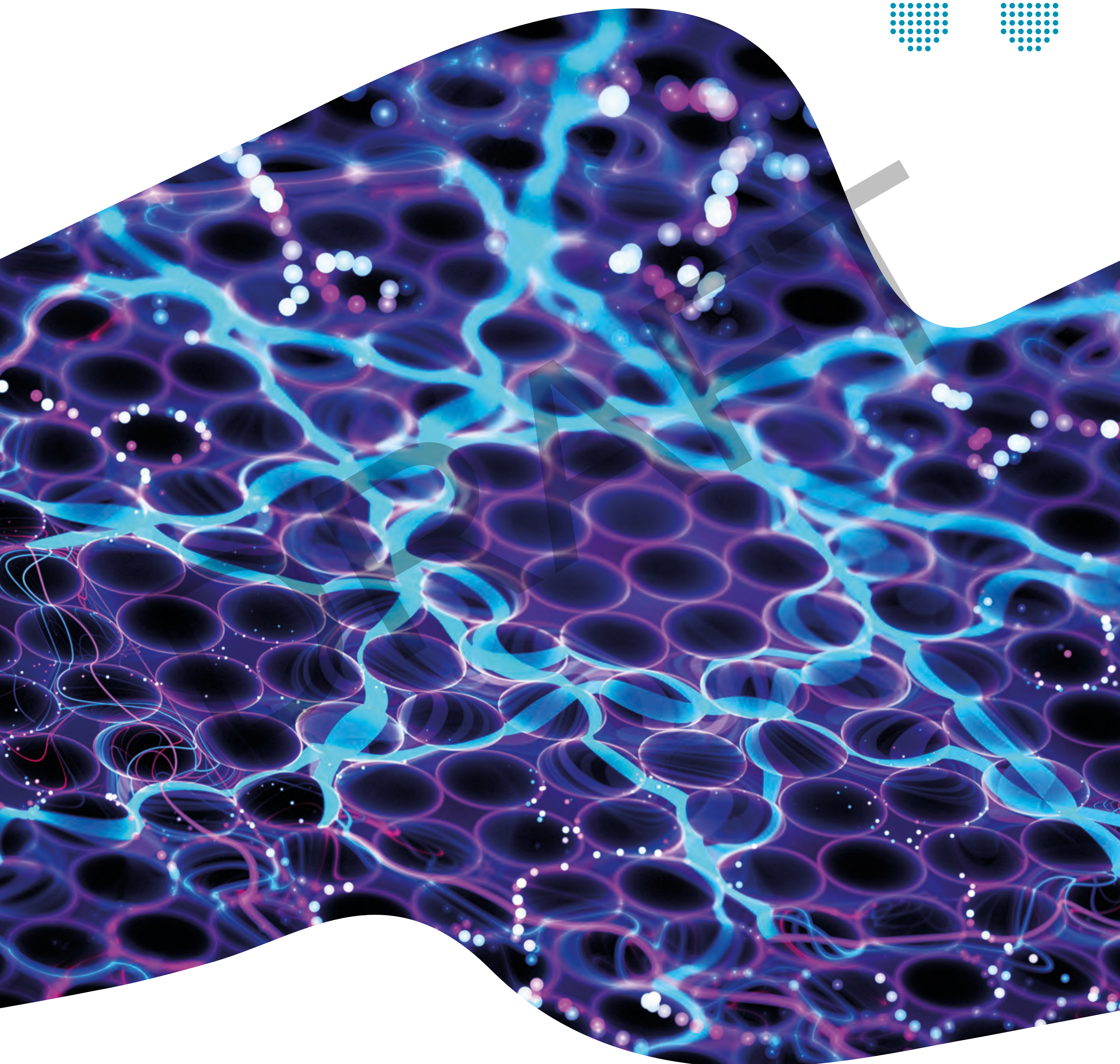
PEARSON

PHYSICS

WESTERN AUSTRALIA

11



2ND EDITION

CHAPTER 03

Linear motion

Motion, from the simple to the complex, is a fundamental part of everyday life. The motion of a gymnast performing a floor routine is a complex form of motion. An Olympic snowboarder competing in a half-pipe event also exhibits a complex form of motion. Simpler examples include a skier travelling in a straight line down a ski run, a train pulling into a station and a swimmer completing a lap of a pool.

Science Understanding

Linear motion and force

- change (final – initial) in a variable is represented by the symbol Δ, e.g. $\Delta t = t_f - t_i$
- displacement is defined as the change in position of an object, including applying the relationship

$$s = \Delta x = x_f - x_i$$

- velocity is defined as the rate of change in displacement of an object, including applying the relationship

$$v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

- uniformly accelerated motion (a is constant) is described in terms of relationships between measurable scalar and vector quantities, such as displacement, velocity and time, including applying the relationships

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad v_f = v_i + a\Delta t \qquad s = v_i\Delta t + \frac{1}{2}a\Delta t^2 \qquad v_f^{\,2} = v_i^{\,2} + 2as$$

- motion can be represented graphically to describe linear motion, including the determination, manipulation, and use of gradients of curves and areas under graphs of
  - displacement–time
  - velocity–time
  - acceleration–time
- vertical motion is analysed by assuming the acceleration due to gravity is constant near Earth's surface.

3.1 Displacement, speed and velocity

In order to analyse and communicate ideas about motion, it is important to understand the terms used to describe motion, even in its simplest form. In this section you will learn about some of the terms used to describe straight-line motion, such as position, distance, displacement, speed and velocity.

CENTRE OF MASS

When analysing motion, things are often more complicated than they first appear. As a freestyle swimmer travels at a constant speed of $2.00\,\mathrm{m\,s^{-1}}$, their head and torso move forwards at this speed. The motion of their arms, however, is more complex. At times their arms move forwards through the air faster than $2.00\,\mathrm{m\,s^{-1}}$, and at other times they are actually moving backwards through the water.

It is beyond the scope of this course to analyse such a complex motion. However, the motion of the swimmer can be simplified by treating the swimmer as a simple object located at a single point called the **centre of mass** or centre of gravity. The centre of mass is the balance point of an object. For a person, the centre of mass is located near the waist. The centres of mass of some everyday objects are shown in Figure 3.1.1. The concept of centre of mass and centre of gravity is discussed in more detail in Year 12.

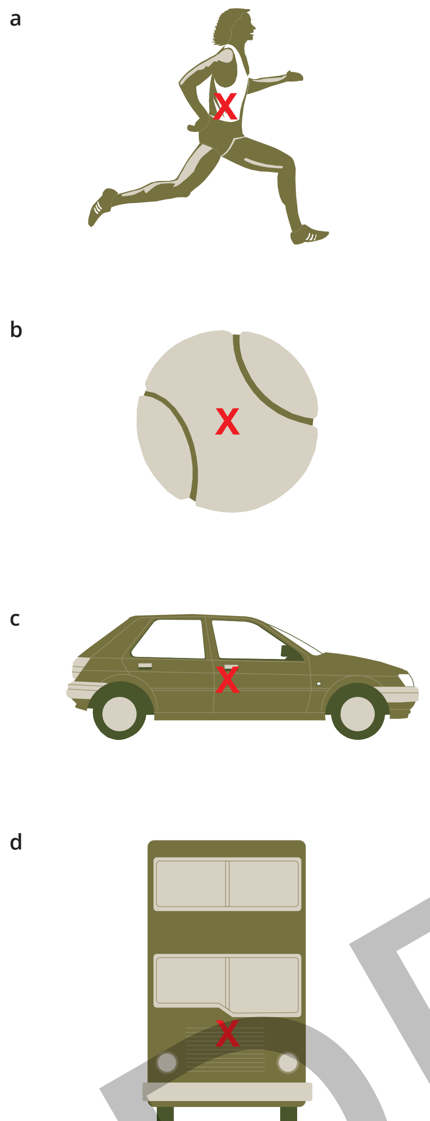


FIGURE 3.1.1 The centre of mass of each object is indicated by a cross.

POSITION, DISTANCE AND DISPLACEMENT

Position, distance and displacement are three essential concepts for analysing straight-line motion and understanding how objects move.

Position

One important term to understand when analysing straight-line motion is **position,** which is given the symbol x.

- Position, x, describes the location of an object at a certain point in time with respect to the origin.
- Position is a vector quantity and therefore requires a direction.
- Change in position, Δx, is the difference between the final position and the initial position, $x_f - x_i$.

Consider a swimmer, Sophie, doing laps in a 50.0 m pool, as shown in Figure 3.1.2. To simplify her motion, Sophie is treated as a simple point object. The pool can be treated as a one-dimensional number line, with the starting block as the origin. The direction to the right of the starting block is taken to be positive.

Sophie's position as she is warming up behind the starting block in Figure 3.1.2a is $x = -10.0\,\mathrm{m}$. The negative sign indicates the direction from the origin; that is, to the left. Her position could also be given as $x = 10.0\,\mathrm{m}$ to the left of the starting block.

At the starting block (Figure 3.1.2b), Sophie's position is $x = 0.0\,\mathrm{m}$. After swimming half the length of the pool, her centre of mass is at a position where $x = +25.0\,\mathrm{m}$ or 25.0 m to the right of the origin (Figure 3.1.2c).

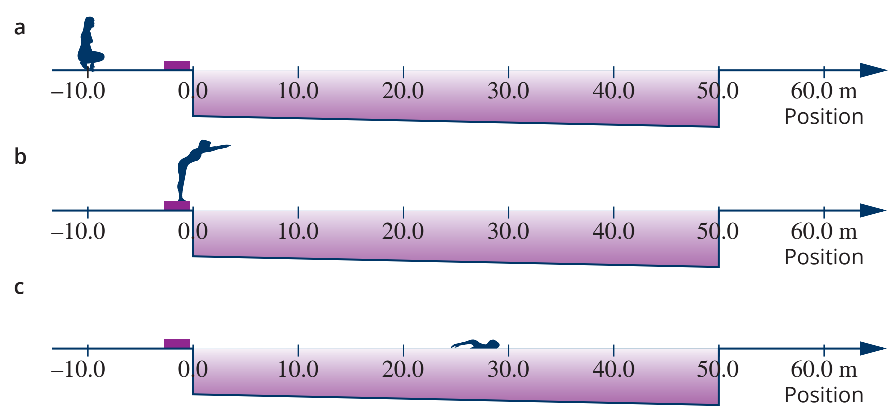


FIGURE 3.1.2 The position, x, of the swimmer is given with reference to the starting block. (a) While warming up, Sophie is at $x = -10.0\,\mathrm{m}$. (b) When she is on the starting block, her position is at $x = 0.0\,\mathrm{m}$. (c) After swimming for a short time, her centre of mass is at a position where $x = +25.0\,\mathrm{m}$.

Distance travelled

Position, x, describes where an object is at a certain point in time. However, **distance travelled**, d, is how far a body travels during a journey.

- Distance travelled, d, describes the length of the path covered during an object's entire journey.
- Distance travelled is a scalar quantity and is measured in metres (m).

For example, if Sophie completes three lengths of the pool, the distance, d, travelled during her swim will be 50.0 + 50.0 + 50.0 = 150.0 m.

The distance travelled is not affected by the direction of the motion; that is, the distance travelled by an object always increases as it moves, regardless of its direction. The tripmeter or odometer of a car or bike, for example, measures distance travelled. To better understand this, consider tying one end of a spool of string to your letterbox and letting it unravel behind you as you walk to school. The length of the unwound string would represent the distance you have travelled.

Displacement

Displacement, s, is defined as the *change in position,* Δx, of an object. Displacement considers only the initial or starting position, x_i, and the final or finishing position, x_f; the route taken between these two points has no effect on displacement. The sign of the displacement indicates the direction in which the position has changed from the start to the end.

- Change (final – initial) in a variable is represented by the symbol Δ.
- Displacement, s, is the change in position of an object in a given direction, $s = \Delta x$.
- Displacement = final position – initial position or $s = \Delta x = x_f - x_i$.
- Displacement is a vector quantity and is measured in metres (m).

Consider the example of Sophie completing one length of the pool. During her swim, the distance travelled is 50.0 m. Her final position, x_f, is +50.0 m and her initial position, x_f, is 0.0 m. Her displacement can therefore be shown as:

s = final position – initial position

$s = x_f - x_i$

$s = (+50.0) - (0.0)$ $s = +50.0$ m or 50.0 m in a positive direction.

Notice that **magnitude**, units and direction are required for a vector quantity. The distance will be equal to the magnitude of displacement only if the body is moving in a straight line and does not change direction. If Sophie swims two lengths, her distance travelled will be 100.0 m: 50.0 m out and 50.0 m back. However, her displacement during this swim will be:

s = final position – initial position

$s = (0.0) - (0.0)$

$s = 0.0$ m

Even though Sophie has swum 100.0 m, the displacement is zero because her initial and final positions are the same.

The above formula for displacement is useful if you already know the initial and final positions of a body's motion. An alternative method to determine total displacement, if you know the displacement of each section of the motion, is to sum or add up the individual displacements for each section of motion.

total displacement = sum of individual displacements, or $s = s_1 + s_2 + s_3 + \ldots$

It is important to remember that displacement is a vector and so, when adding displacements, you must obey the rules of vector addition (discussed in Chapter 2).

In the example above, therefore, in which Sophie completed two laps of the pool, overall displacement could also have been calculated by adding the displacement of each lap:

s = sum of displacements for each lap

s = 50.0 m in the positive direction + 50.0 m in the negative direction

$s = (50.0) + (-50.0)$

$s = 0.0$ m

PHYSICS IN ACTION

Timing and false starts in athletics

Until 1964, all timing of events at the Olympic Games was recorded by handheld stopwatches (Figure 3.1.3). The reaction times of the judges meant an uncertainty of 0.2 s for any measurement. An electronic quartz timing system introduced in 1964 improved accuracy to 0.01 s, but in close finishes the judges still had to wait for a photograph of the finish before they could announce the places.

