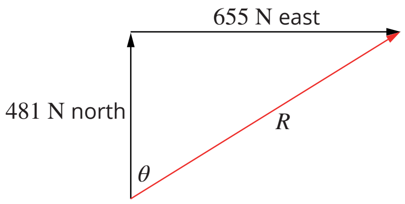
$\tan\theta = \dfrac{55.0}{35.0}$

$\theta = \tan^{-1} 1.5278$

$= 56.8°$

Therefore, the addition of 36.0 m south and 55.0 m west gives a resultant vector to three significant figures of 65.7 m S 56.8° W.

12



$R^2 = 481^2 + 655^2$

$= 231\,361 + 429\,025$

$R = \sqrt{660386}$

$= 813\,\text{N}$

$\tan\theta = 655 \div 481$

$\theta = \tan^{-1} 1.3617$

$= 53.7°$

Therefore, the resultant vector is $R = 813\,\text{N}$, N 53.7° E.

13 Taking right as positive:

$\Delta v = v - u$

$= (-3.00) + (-3.00)$

$= -6.00$

$= 6.00\,\text{m}\,\text{s}^{-1}$ left

14 $\Delta v^2 = (v_2)^2 + (-v_2)^2$

$= 18.7^2 + 13.0^2$

$\Delta v = \sqrt{349.69 + 169}$

$= \sqrt{518.69}$

$= 22.8\,\text{m}\,\text{s}^{-1}$

$\tan\theta = \dfrac{18.7}{13.0}$

$\theta = \tan^{-1} 1.4385$

$= 55.2°$

$\Delta v = 22.8\,\text{m}\,\text{s}^{-1}$ N 55.2° W

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15 $\Delta v^2 = (v_2)^2 + (-v_2)^2$

$= 55.5^2 + 38.8^2$

$\Delta v = \sqrt{3080.25 + 1505.4416}$

$= \sqrt{4585.69}$

$= 67.7\,\text{m}\,\text{s}^{-1}$

$\tan\theta = \frac{38.8}{55.5}$

$\theta = \tan^{-1} 0.6991$

$= 35.0°$

$\Delta v = 67.7\,\text{m}\,\text{s}^{-1}$ N 35.0° W

16 $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

opp = hyp × sin θ

$F_E = (45.5)(\sin 60.0°)$

$= 39.4$ N east

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

adj = hyp × cos θ

$F_S = (45.5)(\cos 60.0°)$

$= 22.8$ N south

17 $u = 422\,\text{m}\,\text{s}^{-1}$

$\theta = 50.0°$

$u_v = u \sin\theta$

$= (422)(\sin 50.0°)$

$= 323\,\text{m}\,\text{s}^{-1}$

18 Findlay horizontal = (212)(cos 60.0°) = 106 N

Dougie horizontal = (424)(cos 50.0°) = 273 N

Total horizontal force = 379 N to the right

19 Note: 5.0 m distance is not needed.

Vertical component of velocity = v cos 20.0° = 3.00

$v = 3.19\,\text{m}\,\text{s}^{-1}$

20 $v = 10.0\,\text{m}\,\text{s}^{-1}$

$\theta = 45.0°$

$v_v = v \sin\theta = (10.0)(\sin 45.0°) = 7.07\,\text{m}\,\text{s}^{-1}$ down

$v_h = v \cos\theta = (10.0)(\cos 45.0°) = 7.07\,\text{m}\,\text{s}^{-1}$ to the right

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