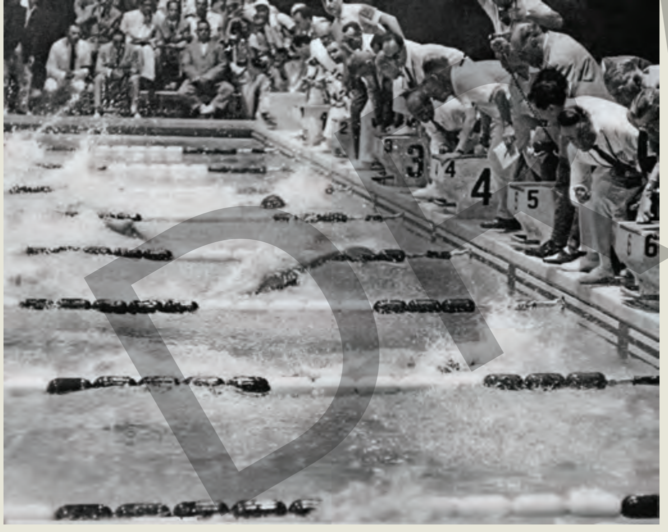


FIGURE 3.1.3 Using stopwatches to time a swimming race at the 1960 Olympic Games in Rome.

The current timing system used in athletics is a vertical line-scanning video system (VLSV). Introduced in 1991, this electronic timing system is completely automatic. The starting pistol triggers a computer to begin timing. At the finish line, a high-speed video camera records the image of each athlete and indicates the time at which each one crosses the line. This system enables the times of all athletes in the race to be precisely measured to one-thousandth of a second.

Another feature of this system is that it indicates when a runner 'breaks' at the start of the race. Each starting block is connected by electronic cables to the timing computer and a pressure sensor indicates if a runner has left the blocks early (Figure 3.1.4). A reaction time of 0.10 s has been incorporated into the system since 2002. This ensures that a runner has not anticipated the pistol. It also means that a runner can still commit a false start even if their start was after the pistol. A start that is less than 0.10 s after the pistol is registered as false.

FIGURE 3.1.4 Starting blocks are fitted with pressure sensors to detect false starts.

SPEED AND VELOCITY

For thousands of years, humans have tried to travel at ever greater speeds. This desire has contributed to the development of all sorts of competitive activities, as well as major advances in engineering and design. World records for some of these pursuits are given in Table 3.1.1.

TABLE 3.1.1 World record speeds for a variety of sports or modes of transport (as of June 2024).

Activity	World record speed ($m\,s^{-1}$)	World record speed ($km\,h^{-1}$)
street luge	45.522	163.88
maglev train	128	460
tennis serve	73.06	263.0
waterskiing (barefoot)	60.678	218.44
cricket delivery	44.81	161.3
electric formula 1	89.4	322

- Speed, v, is defined in terms of the rate, Δt, at which the distance, d, is travelled. Like distance, speed is a scalar quantity. A direction is not required when describing the speed of an object.
- Velocity, v, is defined in terms of the rate, Δt, at which displacement, s, changes and so is a vector quantity. A direction should always be given with a velocity.
- The SI unit for speed and velocity is metres per second ($m\,s^{-1}$), but kilometres per hour ($km\,h^{-1}$) is also commonly used.

Speed and **velocity** are both quantities that give an indication of how quickly the position of an object is changing. Both terms are in common use and are often assumed to have the same meaning. In physics, however, these two terms have different definitions.

Instantaneous speed and velocity

Instantaneous speed and instantaneous velocity give a measure of how fast something is moving at a particular point in time. The speedometer on a car or bike indicates instantaneous speed.

If a speeding car is travelling north and is detected on a police radar gun at $150\,km\,h^{-1}$, it indicates that this car's instantaneous speed is $150\,km\,h^{-1}$, while its instantaneous velocity is $150\,km\,h^{-1}$ north. Notice that the instantaneous speed is equal to the magnitude of the instantaneous velocity.

Average speed and velocity

Average speed and *average velocity* both give an indication of how fast an object is moving over a period of time.

average speed, $v_{av} = \dfrac{\text{distance travelled}}{\text{time taken}} = \dfrac{d}{\Delta t}$

average velocity, $v_{av} = \dfrac{\text{change in displacement}}{\text{time taken}} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t_f - t_i}$

A direction (such as north, south, up, down, left, right, positive, negative) must be given when describing a velocity. The direction will always be the same as that of the displacement.

For example, the average speed of a car that takes 30.0 minutes to travel 20.0 km from Perth to Sorrento is $40.0\,km\,h^{-1}$. However, this does not mean that the car travelled the whole distance at this speed. In fact, it is more likely that the car was moving at $60.0\,km\,h^{-1}$ for a significant amount of time, while for some of the time it may not be moving at all.

Like the relationship between distance and displacement, average speed will be equal to the magnitude of average velocity only if the body is moving in a straight line and does not change direction. In a race around a circular track like the velodrome shown in Figure 3.1.5, regardless of the average speed for one complete lap, the magnitude of the average velocity at the end of that lap will be zero because the displacement is zero.

FIGURE 3.1.5 Anna Meares won the UCI world championship in 2013. She rode 500 m in a world record time of 32.836 s. Her average speed was $55.6\,km\,h^{-1}$ but her average velocity was zero at the start–finish line.

EXTENSION

How police measure the speeds of cars

Road accidents cause the deaths of about 1200 people in Australia each year and many times this number are seriously injured. Numerous steps have been taken to reduce the number of road fatalities. Some of these include random alcohol and drug testing, speed cameras, mandatory wearing of bicycle helmets and the zero blood alcohol level for probationary drivers.

One of the main causes of road trauma is speeding. In their efforts to combat speeding motorists, police employ a variety of speed-measuring devices. One such device is shown in Figure 3.1.6.

FIGURE 3.1.6 Speed cameras on poles.

Speed camera radar

Camera radar units are usually placed in unmarked vehicles parked by the side of the road. These units emit a radar signal frequency of 24.15 GHz (2.415×10^{10} Hz). The radar antenna has a parabolic reflector that enables the unit to produce a directional radar beam that is 5° wide, allowing individual vehicles to be targeted. The radar range and field of vision for a camera is shown in Figure 3.1.7. The radar signal allows speeds to be determined by the Doppler effect, where the reflected radar signal from an approaching vehicle has a higher frequency than the original signal. Similarly, the reflected signal from a receding vehicle has a lower frequency. This change in frequency or 'Doppler shift' is processed by the unit and gives a measurement of the instantaneous speed of the target vehicle.

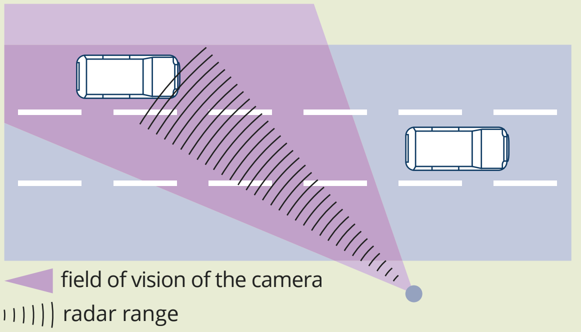


FIGURE 3.1.7 Diagram showing the visual range of a speed camera.

Camera radar units are capable of targeting a single vehicle up to 1.2 km away. In traffic, the units can distinguish between individual cars and take two photographs per second. The photographs and infringement notices are mailed to the offending motorists.

Laser speed guns

Speed guns are used by police to obtain an instant measure of the speed of an approaching or receding vehicle. The unit is usually handheld and is aimed directly at a vehicle using a target sight. It emits a pulse of infra-red radiation frequency of 331 THz (3.31×10^{14} Hz). As with camera radar units, the speed is determined by the Doppler shift produced by the target vehicle. The infra-red pulse is very narrow and directional, being just 0.17° wide. This allows vehicles to be targeted with great precision. Handheld units can be used at distances up to 800 m. If the vehicle's speed registers over the limit, police are likely to pull the driver over.

Fixed speed cameras

Fixed speed cameras obtain their readings by using a system of three strips with piezoelectric sensors in them across the road (see Figure 3.1.8). The strips respond to the pressure exerted as the car drives over them and create an electrical pulse that is detected by the unit. By knowing the precise distance between the strips and measuring the time that the car takes to travel across them, the speed of the car can be determined. This is actually measuring the average speed of the car, but by placing the strips close together the average speed gives a very good approximation of the instantaneous speed.

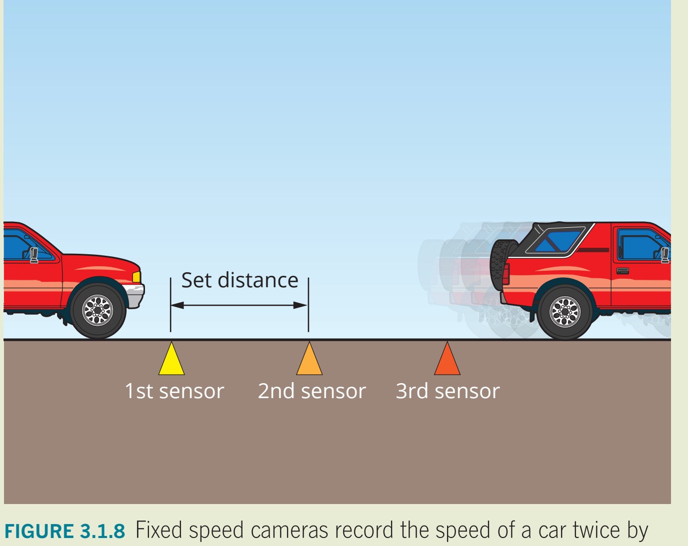


FIGURE 3.1.8 Fixed speed cameras record the speed of a car twice by measuring the time the car takes to travel over a series of three sensor strips embedded in the roadway.

Converting $km\,h^{-1}$ to $m\,s^{-1}$

You should be familiar with $100.0\,km\,h^{-1}$ as it is the speed limit for most freeways and country roads in Australia. Cars that maintain this speed would travel 100.0 km in 1.00 hour. Since there are 1000 m in 1.00 km and 3600 s in 1.00 hour (60.0 s × 60.0 min), this is the same as travelling 100 000 m in 3600 s.

$$\begin{aligned} 100.0\,km\,h^{-1} &= 100.0 \times 1000\,m\,h^{-1} \\ &= 100\,000\,m\,h^{-1} \\ &= \frac{100\,000}{3600}\,m\,s^{-1} \\ &= 27.8\,m\,s^{-1} \end{aligned}$$

So, $km\,h^{-1}$ can be converted to $m\,s^{-1}$ by multiplying by $\frac{1000}{3600}$ (or simply dividing by 3.60).

Converting $m\,s^{-1}$ to $km\,h^{-1}$

A champion Olympic sprinter can run at an average speed of close to $10.0\,m\,s^{-1}$. Each second, the athlete will travel approximately 10.0 m. At this rate, in 1.00 hour the athlete would travel 10.0 × 3600 = 36 000 m = 36.0 km.

$$\begin{aligned} 10.0\,m\,s^{-1} &= 10.0 \times 3600\,m\,h^{-1} \\ &= 36\,000\,m\,h^{-1} \\ &= \frac{36\,000}{1000}\,km\,h^{-1} \\ &= 36.0\,km\,h^{-1} \end{aligned}$$

So, $m\,s^{-1}$ can be converted to $km\,h^{-1}$ by multiplying by $\frac{3600}{1000}$ (or simply multiplying by 3.60).

When converting a speed from one unit to another, it is important to compare the speeds to ensure that your answers make sense. To do so, a good rule to remember is that the number in front of $km\,h^{-1}$ is always larger than the number in front of $m\,s^{-1}$. The diagram in Figure 3.1.9 summarises the conversion between units for speed.

PHYSICSFILE

Reaction time

Drivers are often distracted by loud music or phone calls. These distractions result in many accidents on the road. If cars are moving at high speeds, they can travel a considerable distance in the short time that the driver takes just to recognise a hazard and apply the brakes. This is known as the reaction distance, which adds to the stopping distance. Lower speeds and short reaction times are very important in helping all road users to avoid collisions. This is easy to understand given that distance is directly proportional to both speed and time, $d = v\Delta t$.

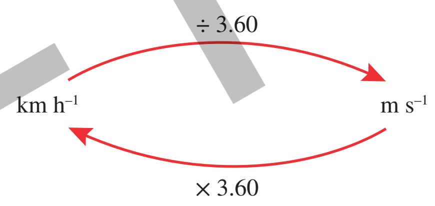


FIGURE 3.1.9 Rules for converting between $m\,s^{-1}$ and $km\,h^{-1}$.

PHYSICS IN ACTION

Alternative units for speed and distance

Metres per second is the standard unit for measuring speed as it is derived from the standard unit for distance (metres) and the standard unit for time (seconds). However, alternative units are often used that better suit certain applications.

The speed of a boat is usually measured in knots, where 1.00 knot = $0.510\,m\,s^{-1}$. This unit originated in the nineteenth century when the speed of sailing ships would be measured by allowing a rope, with knots tied at regular intervals, to be dragged by the water through a sailor's hands. By counting the number of knots that passed through the sailor's hands, and measuring the time taken for this to happen, the average speed formula could be applied to estimate the speed of the ship.

The speed of very fast jet planes, such as the one in Figure 3.1.10, can be measured in Mach numbers, which are related to the speed of sound. One Mach (referred to as Mach 1) is equal to the speed of sound, which is $340\,m\,s^{-1}$. Mach 2 is equal to $680\,m\,s^{-1}$, or twice the speed of sound.

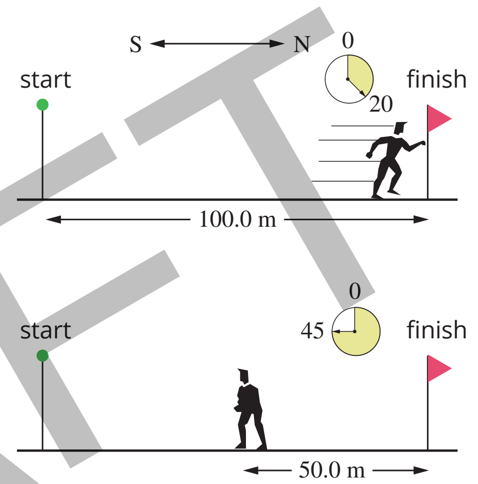
The light-year is an alternative unit for measuring distance. The speed of light in a vacuum is $3.00 \times 10^8\,m\,s^{-1}$, which is three hundred million metres every second. One light-year is the distance that light travels in 1 year, which is 3.15576×10^7 s. Because distances between objects in the universe are so large, astronomers use the light-year to measure distances in space. For example, it takes 4.24 years for light to travel 4.014×10^{16} m to us from Proxima Centauri, the nearest star to Earth other than our own star, Sol (the Sun). It is much easier to say that the distance from Earth to our nearest star is 4.24 light-years than it is to use units that work on a smaller scale. Light takes approximately 8.5 minutes to travel from the Sun to Earth, so it could be said that the Sun is 8.5 light-minutes away.

FIGURE 3.1.10 Modern fighter aeroplanes can fly at speeds above Mach 1.

Worked example 3.1.1

CALCULATING VELOCITY AND CONVERTING UNITS

Sam is an athlete performing a training routine by running back and forth along a straight stretch of running track. Sam jogs 100.0 m north in a time of 20.0 s, then turns and walks 50.0 m south in a further 25.0 s before stopping.

a Calculate Sam's velocity in $m\,s^{-1}$.	
Thinking	**Working**
Calculate the displacement. Remember that total displacement is the sum of individual displacements. Sam's total journey consists of two displacements: 100.0 m north and then 50.0 m south. Take north to be the positive direction.	s = sum of displacements s = 100.0 m north + 50.0 m south $s = (100.0) + (-50.0)$ s = +50.0 m or 50.0 m north 
Work out the total time taken for the journey.	$\Delta t = (20.0) + (25.0) = 45.0$ s
Substitute the values into the velocity equation.	Displacement, s, is 50.0 m north. Time taken, Δt, is 45.0 s. $v = \frac{s}{\Delta t}$ $v = \frac{(50.0)}{(45.0)}$ $v = 1.11111$
Velocity is a vector, so a direction must be given.	$v = 1.11\ m\,s^{-1}$ north

b Determine the magnitude of Sam's velocity in $km\,h^{-1}$.	
Thinking	**Working**
Convert from $m\,s^{-1}$ to $km\,h^{-1}$ by multiplying by 3.60.	$v = 1.11111\ m\,s^{-1}$ $v = (1.11111)(3.60)$ $v = 4.00000$
As the magnitude of the velocity is needed, the direction is not required in this answer.	magnitude of $v = 4.00\ km\,h^{-1}$

c What is Sam's speed in m s^{-1}?

Thinking	Working
Calculate the distance. Remember that distance is the length of the path covered in the entire journey. The direction does not matter. Sam travels 100.0 m in one direction and then 50.0 m in the other direction.	$d = (100.0) + (50.0)$ $d = 150.0$ m
Work out the total time taken for the journey.	$\Delta t = (20.0) + (25.0) = 45.0$ s
Substitute the values into the speed equation.	Distance, d, is 150.0 m. Time taken, Δt, is 45.0 s. $v = \frac{d}{\Delta t}$ $v = \frac{(150.0)}{(45.0)}$ $v = 3.33333$ $v = 3.33$ m s^{-1}

d What is Sam's speed in km h^{-1}?

Thinking	Working
Convert from m s^{-1} to km h^{-1} by multiplying by 3.60.	$v = 3.33333$ m s^{-1} $v = (3.33333)(3.60)$ $v = 12.0000$ $v = 12.0$ km h^{-1}

Worked example: Try yourself 3.1.1

CALCULATING VELOCITY AND CONVERTING UNITS

Sally is an athlete performing a training routine by running back and forth along a straight stretch of running track. Sally jogs 108.0 m west in a time of 20.0 s, then turns and walks 165.0 m east in a further 45.0 s before stopping.

a Calculate Sally's velocity in m s^{-1}.

b Calculate the magnitude of Sally's velocity in km h^{-1}.

c What is Sally's speed in m s^{-1}?

d What is Sally's speed in km h^{-1}?

PHYSICSFILE

Breaking the speed limit

Over the past 100 years, advances in engineering and technology have led to the development of faster machines. Cars, planes and trains can now move people at speeds that were thought to be both unattainable and life-threatening a century ago.

The 1 mile land-speed record is 1220 km h^{-1} (339 m s^{-1}). This was set in 1997 in Nevada by Andy Green driving his jet-powered *Thrust* SSC.

The fastest combat jet is the MiG-25. In 1976 it reached a speed of 3800 km h^{-1} (1056 m s^{-1}), which is more than three times the speed of sound.

In 2007, Markus Stoeckl of Austria set a new speed record for mountain biking. He reached a speed of 210 km h^{-1} racing down a ski slope in Chile, pictured in Figure 3.1.11. This record was broken by Eric Barone in 2017 with a speed of 227.720 km h^{-1}.

FIGURE 3.1.11 Markus Stoeckl set a new speed record for mountain biking in 2007.

3.1 Review

SUMMARY

- Position, x, defines the location of an object with respect to a defined origin.
- Distance travelled, d, tells us how far an object has actually travelled. Distance travelled is a scalar quantity.
- Displacement, s, is a vector quantity and is defined as the change in position of an object, Δx, in a given direction: $s = \Delta x = x_f - x_i$
- The average speed of a body, v_{av}, is defined as the rate of change of distance and is a scalar quantity:

$$v_{av} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{d}{\Delta t}$$

- The average velocity of a body, v_{av}, is defined as the rate of change of displacement and is a vector quantity:

$$v_{av} = \frac{\text{displacement}}{\text{time taken}} = \frac{s}{\Delta t}$$

- To convert from m s^{-1} to km h^{-1}, multiply by 3.60.
- To convert from km h^{-1} to m s^{-1}, divide by 3.60.
- The SI unit for both speed and velocity is metres per second (m s^{-1}).

KEY QUESTIONS

1 A student jogs one lap of a 400.0 m track in 2.00 minutes. Calculate:

a their average speed

b their average velocity.

2 A person swims ten lengths of a 25.0 m pool. Which one or more of the following statements correctly describes their distance travelled and displacement?

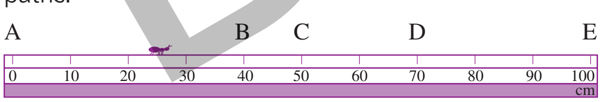
A Their distance travelled is zero.

B Their displacement is zero.

C Their distance travelled is 250.0 m.

D Their displacement is 250.0 m.

3 An ant is walking back and forth along a metre ruler, as shown in the figure below. Using the sign convention that right is positive and left is negative, determine both the displacement and the distance travelled by the ant as it moves along the following paths.



a A to B

b C to B

c C to D

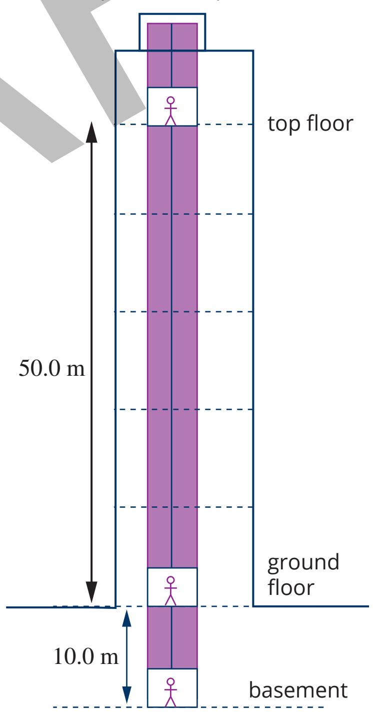
d C to E and then to D

4 During a training ride, a cyclist rides 50.0 km north then 30.0 km south.

a What is the distance travelled by the cyclist during the ride?

b What is the displacement of the cyclist for this ride?

5 A lift in a city building, shown in the figure below, carries a passenger from the ground floor down to the basement, then up to the top floor.



a What is the displacement of the lift as it travels from the ground floor to the basement?

b What is the displacement of the lift as it travels from the basement to the top floor?

c What is the distance travelled by the lift during this entire trip?

d What is the displacement of the lift during this entire trip?

6 A car travelling at a constant speed was timed over a distance of 400.0 m and was found to cover that distance in 12.0 s.

a What was the car's average speed?

b The driver was distracted when they encountered a hazard, which meant that their reaction time was 0.750 s before applying the brakes. How far did the car travel during this period of time?

7 A cyclist travels 25.0 km in 90.0 minutes.

a What is their average speed in $km\,h^{-1}$?

b What is their average speed in $m\,s^{-1}$?

8 Ali pushes a toy truck 5.00 m east, then stops it and pushes it 4.00 m west. The entire motion takes 10.0 s.

a What is the truck's average speed?

b What is the truck's average velocity?

9 Jackie rides a bicycle to school and travels 2.50 km south in 15.0 min.

a Calculate Jackie's average speed in kilometres per hour ($km\,h^{-1}$).

b What was Jackie's average velocity in metres per second ($m\,s^{-1}$)?

10 An athlete in training for a marathon runs 10.0 km north along a straight road before realising that they have dropped their drink bottle. The athlete turns around and runs back 3.0 km to find the bottle, then resumes running in the original direction. After running for 1.50 h, the athlete reaches a point 15.0 km from the starting position and stops.

a What is the distance travelled by the athlete during the run?

b What is the athlete's displacement during the run?

c What is the average speed of the athlete in $km\,h^{-1}$?

d What is the athlete's average velocity in $km\,h^{-1}$?

3.2 Acceleration

If you have been on a train as it pulled out of the station, you have experienced acceleration. If you have been in an aeroplane as it has taken off along a runway, you will have experienced a much greater acceleration. Astronauts and fighter pilots experience enormous accelerations that would make an untrained person lose consciousness. **Acceleration**, which is a measure of how quickly velocity changes, will be discussed in this section.

FINDING THE CHANGE IN VELOCITY AND SPEED

The velocity and speed of everyday objects are changing all the time. Examples of these are when a car moves away as the traffic lights turn green, when a tennis ball bounces or when you travel on a rollercoaster. If the initial and final velocity of an object are known, its change in velocity can be calculated.

To find the change, Δ, in any physical quantity, including speed and velocity, the initial value is taken away from the final value:

$$\Delta v = v_f - v_i$$

Change in speed is the final speed minus the initial speed:

$\Delta v = v_f - v_i$

where v_i is the initial speed (m s^{-1})

v_f is the final speed (m s^{-1})

Δv is the change in speed (m s^{-1}).

Since speed is a scalar, direction is not required.

Change in velocity is the final velocity minus the initial velocity:

$\Delta v = v_f - v_i$

where v_i is the initial velocity (m s^{-1})

v_f is the final velocity (m s^{-1})

Δv is the change in velocity (m s^{-1}).

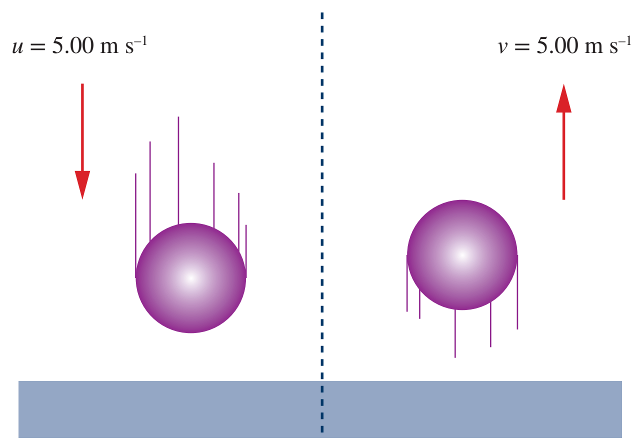
Since velocity is a vector, this should be done by performing a vector subtraction. As for all vectors, direction is required.

Vector subtraction was covered in detail in Section 2.3 on page xxx.

Worked example 3.2.1

CHANGE IN SPEED AND VELOCITY 1

A golf ball is dropped vertically onto a concrete floor and strikes the floor at $5.00\ \text{m s}^{-1}$. It then rebounds upwards at $5.00\ \text{m s}^{-1}$.



a Calculate the change in speed of the ball.

Thinking	**Working**
Find the values for the initial speed and the final speed of the ball.	$v_i = 5.00\,\mathrm{m\,s^{-1}}$ $v_f = 5.00\,\mathrm{m\,s^{-1}}$
Substitute the values into the change in speed equation: $\Delta v = v_f - v_i$	$\Delta v = v_f - v_i$ $\Delta v = (5.00) - (5.00)$ $\Delta v = 0.0\,\mathrm{m\,s^{-1}}$

b Calculate the change in velocity of the ball.

Thinking	**Working**
Velocity is a vector. Apply the sign convention up for positive and down for negative to replace the directions.	$v_i = 5.00\,\mathrm{m\,s^{-1}}$ down $v_i = -5.00\,\mathrm{m\,s^{-1}}$ $v_f = 5.00\,\mathrm{m\,s^{-1}}$ up $v_f = +5.00\,\mathrm{m\,s^{-1}}$
As the change in velocity equation is a vector subtraction equation, reverse the direction of v_i to get $-v_i$, then add the two vectors.	$v_i = -5.00\,\mathrm{m\,s^{-1}}$, therefore $-v_i = +5.00\,\mathrm{m\,s^{-1}}$
Substitute the values into the vector addition equation: $\Delta v = v_f + (-v_i)$	$\Delta v = v_f + (-v_i)$ $\Delta v = (+5.00) + (+5.00)$ $\Delta v = +10.0\,\mathrm{m\,s^{-1}}$
Apply the sign convention to describe the direction.	$\Delta v = 10.0\,\mathrm{m\,s^{-1}}$ up

Worked example: Try yourself 3.2.1

CHANGE IN SPEED AND VELOCITY 1

A golf ball is dropped onto a wooden floor and strikes the floor at $9.00\,\mathrm{m\,s^{-1}}$. It then rebounds at $7.00\,\mathrm{m\,s^{-1}}$.

a Calculate the change in speed of the ball.

b Calculate the change in velocity of the ball.

ACCELERATION

Consider the following information about the instantaneous velocity of a car that starts from rest as shown in Figure 3.2.1.

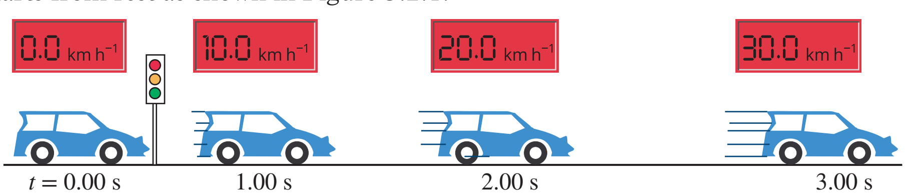


FIGURE 3.2.1 A car's acceleration as it increases in velocity from $0.0\,\mathrm{km\,h^{-1}}$ to $30.0\,\mathrm{km\,h^{-1}}$.

The velocity of the car pictured above increases by $10.0\,\mathrm{km\,h^{-1}}$ each second. In other words, its velocity changes by $+10\,\mathrm{km\,h^{-1}}$ per second. This is stated as an acceleration, $a = +10.0$ kilometres per hour per second or $+10.0\,\mathrm{km\,h^{-1}\,s^{-1}}$. More commonly in physics, velocity information is given in metres per second.

The athlete in Figure 3.2.2 takes 3.00 s to come to a stop at the end of a race.

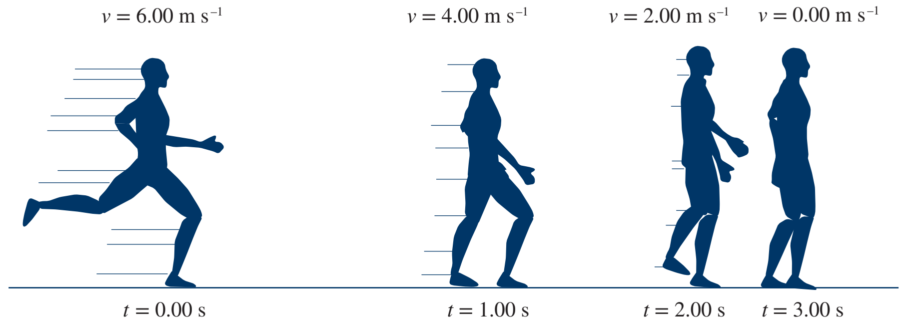


FIGURE 3.2.2 The velocity of the athlete changes by $-2.00\,\text{m}\,\text{s}^{-1}$ each second. Therefore, the acceleration, *a*, is $-2.00\,\text{m}\,\text{s}^{-2}$.

$$a = \frac{\text{change in velocity}}{\text{time taken}}$$

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{v_f - v_i}{t_f - t_i}$$

where a is the acceleration ($\text{m}\,\text{s}^{-2}$)

v_f is the final velocity ($\text{m}\,\text{s}^{-1}$)

v_i is the initial velocity ($\text{m}\,\text{s}^{-1}$)

$\Delta t = t_f - t_i$ is the time interval (s).

The velocity of the athlete changes by $-2.00\,\text{m}\,\text{s}^{-1}$ each second, so the acceleration is –2.00 metres per second per second. This is usually expressed as $a = -2.00$ metres per second squared or $a = -2.00\,\text{m}\,\text{s}^{-2}$.

A negative acceleration can mean that the object is slowing down in the direction of travel, as is the case with the athlete in Figure 3.2.2. A negative acceleration can also mean speeding up but in the opposite direction.

As acceleration is a vector quantity, vector diagrams can be used to calculate resultant accelerations of an object. Vector diagrams were covered in Chapter 2.

Average acceleration

As with speed and velocity, the average acceleration of an object can also be calculated. As all calculations of acceleration in this course are calculations of average acceleration, the av subscript can be assumed and so it can be omitted from your working.

Average acceleration, a_{av}, is the rate of change of velocity:

Worked example 3.2.2

CHANGE IN SPEED AND VELOCITY 2

A golf ball is dropped vertically onto a concrete floor and strikes the floor at $7.50\,\text{m}\,\text{s}^{-1}$. It then rebounds upwards at $7.50\,\text{m}\,\text{s}^{-1}$. The contact with the floor lasts for 25.0 milliseconds.

Calculate the average acceleration of the ball during its contact with the floor.

Thinking	Working
Note that the values you will need to calculate the average acceleration are initial velocity, final velocity and period of time. Use the convention that up is positive and down is negative. Convert 25.0 ms into s by multiplying by 10^{-3}, as the symbol m, for milli, represents 10^{-3}.	$v_i = -7.50\,\text{m}\,\text{s}^{-1}$ $-v_i = +7.50\,\text{m}\,\text{s}^{-1}$ $v_f = +7.50\,\text{m}\,\text{s}^{-1}$ $\Delta v = v_f + (-v_i)$ $\Delta v = (+7.50) + (+7.50)$ $\Delta v = +15.00\,\text{m}\,\text{s}^{-1}$ $\Delta t = 25.0\,\text{ms}$ $\Delta t = 25.0 \times 10^{-3}$ $\Delta t = 2.50 \times 10^{-2}\,\text{s}$

Substitute the values into the average acceleration equation.	$a = \frac{\text{change in velocity}}{\text{time taken}}$ $a = \frac{\Delta v}{\Delta t}$ $a = \frac{(+15.00)}{(2.50 \times 10^{-2})}$ $a = +600.00$ $a = +6.00 \times 10^{2}\,\text{m s}^{-2}$
Acceleration is a vector, so you must include a direction in your answer.	$a = 6.00 \times 10^{2}\,\text{m s}^{-2}$ up

Worked example: Try yourself 3.2.2

CHANGE IN SPEED AND VELOCITY 2

A netball is dropped vertically onto a court and strikes the surface at $9.00\,\text{m s}^{-1}$. It then rebounds upwards at $7.00\,\text{m s}^{-1}$. The contact time with the court is 35.0 milliseconds.

Calculate the average acceleration of the ball during its contact with the court.

PHYSICSFILE

Human acceleration

In the 1950s, the United States Air Force used a rocket sled to determine the effect of extremely large accelerations on humans. One of these sleds is shown in Figure 3.2.3. The aim was to find out the greatest accelerations that humans could safely withstand to help develop ejector seats for pilots.

The testing site consisted of an 800 m long railway track and a sled with nine rocket motors. One volunteer, Colonel John Stapp, was strapped into the sled and accelerated to speeds of over $1000\,\text{km h}^{-1}$ in a very short period of time. Water scoops were used to stop the sled abruptly in just 0.35 s. This equates to a deceleration of greater than $400\,\text{m s}^{-2}$. The effects of these massive accelerations are evident on his face (Figure 3.2.4).

Colonel John Stapp was a human guinea pig who suffered a great deal of discomfort so that other pilots would benefit. Safer ejector seats and non-human crash test dummies were developed because of these experiments.

FIGURE 3.2.3 The rocket-powered sled used to test the effects of acceleration on humans.

FIGURE 3.2.4 Photos showing the distorted face of Colonel John Stapp.

3.2 Review

SUMMARY

- Change in speed is a scalar calculation:
  Δv = final speed – initial speed = $v_f - v_i$
- Change in velocity is a vector calculation:
  Δv = final velocity – initial velocity = $v_f - v_i$
- Acceleration is a vector. The acceleration of a body, a, is defined as the rate of change of velocity:
  $$a = \frac{\text{change in velocity}}{\text{time taken}}$$
  $$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$
- The standard unit of acceleration is metres per second per second ($m\,s^{-2}$).

KEY QUESTIONS

1 A radio-controlled car is travelling east at 10.0 $km\,h^{-1}$. After hitting some sand, it slows down to 3.00 $km\,h^{-1}$ east. Determine its change in speed.

2 A lump of Blu Tack is falling vertically at 5.00 $m\,s^{-1}$ and when it hits the floor it stops without rebounding. Calculate its change in velocity during the collision.

3 A ping pong ball is falling vertically at 6.00 $m\,s^{-1}$. As it hits the floor, it rebounds at 3.00 $m\,s^{-1}$ up. Calculate its change in velocity during the bounce.

4 While playing a 90 minute soccer match, Ashley is running north at 7.50 $m\,s^{-1}$. Ashley slides along the ground with 1.50 seconds remaining in the match and stops at the same time the referee ends the game. Calculate Ashley's average acceleration as they slide to a stop.

5 Dee launches a model rocket at t = 0.00 s vertically and it reaches a speed of 155 $km\,h^{-1}$ at t = 3.50 s. What is the magnitude of its average acceleration in $km\,h^{-1}\,s^{-1}$?

6 A squash ball travelling east at 25.7 $m\,s^{-1}$ strikes the front wall of the court and rebounds at 15.2 $m\,s^{-1}$ west. The contact time between the wall and the ball is 0.0535 s. Calculate:

a the change in speed of the ball
b the change in velocity of the ball
c the average acceleration of the ball during its contact with the wall.

7 A greyhound starts from rest at t = 0.00 s and accelerates uniformly. Its velocity at t = 1.25 s is 8.08 $m\,s^{-1}$ south. Determine:

a the change in speed of the greyhound
b the change in velocity of the greyhound
c the acceleration of the greyhound.

8 How long does it take a vehicle travelling at 10.0 $m\,s^{-1}$ to reach 30.0 $m\,s^{-1}$ if it accelerates at 3.00 $m\,s^{-2}$?

9 A car travelling at 20.0 $m\,s^{-1}$ decelerates at 2.50 $m\,s^{-2}$. Calculate the time taken to stop.

10 A cyclist takes 4.00 s to slow down at −3.00 $m\,s^{-2}$ and completely stop. Calculate the initial velocity of the cyclist.

3.3 Graphing position, velocity and acceleration over time

At times, even the motion of an object travelling in a straight line can be complicated. The object may travel forwards or backwards, speed up or slow down, or even stop. Where the motion remains in one dimension, however, the information can be more easily understood when presented in graphical form.

The main advantage of a graph compared with a table is that it allows the nature of the motion to be seen clearly. Information that is contained in a table is not as readily accessible or as easy to interpret as information presented graphically. This section examines position–time, velocity–time, and acceleration–time graphs.

POSITION–TIME (*X–T*) GRAPHS

A position–time graph indicates the position, x, of an object at any time, t, for motion that occurs over an extended time interval. However, the graph can also provide additional information.

Consider Sophie, shown in Figure 3.3.1, swimming laps of a 50.0 m pool. Her position–time data are shown in Table 3.3.1. The starting point is treated as the origin for this motion.

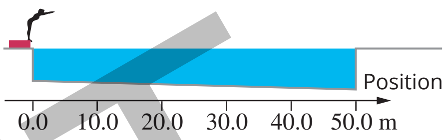


FIGURE 3.3.1 Swimmer standing at the end of a 50.0 m swimming pool.

TABLE 3.3.1 Positions and times of a swimmer completing 1.5 laps of a pool.

Time (s)	0.0	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	50.0	55.0	60.0
Position (m)	0.0	10.0	20.0	30.0	40.0	50.0	50.0	50.0	45.0	40.0	35.0	30.0	25.0

Analysis of Table 3.3.1 reveals several features of Sophie's swim. For the first 25.0 s, she swims at a constant rate. Every 5.00 s she travels 10.0 m in a positive direction, i.e. her velocity is $+2.00\,\mathrm{m\,s^{-1}}$. Then, from 25.0 s to 35.0 s, her position does not change. She seems to be resting, as she is stationary for this 10.0 s interval. Finally, from 35.0 s to 60.0 s, she swims back towards the starting point, in a negative direction. On this return lap, she maintains a more leisurely rate of 5.00 m every 5.00 s, so her velocity is $-1.00\,\mathrm{m\,s^{-1}}$. However, Sophie does not complete this lap, finishing 25.0 m from the start. This data is shown more conveniently on the position–time graph in Figure 3.3.2.

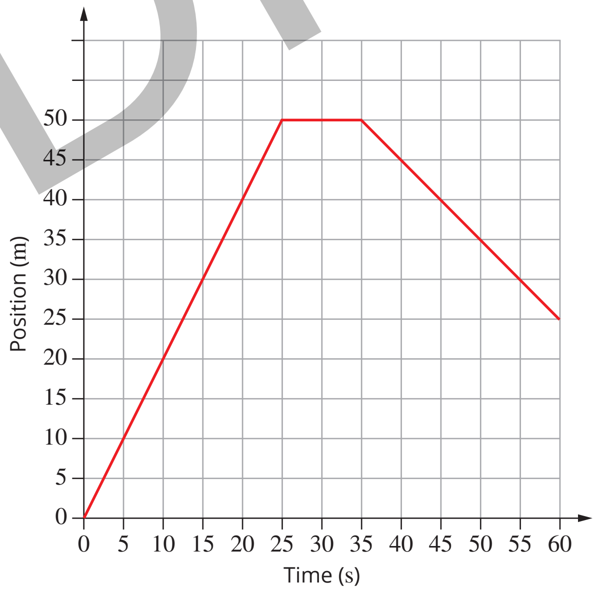


FIGURE 3.3.2 This position–time graph represents the motion of a swimmer travelling 50.0 m along a pool, then resting and swimming back towards the starting position. The swimmer finishes halfway along the pool.

The displacement, s, of the swimmer can be determined by comparing the initial and final positions. Her displacement between 20.0 s and 60.0 s is, for example:

$$s = \text{final position} - \text{initial position}$$
$$s = (25.0) - (40.0)$$
$$s = -15.0\,\text{m}$$

By further examining the graph, it can be seen that during the first 25.0 s the swimmer has a displacement of +50.0 m. Therefore, her average velocity is +2.00 m s^{-1}, i.e. 2.00 m s^{-1} to the right, during this time. This value can also be obtained by finding the gradient of this section of the graph.

A straight-line in a position–time graph (Figure 3.3.3) indicates a uniform velocity. The slope (gradient) of the line is equal to the velocity of the object.

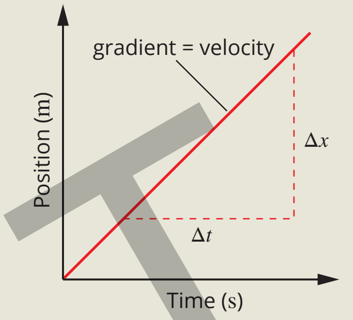


FIGURE 3.3.3 Position–time graph with gradient.

A positive velocity indicates that the object is moving in a positive direction and negative velocity indicates motion in a negative direction.

To confirm that the gradient of a position–time graph is a measure of velocity you can use **dimensional analysis**:

$$\text{gradient of } x\text{–}t \text{ graph} = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$$

The units of this gradient will be metres per second (m s^{-1}) so gradient is a measure of velocity. Note that the rise in the graph is the change in position, which is the definition of displacement; that is, $\Delta x = s$.

Non-uniform velocity

For motion with uniform (constant) velocity, the position–time graph will be a straight line, but if the velocity is non-uniform the graph will be curved. If the position–time graph is curved, the instantaneous velocity will be the gradient of the tangent to the line at the point of interest; the average velocity will be the gradient of the chord between two points. This is illustrated in Figure 3.3.4.

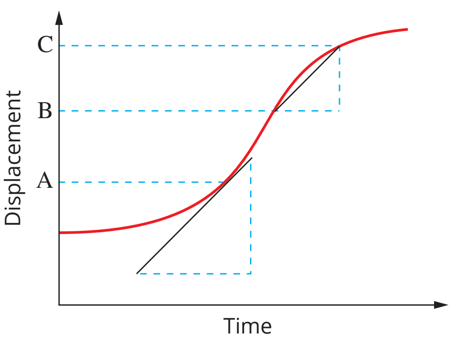
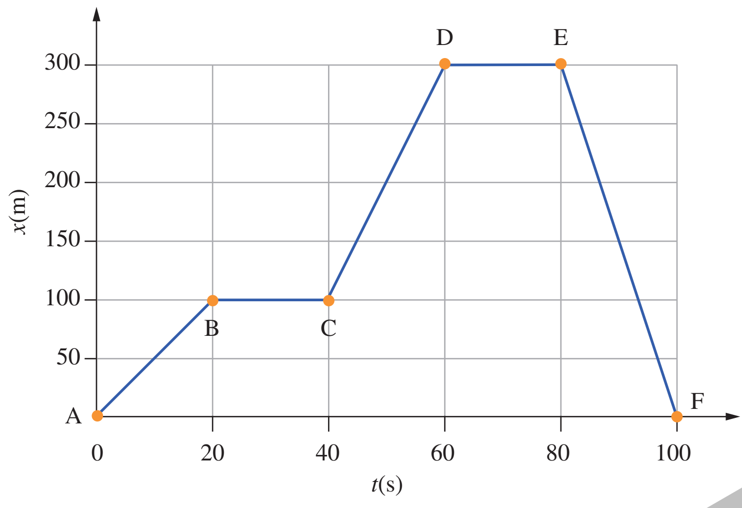


FIGURE 3.3.4 The instantaneous velocity at point A is the gradient of the tangent at that point. The average velocity between points B and C is the gradient of the chord between these points on the graph.

Worked example 3.3.1

ANALYSING A POSITION–TIME GRAPH

The motion of a cyclist is represented by the position–time graph below, with important features of the motion labelled A, B, C, D, E and F.



a What is the velocity of the cyclist between A and B?	
Thinking	**Working**
Determine the change in position (displacement) of the cyclist between A and B using: s = final position − initial position $s = \Delta x = x_f - x_i$	At A, $x_i = 0.0\,\text{m}$ At B, $x_f = 100.0\,\text{m}$ $s = (100.0) - (0.0)$ $s = +100.0\,\text{m}$ or $100.0\,\text{m}$ forwards (that is, away from the starting point)
Determine the time taken to travel from A to B. $\Delta t = t_f - t_i$	At A, $t_i = 0.0\,\text{s}$ At B, $t_f = 20.0\,\text{s}$ $\Delta t = (20.0) - (0.0)$ $\Delta t = 20.0\,\text{s}$
Calculate the gradient of the graph between A and B using: gradient of x–t graph $= \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$ Remember that $\Delta x = s$.	gradient $= \frac{\Delta x}{\Delta t}$ gradient $= \frac{(+100.0)}{(20.0)}$ gradient $= +5.00$
State the velocity, using: gradient of x–t graph = velocity Velocity is a vector so a direction must be given.	Since the gradient is +5.00, the velocity is $+5.00\,\text{m}\,\text{s}^{-1}$ or $5.00\,\text{m}\,\text{s}^{-1}$ forwards.

b Describe the motion of the cyclist between B and C.	
Thinking	**Working**
Interpret the shape of the graph between B and C.	The graph is flat between B and C, indicating that the cyclist's position is not changing during this time. So, the cyclist is not moving. If the cyclist is not moving, the velocity is $0\,\text{m}\,\text{s}^{-1}$.

You may confirm the result by calculating the gradient of the graph between B and C using: gradient of x–t graph $= \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$ Remember that $\Delta x = s$.	gradient $= \frac{(0.00)}{(20.0)}$ gradient = 0.00 Since the gradient is 0.00, the velocity is $0.00\,\text{m}\,\text{s}^{-1}$.

Worked example: Try yourself 3.3.1

ANALYSING A POSITION–TIME GRAPH

Use the graph shown in Worked example 3.3.1 to answer the following questions.

a What is the velocity of the cyclist between E and F?

b Describe the motion of the cyclist between D and E.

VELOCITY–TIME (v–t) GRAPHS

A graph of velocity, v, against time, t, shows how the velocity of an object changes with time.

Analysing motion

A velocity–time graph is useful for analysing the motion of an object moving in a complex manner.

Consider the example of the girl in Figure 3.3.5. Aliyah is running back and forth along an aisle in a supermarket. A study of the velocity–time graph reveals that Aliyah is moving with a positive velocity, i.e. in a positive direction, for the first 6.0 s. Between the 6.0 s mark and the 7.0 s mark she is stationary, then she runs in the reverse direction, i.e. has a negative velocity, for the final 3.0 s.

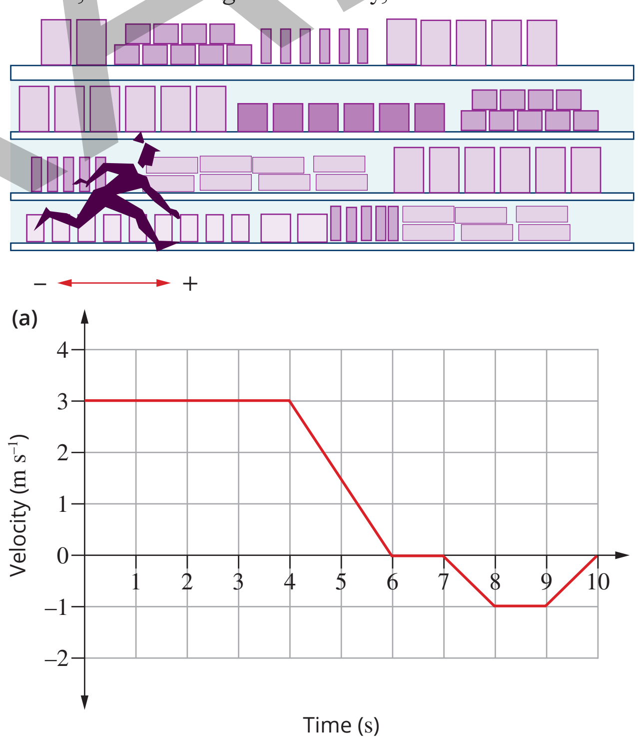


FIGURE 3.3.5 Diagram and v–t graph for a girl running along an aisle.

The graph shows Aliyah's velocity at each instant in time. She moves in a positive direction with a constant speed of $3.00\,\text{m}\,\text{s}^{-1}$ for the first 4.0 s. From 4.0 s to 6.0 s, she continues moving in a positive direction but slows down. At 6.0 s, she comes to a stop for 1.0 s. During the final 3.0 s, she accelerates in the negative direction for

1.0 s, then travels at a constant velocity of $-1.0\,\mathrm{m\,s^{-1}}$ for 1.0 s. She then slows down and comes to a stop at 10.0 s. Remember that whenever the graph is below the time axis, velocity is negative, which indicates travel in the reverse direction. So, she is travelling in the reverse direction for the last 3.0 s of her journey.

Finding displacement

A velocity–time graph can also be used to find the displacement of the object under consideration, as shown in Figure 3.3.6.

Displacement, s, is given by the area under a velocity–time graph, i.e. the area between the graph and the time axis. It is important to note that an area below the time axis indicates a negative displacement, i.e. motion in a negative direction.

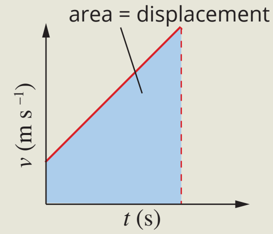


FIGURE 3.3.6 The area under a v–t graph gives displacement.

It is easier to see why the displacement is given by the area under the v–t graph when velocity is constant. For example, the graph in Figure 3.3.7 shows that in the first 6.0 s of motion, Aliyah moves with a constant velocity of $+3.0\,\mathrm{m\,s^{-1}}$ for 4.0 s. Note that the area under the graph for this period of time is a rectangle. Her displacement, s, during this time can be determined by rearranging the formula for velocity:

$$v = \frac{s}{\Delta t}$$
$$\therefore s = v \times \Delta t$$
$$s = \text{height} \times \text{base}$$
$$s = \text{area under } v\text{–}t \text{ graph}$$

Aliyah then slows from $3.0\,\mathrm{m\,s^{-1}}$ to zero in the next 2.0 s. To understand why the displacement for this period of time is given by the triangular area under the graph requires more complicated mathematics known as calculus, which is beyond the scope of this book.

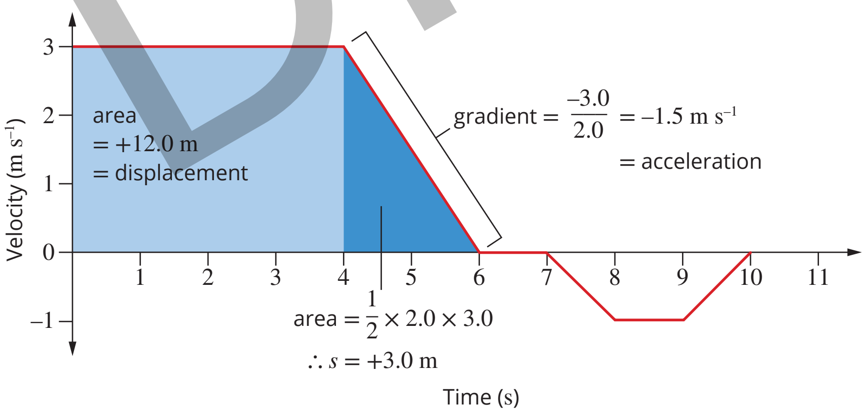


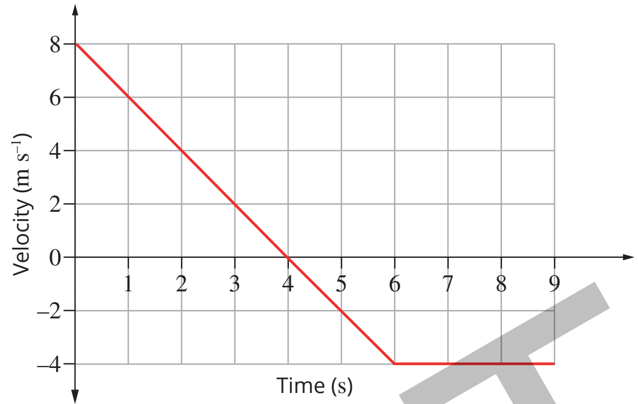
FIGURE 3.3.7 Area values as shown in a v–t graph.

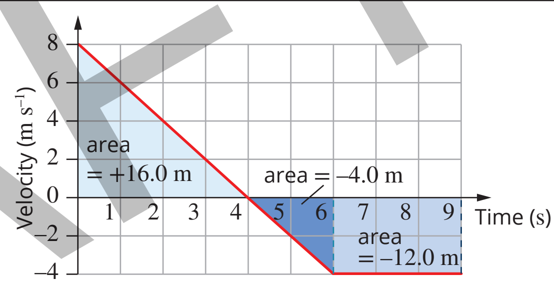
From Figure 3.3.7, the area under the graph for the first 4.0 s gives Aliyah's displacement during this time, i.e. +12.0 m. The displacement from 4.0 s to 6.0 s is represented by the area of the darker blue triangle and is equal to +3 m. The total displacement during the first 6 s is (+12.0 m) + (+ 3.0 m) = +15.0 m.

Worked example 3.3.2

ANALYSING A VELOCITY–TIME GRAPH

The motion of a radio-controlled car initially travelling east across a driveway in a straight line is represented by the graph below.



a What is the displacement of the car during the first 4.0 s?	
Thinking	**Working**
Displacement is the area under the graph. You must therefore find the area under the graph for the period of time for which you want to calculate the displacement. As $s = v\Delta t$, or $s = \Delta t \times v$, the base, b, is Δt and the height, h, is v. Use $s = b \times h$ for squares and rectangles. Use $s = \frac{1}{2}(b \times h)$ for triangles.	 The area from 0.0 to 4.0 s is a triangle, so: $s = \frac{1}{2}(b \times h)$ $s = \frac{1}{2}(4.0)(+8.0)$ $s = +16.0\,\text{m}$
Displacement is a vector quantity, so a direction is needed.	displacement = 16.0 m east

b What is the average velocity of the car for the first 4.0 s?	
Thinking	**Working**
Identify the equation and variables, and apply the sign convention.	$v = \frac{s}{\Delta t}$ $s = +16.0\,\text{m}$ $\Delta t = 4.0\,\text{s}$
Substitute values into the equation: $v = \frac{s}{\Delta t}$	$v = \frac{s}{\Delta t}$ $v = \frac{(+16.0)}{(4.0)}$ $v = +4.0000$
Velocity is a vector quantity, so a direction is needed.	$v = 4.0\,\text{m s}^{-1}$ east

Worked example: Try yourself 3.3.2

ANALYSING A VELOCITY–TIME GRAPH

Use the graph shown in Worked example 3.3.2 to answer the following questions.

a What is the displacement of the car from 4.0 to 6.0 s?

b What is the average velocity of the car from 4.0 to 6.0 s?

ACCELERATION FROM A VELOCITY–TIME (*v*–*t*) GRAPH

The acceleration of an object can also be found from a velocity–time graph, as shown in Figure 3.3.8.

The gradient of a velocity–time graph gives the average acceleration of the object over the time interval.

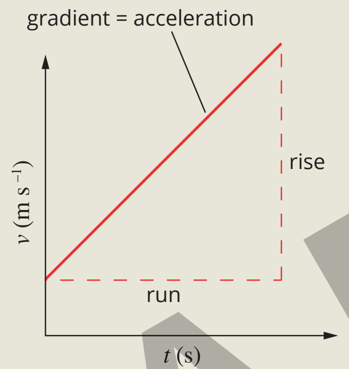


FIGURE 3.3.8 Gradient as displayed in a *v*–*t* graph.

Consider the motion of Aliyah in the 2.0 s interval between 4.0 s and 6.0 s on the graph in Figure 3.3.9. She is moving in a positive direction but slowing down from 3 m s^{-1} to rest.

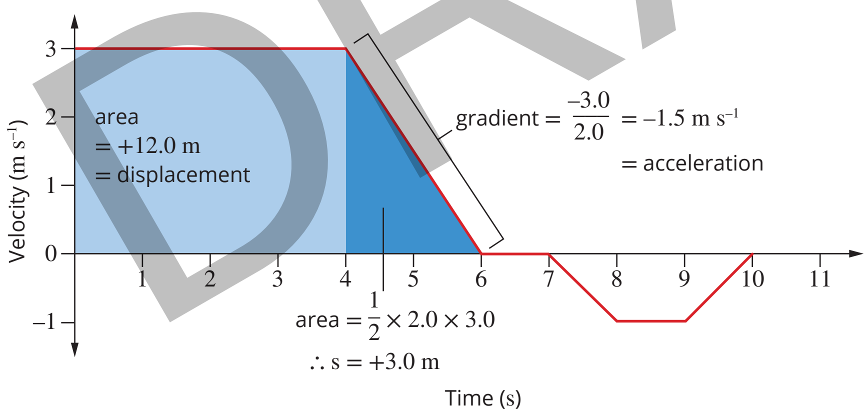


FIGURE 3.3.9 Acceleration as displayed in a *v*–*t* graph.

The gradient of the line from $t_i = 4.0$ s to $t_f = 6.0$ s is equal to her acceleration, as:

$$\text{gradient} = \frac{\Delta v}{\Delta t}$$

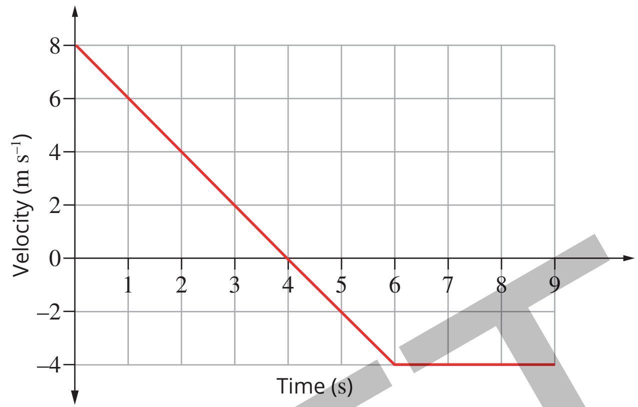
$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{(0.0) - (3.0)}{(2.0)} = -1.5\,\text{m s}^{-2}$$

Acceleration is the change in velocity divided by the period of time taken, which is equal to the gradient of the *v*–*t* graph. As can be seen from Figure 3.3.9 and the calculation above, the gradient of the line between 4.0 s and 6.0 s is $-1.5\,\text{m s}^{-2}$.

Worked example 3.3.3

FINDING ACCELERATION USING A VELOCITY–TIME GRAPH

Consider the motion of the radio-controlled car described in Worked example 3.3.2. The car initially travels east in a straight line across a driveway as shown by the graph below.



What is the acceleration of the car during the first 4.0 s?

Thinking	Working
Acceleration is the gradient of a *v*–*t* graph. Calculate the gradient using: $\text{gradient} = \frac{\text{rise}}{\text{run}}$ $a = \text{gradient} = \frac{\Delta v}{\Delta t}$	gradient from 0.0 to 4.0 s $= \frac{\text{rise}}{\text{run}}$ $a = \frac{\Delta v}{\Delta t}$ $a = \frac{v_f - v_i}{t_f - t_i}$ $a = \frac{(0.0) - (8.0)}{(4.0) - (0.0)}$ $a = -2.0000$ $a = -2.0\ \text{m s}^{-2}$
Acceleration is a vector quantity, so a direction is needed. Note: In this case, the car is moving in the easterly direction and slowing down.	$a = 2.0\ \text{m s}^{-2}$ west

Worked example: Try yourself 3.3.3

FINDING ACCELERATION USING A VELOCITY–TIME GRAPH

Use the graph shown in Worked example 3.3.3 to answer the following question.
What is the acceleration of the car during the period from 4.0 to 6.0 s?

DISTANCE TRAVELLED

A velocity–time graph can also be used to calculate the distance travelled by a moving object. The process of determining distance requires you to calculate the area under the v–t graph, as you would when calculating displacement. However, since distance travelled by an object always increases as the object moves, regardless of direction, you must add up all the areas between the graph and the time axis, regardless of whether the area is above or below the axis.

For example, Figure 3.3.10 shows the velocity–time graph of the radio-controlled car from Worked example 3.3.3. The area above the time-axis, which corresponds to motion in the positive direction, is +16.0 m, while the area below the axis, which corresponds to negative motion, consists of –4.0 m and –12.0 m. To calculate the total displacement, you would add up each displacement:

total displacement $s = (+16.0) + (-4.0) + (-12.0)$

$s = (+16.0) + (-16.0)$

$s = 0.0\,\text{m}$

To calculate the total distance, you would add up the magnitude of the areas, ignoring whether they are positive or negative:

total distance $d = (16.0) + (4.0) + (12.0)$

$d = 32.0\,\text{m}$

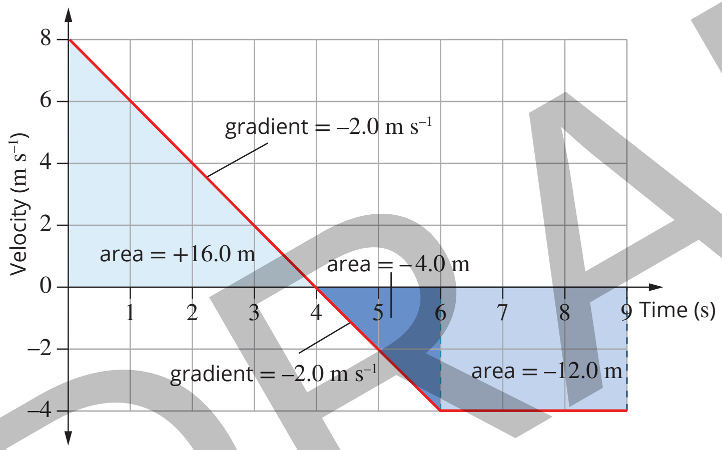


FIGURE 3.3.10 Both distance and displacement can be calculated by using the areas under the velocity–time graph.

Non-uniform acceleration

For motion with uniform (constant) acceleration, the velocity–time graph will be a straight line. For non-uniform acceleration, the velocity–time graph will be curved. If the velocity–time graph is curved, the instantaneous acceleration will be the gradient of the tangent to the curve at the point of interest; the average acceleration will be the gradient of the chord between two points. The displacement can still be calculated by finding the area under the graph; however, you will need to make some estimations.

PHYSICSFILE

Area under graphs

The calculation of the area under a graph is useful in many areas of physics.

Some examples include:

- power–time graphs, where the area represents the energy used over that period of time
- force–time graphs, where the area represents the impulse or change in momentum over a period time (see section xxx)
- force–displacement graphs, where the area represents the work done or energy transferred while the forces are acting.

ACCELERATION–TIME (*a*–*t*) GRAPHS

An acceleration–time graph simply indicates the acceleration of the object as a function of time. The area under an acceleration–time graph is found by multiplying an acceleration, a, and the period of time, Δt, over which the acceleration changes. The area gives a change in velocity, Δv, value:

area $= \Delta v = a \times \Delta t$

In order to establish the actual velocity of the object, its initial velocity must be known. Figure 3.3.11 shows both Aliyah's velocity versus time (v–t) and acceleration versus time (a–t) graphs.

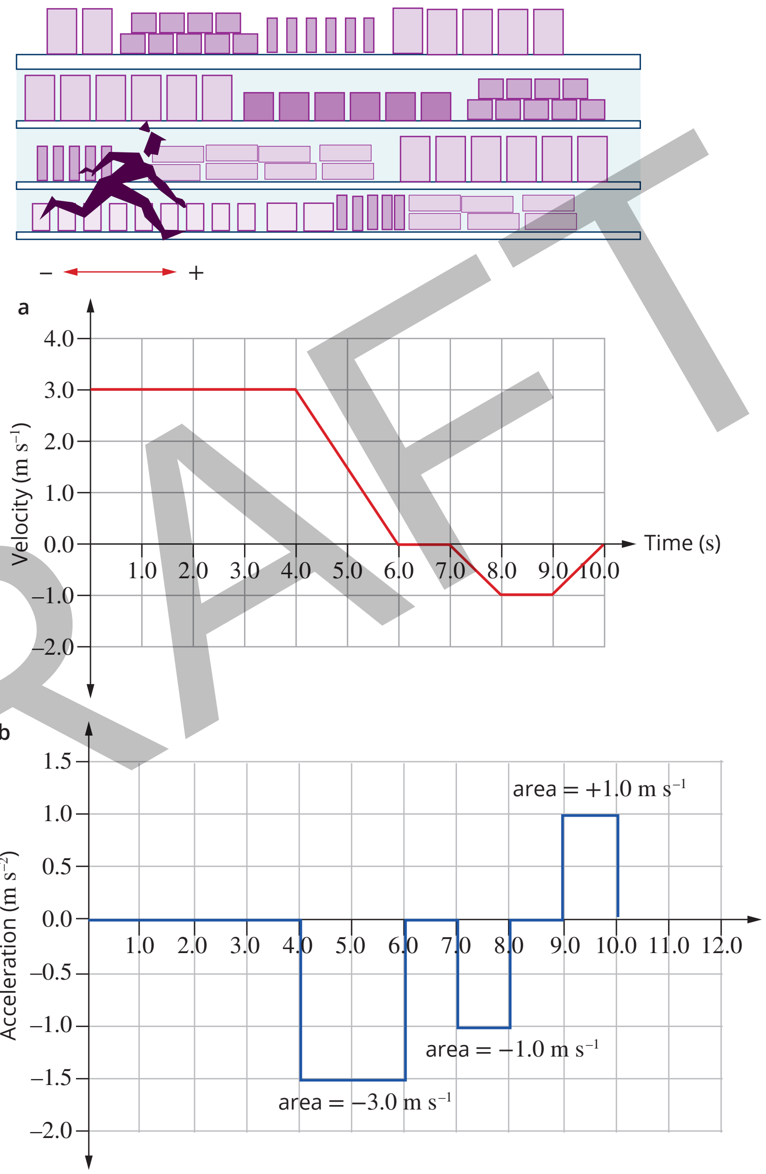


FIGURE 3.3.11 (a) Aliyah's velocity versus time (v–t) graph. (b) Aliyah's acceleration versus time (a–t) graph.

From $t_i = 4.0$ to $t_f = 6.0$ s, the area under the a–t graph shows that $\Delta v = -3.0\,\text{m}\,\text{s}^{-1}$. This indicates that she has slowed down by $3.0\,\text{m}\,\text{s}^{-1}$ during this period of time. Aliyah's v–t graph confirms this fact. Her initial speed is $3.0\,\text{m}\,\text{s}^{-1}$, so she must be stationary ($v = 0$) after 6.0 s. This calculation could not be made without knowing her initial velocity.

EXTENSION

Graphing in physics

Graphs in physics can be useful in solving problems as an alternative to using equations. An advantage of graphs is that they give a quick and easy picture of the relationship between the data that has been measured.

In this course, you will mainly analyse straight-line or linear graphs. A higher level of mathematical skills is required to analyse nonlinear graphs or curves.

Differentiation is used to find the gradients of curves, where the intervals of change are infinitely small.

For a straight line, gradient $= \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$

If you consider a curve to be a series of infinitesimally small straight lines, then Δx and Δt are also extremely small. The gradient then becomes $\frac{\delta x}{\delta t}$, where the lower-case Greek letter delta, δ, denotes an infinitesimally small change. This is often written as $\frac{dx}{dt}$, i.e. the derivative of *x* with respect to *t*.

For a velocity–time graph, the gradient gives the acceleration—i.e. $a = \frac{dv}{dt}$ You have already seen that displacement can be found by calculating the area under a velocity–time graph, for example by breaking up the area under the graph into rectangles and triangles. Similarly, taking extremely small sections and adding them all together again gives the area under a non-linear graph, as shown in Figure 3.3.12. This is called integration, and for a curve given by a function *f(x)* can be written as:

$$\text{area} = \int_a^b f(x)dx$$

So, for a velocity–time graph, finding the area under the graph gives you the displacement:

$$s = \int_{t_1}^{t_2} \text{v}\,\text{dt}$$

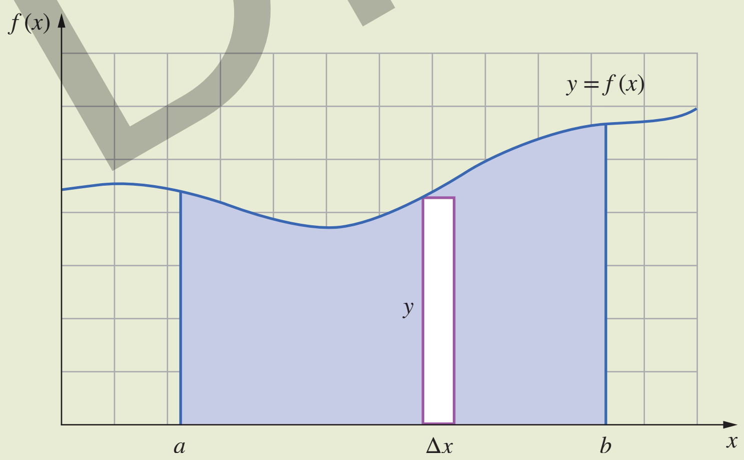


FIGURE 3.3.12 The small areas are added together by integration.

3.3 Review

SUMMARY

- A position–time (x–t) graph can be used to determine the location of an object at any given time. Additional information can also be derived from the graph:
  - displacement, s, is given by the change in position of an object
  - the velocity, v, of an object is given by the gradient of the position–time graph
  - if the position–time graph is curved, the gradient of the tangent at a point gives the instantaneous velocity, v.
- The gradient of a velocity–time (v–t) graph is the acceleration, a, of the object.
- The area under a velocity–time (v–t) graph is the displacement, s, of the object.
- The area under an acceleration–time graph (a–t) is the change in velocity, Δv, of the object.

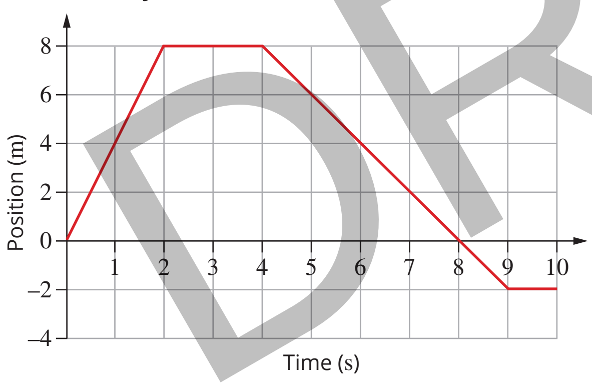
KEY QUESTIONS

1 Which of the following does the gradient of a position–time graph represent?

A displacement
B acceleration
C time
D velocity

The following information relates to questions 2–6.

The graph represents the straight-line motion of a radio-controlled toy car.



2 Describe the motion of the car in terms of its position.

3 Find out the position of the toy car after:

a 2.0 s
b 4.0 s
c 6.0 s
d 10.0 s.

4 When did the car return to its starting point?

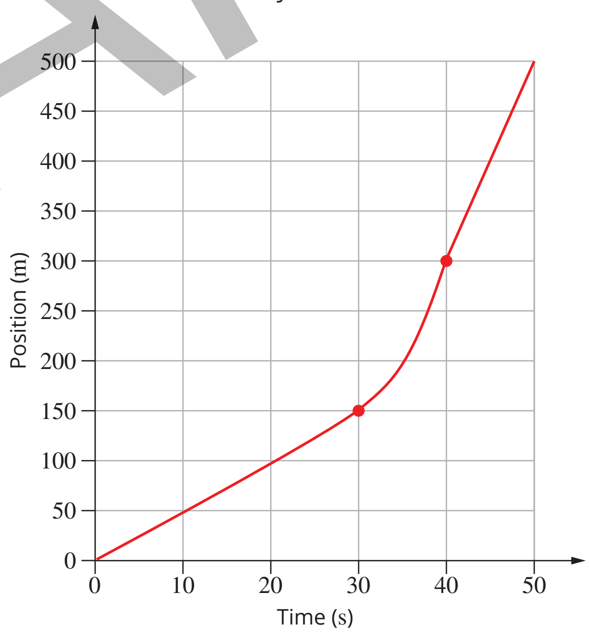
5 Calculate the velocity of the toy car:

a during the first 2.0 s
b at 3.0 s
c from $t_i = 4.0$ s to $t_f = 8.0$ s
d at 8.0 s
e from $t_i = 8.0$ s to $t_f = 9.0$ s.

6 During its total 10.0 s of motion, what was the car's:

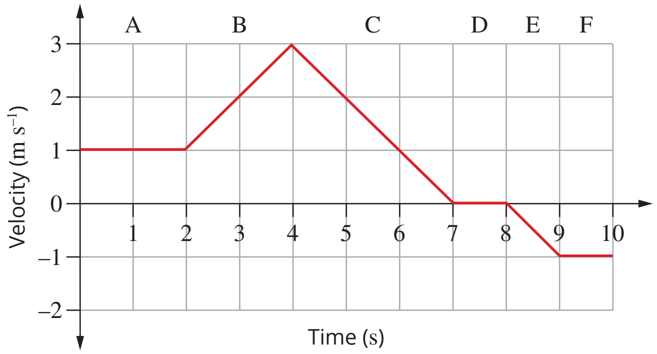
a distance travelled
b displacement?

7 The position–time graph for a cyclist travelling north along a straight road is shown. Calculate the following information about the cyclist's motion.



a What was the average speed of the cyclist during the first 30.0 s?
b What was the average velocity of the cyclist during the final 10.0 s?
c What was the average velocity of the cyclist for the whole trip?

8 The graph in the figure below shows the motion of a dog running along a footpath.



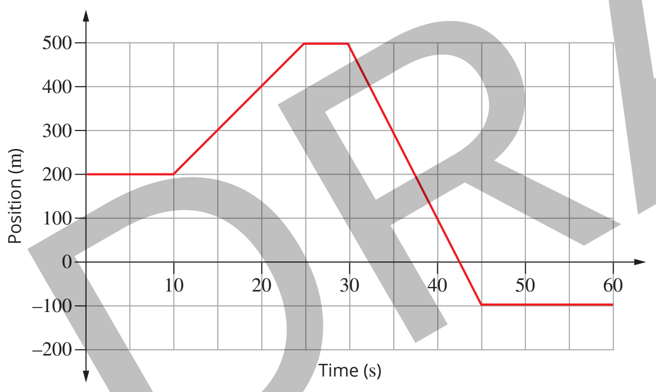
a What is the magnitude of the acceleration of the dog at $t = 1.0$ s?

b What is the magnitude of the acceleration of the dog at $t = 5.0$ s?

c What is the magnitude of the displacement of the dog for the first 7.0 s?

d What is the magnitude of the average velocity of the dog over the first 7.0 s?

9 The graph below shows the position of a motorbike along a straight stretch of road as a function of time. The motorcyclist starts 200.0 m north of a town.

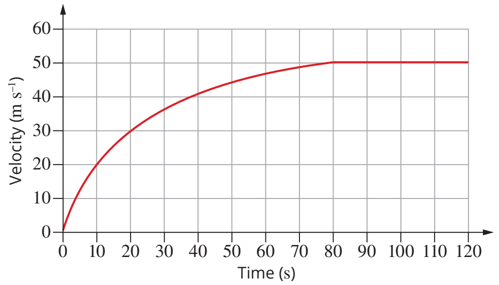


Calculate the instantaneous velocity of the motorcyclist at each of the following times:

a 15.0 s

b 35.0 s.

10 The straight-line motion of a high-speed train is shown in the graph below.



a How long does it take the train to reach its maximum speed?

b What is the acceleration of the train 10.0 s after starting?

c What is the acceleration of the train 40.0 s after starting?

d By counting squares, estimate the displacement of the train after 120.0 s.

3.4 Equations for uniform acceleration

A graph is an excellent way of representing motion as it provides a great deal of information that is easy to visualise and interpret. However, a graph is time-consuming to draw, and sometimes values can only be estimated rather than precisely calculated.

In the previous section, various graphs of motion were used to determine quantities such as displacement, velocity and acceleration. This section examines a more precise method of solving problems involving *constant* or *uniform acceleration*. This method involves the use of a series of equations that can be derived from the basic definitions developed earlier.

DERIVING THE EQUATIONS

Consider an object moving in a straight line with an initial velocity, v_i, and a uniform acceleration, a, for a time interval, Δt. As the variables v_i, v_f, and a are vectors, and the motion is limited to one dimension, the sign and direction convention of right as positive and left as negative can be used. After a period of time, Δt, the object has changed its velocity from an initial velocity of v_i and is now travelling with a final velocity of v_f. Its acceleration will be given by:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

If the initial time is t_i and the final time is t_f, then $\Delta t = t_f - t_i$. The above equation can then be rearranged as:

$$v_t = v_i + a\Delta t \qquad \text{(i)}$$

The average velocity of the object is:

$$v_{av} = \frac{s}{\Delta t}$$

When acceleration is uniform, average velocity, v_{av}, can also be found as the average of the initial and final velocities:

$$v_{av} = \frac{1}{2}(v_i + v_f)$$

This relationship is shown graphically in Figure 3.4.1.

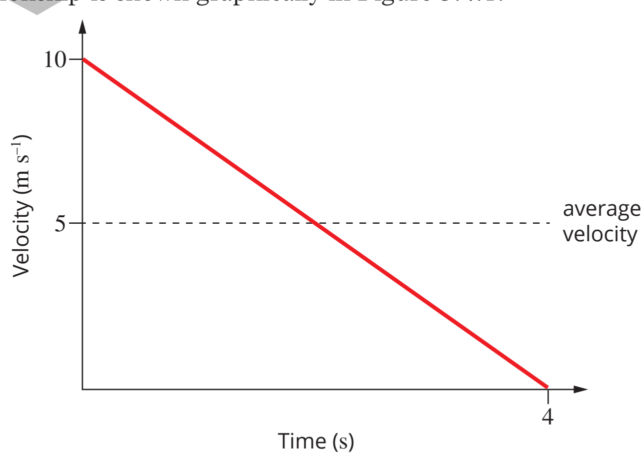


FIGURE 3.4.1 Uniform acceleration as displayed by a v–t graph.

By combining these two equations of average velocity, we get:

$$\frac{s}{\Delta t} = \frac{1}{2}(v_i + v_f)$$

This gives:

$$s = \frac{1}{2}(v_i + v_f)\Delta t \qquad \text{(ii)}$$

A graph describing constant acceleration motion is shown in Figure 3.4.2. For constant acceleration, the velocity is increasing by the same amount in each time interval, so the gradient of the v–t graph is constant. The displacement, which is equal to the area under the v–t graph, is given by the combined area of the rectangle and the triangle:

$$\text{area} = s = s_1 + s_2$$

$$s = (v_i \times \Delta t) + \frac{1}{2}(v_f - v_i) \times \Delta t$$

As $a = \dfrac{v_f - v_i}{\Delta t}$ then $v_f - v_i = a\Delta t$, and this can be substituted for $v_f - v_i$ in the equation above to give:

$$s = (v_i \times \Delta t) + \frac{1}{2}(a\Delta t)\Delta t$$

$$s = (v_i \Delta t) + \frac{1}{2}a\Delta t^2$$

$$s = v_i\Delta t + \frac{1}{2}a\Delta t^2 \qquad \text{(iii)}$$

Another way to calculate the area under the graph is to use the large area $(v_f\Delta t)$ and subtract the triangle component $(\frac{1}{2}a\Delta t^2)$. This will give you:

$$s = v_f\Delta t - \frac{1}{2}a\Delta t^2 \qquad \text{(iv)}$$

Rewriting equation (i) with Δt as the subject gives:

$$\Delta t = \frac{v_f - v_i}{a}$$

Now, if this is substituted into equation (ii):

$$s = \frac{1}{2}(v_f - v_r)\Delta t \qquad \text{(ii)}$$

$$s = \frac{v_i + v_f}{2} \times \frac{v_f - v_i}{a}$$

Multiplying the top line and bottom line gives:

$$s = \frac{v_f^2 - v_i^2}{2a}$$

Finally, transposing this gives:

$$v_f^2 = v_i^2 + 2as \qquad \text{(v)}$$

Equations (i) to (v) are commonly used to solve problems in which acceleration is constant. They are summarised below.

$$v_f = v_i + a\Delta t$$

$$s = \frac{1}{2}(v_i + v_f)\Delta t$$

$$s = v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$s = v_f\Delta t - \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2as$$

where s is the displacement (m)

v_i is the initial velocity (m s^{-1})

v_f is the final velocity (m s^{-1})

a is the acceleration (m s^{-2})

Δt is the time taken (s).

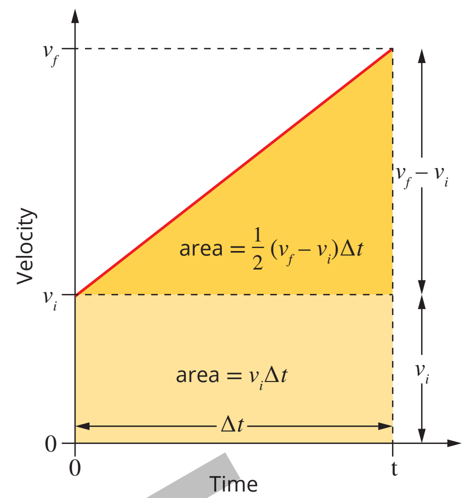


FIGURE 3.4.2 The area under a v–t graph broken up into a rectangle and a triangle.

SOLVING PROBLEMS USING EQUATIONS

When solving problems using these equations, it is important to think about the problem and try to visualise what is happening. Follow the steps below.

Step 1 Draw a simple diagram of the situation.

Step 2 Write down the information that has been given in the question. You might like to use the word 'sifat' as a memory prompt to help you remember the list of variables in the order s, v_i, v_f, a and Δt. Use a sign convention to assign positive and negative values to indicate directions. Convert all units to SI form.

Step 3 Select the equation that matches your data. It should include three values that you know, and the one value that you want to solve.

Step 4 Write your preliminary answer to five or six significant figures, which may then be used in any subsequent questions, i.e. part (b), (c), etc. This will help to reduce rounding errors in subsequent answers.

Step 5 Use the appropriate number of significant figures in your final answer. If your answer is greater than 9999 or less than 0.001, provide your answer in scientific notation.

Step 6 Include units with the final answer and specify a direction if the quantity is a vector.

Worked example 3.4.1

USING THE EQUATIONS OF MOTION

A snowboarder in a race is travelling $10.0\,\text{m}\,\text{s}^{-1}$ north as she crosses the finishing line. She then decelerates uniformly, coming to a stop over a distance of 20.0 m.

a Calculate her acceleration as she comes to a stop.	
Thinking	**Working**
Write down the known quantities as well as the quantity you are finding. (The term 'sifat' may help you to recall them.) Apply the sign convention that north is positive and south is negative.	Take all the information that you can from the question: • constant acceleration, so use equations for uniform acceleration • 'coming to a stop' means that the final velocity is zero. $s = +20.0\,\text{m}$ $v_i = +10.0\,\text{m}\,\text{s}^{-1}$ $v_f = 0.00\,\text{m}\,\text{s}^{-1}$ $a = ?$ $\Delta t = ?$
Identify the correct equation to use.	$v_f^2 = v_i^2 + 2as$
Substitute known values into the equation and solve for a.	$v_f^2 = v_i^2 + 2as$ $a = \dfrac{v_f^2 - v_i^2}{2s}$ $a = \dfrac{(0.0)^2 - (+10.0)^2}{2(+20.0)}$ $a = \dfrac{(-100.0)}{(+40.0)}$ $a = -2.5000$
Use the sign convention to state the answer with its direction, units and the correct number of significant figures.	$a = 2.50\,\text{m}\,\text{s}^{-2}$ south

b How long does she take to come to a stop?	
Thinking	**Working**
Write down the known quantities as well as the quantity you are finding. (The term 'sifat' may help you to recall them.) Apply the sign convention that north is positive and south is negative.	Take all the information that you can from the question: • constant acceleration, so use equations for uniform acceleration • 'coming to a stop' means that the final velocity is zero. $s = +20.0\,\text{m}$ $v_i = +10.0\,\text{m s}^{-1}$ $v_f = 0.00\,\text{m s}^{-1}$ $a = -2.5000\,\text{m s}^{-2}$ $\Delta t = ?$
Identify the correct equation to use. Since you now know four values, any equation involving Δt will work.	$v_f = v_i + a\Delta t$
Substitute known values into the equation and solve for Δt.	$v_f = v_i + a\Delta t$ $\Delta t = \frac{v_f - v_i}{a}$ $\Delta t = \frac{(0.00) - (10.0)}{(-2.5000)}$ $\Delta t = 4.0000\,\text{s}$
State the answer with its units and the correct number of significant figures.	$\Delta t = 4.00\,\text{s}$

c What is the average velocity of the snowboarder as she comes to a stop?	
Thinking	**Working**
Write down the known quantities as well as the quantity that you are finding. Apply the sign convention that north is positive and south is negative.	Take all the information that you can from the question: • constant acceleration, so we only need to find the average of the final and initial speeds. $v_i = +10.0\,\text{m s}^{-1}$ $v_f = 0.00\,\text{m s}^{-1}$ $v_{av} = ?$
Identify the correct equation to use.	$v_{av} = \frac{1}{2}(v_f + v_i)$
Substitute known values into the equation and solve for vav. Include units with the answer.	$v_{av} = \frac{1}{2}(v_f + v_i)$ $v_{av} = \frac{1}{2}(0.00 + 10.0)$ $v_{av} = 5.0000$
Use the sign convention to state the answer with its direction, units and the correct number of significant figures.	$v_{av} = 5.00\,\text{m s}^{-1}$ north

Worked example: Try yourself 3.4.1

USING THE EQUATIONS OF MOTION

A snowboarder in a race is travelling $15.5\,\text{m s}^{-1}$ east as she crosses the finishing line. She then decelerates uniformly until coming to a stop over a distance of 30.0 m.

a Calculate her acceleration as she comes to a stop.
b How long does she take to come to a stop?
c What is the average velocity of the snowboarder as she comes to a stop?

3.4 Review

SUMMARY

- The following equations can be used for situations in which there is a constant acceleration, where:

  s is the displacement (m)
  v_i is the initial velocity (m s^{-1})
  v_f is the final velocity (m s^{-1})
  a is the acceleration (m s^{-2})
  Δt is the period of time (s).
- $v_f = v_i + a\Delta t$
- $s = \frac{1}{2}(v_i + v_f)\Delta t$
- $s = v_i\Delta t + \frac{1}{2}a\Delta t^2$
- $s = v_f\Delta t - \frac{1}{2}a\Delta t^2$
- $v_f^2 = v_i^2 + 2as$
- $v_{av} = \frac{s}{\Delta t} = \frac{v_i + v_f}{2}$
- A sign and direction convention for motion in one dimension needs to be used with these equations.

KEY QUESTIONS

1 A cyclist has a uniform acceleration as they roll down a hill. Their initial speed is 5.09 m s^{-1}. They travel a distance of 32.5 m and their final speed is 18.3 m s^{-1}. Which equation should be used to determine their acceleration?

A $v_f = v_i + a\Delta t$

B $s = \frac{1}{2}(v_i + v_f)\Delta t$

C $s = v_i\Delta t + \frac{1}{2}a\Delta t^2$

D $s = v_f\Delta t - \frac{1}{2}a\Delta t^2$

E $v_f^2 = v_i^2 + 2as$

2 A new-model hydrogen vehicle travels with a uniform acceleration on a racetrack. It starts from rest and covers 445 m in 16.0 s.

a Calculate the magnitude of its average acceleration during this time.

b What is the final speed of the car in m s^{-1}?

c What is the car's final speed in km h^{-1}?

3 An electric hybrid car starts from rest and accelerates uniformly in a positive direction for 3.10 s. It reaches a final speed of 19.9 m s^{-1}.

a Calculate the magnitude of the acceleration of the hybrid car.

b What is the magnitude of the average velocity of the hybrid car during this time?

c What is the distance travelled by the hybrid car during this time?

4 During its launch phase, a space rocket accelerates uniformly from rest to 167 m s^{-1} upwards in 4.02 s, then enters a constant speed phase of 167 m s^{-1} for the next 5.40 s.

a Calculate the acceleration of the rocket in its initial launch phase.

b Calculate the combined distance (in km) the rocket travels during both phases of the flight.

c What is the final speed of the rocket in km h^{-1}?

d What is the average speed of the rocket during the first 4.02 s?

e What is the average speed of the rocket during the total 9.42 seconds of motion?

5 While overtaking another cyclist, Charlie increases their speed uniformly from 4.12 m s^{-1} to 6.07 m s^{-1} east over a time interval of 0.508 s.

a Calculate the magnitude of Charlie's average acceleration during this time.

b How far does Charlie travel while accelerating?

c What is Charlie's average speed during this time?

6 A diver enters a diving pool headfirst while travelling at 18.0 m s^{-1} downwards. The diver hits the water at t_i = 0.00 s and stops after a downwards displacement of 4.06 m. Consider the diver to be a single point located at their centre of mass and assume their acceleration through the water to be uniform.

a What is the magnitude of the average acceleration of the diver as the diver travels through the water?

b How long does the diver take to come to a stop?

c What is the velocity of the diver after they have dived through 2.00 m of water?

7 A car is travelling along a straight road at 75.0 km h^{-1} east. In an attempt to avoid an accident, the motorist has to brake suddenly and stop the car.

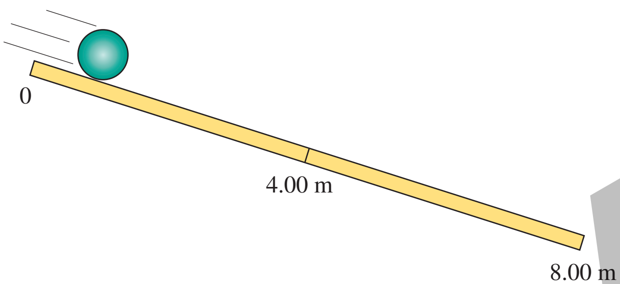
a What is the car's initial speed in m s^{-1}?

b If the reaction time of the motorist is 0.254 s, what is the displacement of the car before they are able to apply the brakes?

c Once the brakes are applied, the car has an acceleration of –6.70 m s^{-2}. How far does the car travel while stopping?

d What is the total displacement of the car from the time the driver first notices the danger to when the car comes to a stop?

8 A billiard ball rolls from rest down a smooth ramp that is 8.00 m long. The acceleration of the ball is constant at 2.60 m s^{-2}.



a What is the velocity of the ball when it is halfway down the ramp?

b What is the final velocity of the ball at the bottom of the ramp?

c How long does the ball take to roll the first 4.00 m?

d How long does the ball take to travel the final 4.00 m?

9 A cyclist, Nolan, is travelling at a constant speed of 12.2 m s^{-1} when they pass a stationary bus. The bus starts moving just as Nolan passes, and it accelerates uniformly at 1.50 m s^{-2}.

a When does the bus reach the same speed as Nolan?

b How long does the bus take to catch Nolan?

c What distance has Nolan travelled before the bus catches up?

3.5 Vertical motion

Until 500 years ago, it was widely believed that the heavier an object was, the faster it would fall. This was the theory proposed by Aristotle, and it lasted for 2000 years until the end of the Middle Ages. In the seventeenth century, the Italian scientist Galileo conducted experiments that showed that the mass of the object did not affect the rate at which it fell, as long as **air resistance** was not a factor.

It is now known that falling objects speed up because of gravity; however, many people still think that objects with a greater mass fall faster than objects of a lesser mass. This confusion often arises because they fail to consider the effects of air resistance. This section examines the motion of falling objects..

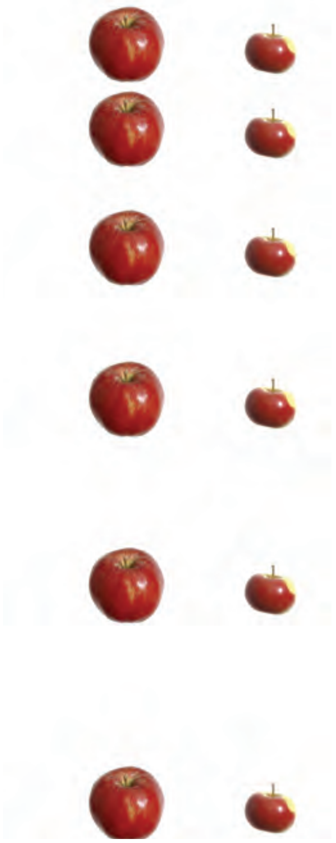


FIGURE 3.5.1 A stroboscopic image of a free-falling apple. The time elapsed between each image of the apple is the same but the vertical displacement increases during each period of time, which shows the apple is accelerating. Without air resistance, the two different mass apples accelerate at the same rate.

ANALYSING VERTICAL MOTION

Some falling objects are affected by air resistance more than others; for example, feathers and balloons. This is why these objects do not speed up much as they fall. However, if air resistance can be ignored, all bodies in **free fall** near the Earth's surface will move with an equal downwards acceleration. The stroboscopic image in Figure 3.5.1 clearly shows an apple accelerating as it falls, since the vertical displacement of the apple between each photograph increases. In a vacuum, this acceleration would be the same for a feather, a bowling ball, or any other object. The mass of the object does not matter if air resistance is removed.

At the Earth's surface, the acceleration due to gravity, g, is $-9.80\,\text{m}\,\text{s}^{-2}$, where the negative sign indicates a downwards direction. The acceleration of a body due to gravity is independent of its initial velocity and is the same whether the object has been thrown vertically upwards or is falling vertically downwards.

For example, a coin that is dropped from rest at $t = 0.00\,\text{s}$ will have an initial velocity of $0.00\,\text{m}\,\text{s}^{-1}$. At $t = 1.00\,\text{s}$ it will be falling with a velocity of $-9.80\,\text{m}\,\text{s}^{-1}$ and at $t = 2.00\,\text{s}$ with a velocity of $-19.6\,\text{m}\,\text{s}^{-1}$, and so on. As Δt increases with each second, the coin's velocity increases by $-9.80\,\text{m}\,\text{s}^{-1}$. The motion of a falling coin is illustrated in Figure 3.5.2.

However, if the coin was launched straight up at $t = 0.00\,\text{s}$ with an initial velocity of $+19.6\,\text{m}\,\text{s}^{-1}$, then at $t = 1.00\,\text{s}$ its velocity would be $+9.80\,\text{m}\,\text{s}^{-1}$ and at $t = 2.00\,\text{s}$ its velocity would be $0.00\,\text{m}\,\text{s}^{-1}$. In other words, with each second of the coin's upwards journey, its velocity would decrease by $-9.80\,\text{m}\,\text{s}^{-1}$. At the instant in time the velocity reaches $0.00\,\text{m}\,\text{s}^{-1}$, the motion of the coin changes from upwards to downwards, which means it would accelerate downwards as described in the previous paragraph. This point in time also represents the time at which the coin reaches its maximum vertical displacement. The motion of a coin thrown vertically upwards is shown in Figure 3.5.3.

So, regardless of whether the coin is falling vertically downwards or is flipped vertically upwards, its speed changes at the same rate. The speed of the falling coin *increases* by $-9.80\,\text{m}\,\text{s}^{-1}$ each second and the speed of the rising coin *decreases* by $-9.8\,\text{m}\,\text{s}^{-1}$ each second. That means that the acceleration of the coin due to gravity is $-9.80\,\text{m}\,\text{s}^{-2}$, or $9.80\,\text{m}\,\text{s}^{-2}$ downwards, in both cases.

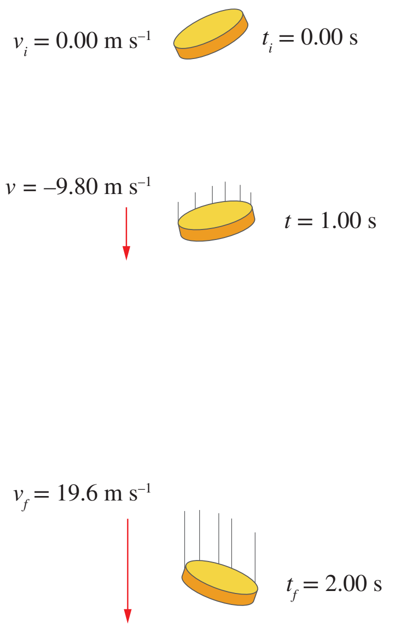


FIGURE 3.5.2 A falling coin.

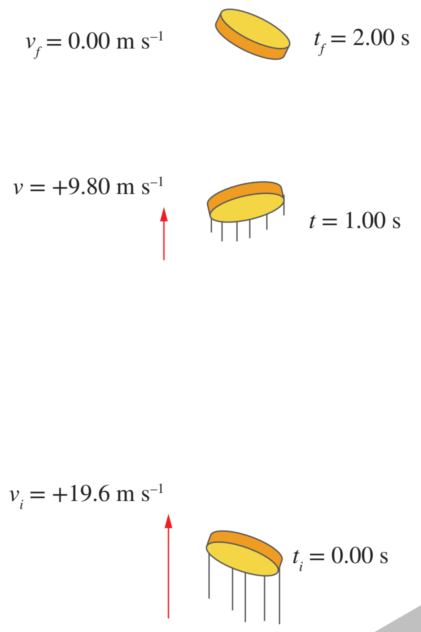


FIGURE 3.5.3 A coin thrown vertically upwards.

PHYSICSFILE

Galileo's experiment carried out on the Moon

In 1971, David Scott went to great lengths to show that Galileo's prediction was correct. As an astronaut on the Apollo 15 Moon mission, he took a hammer and a feather on the voyage. He stepped onto the lunar surface, held the feather and hammer at the same height and dropped them together. As Galileo had predicted 400 years earlier, in the absence of any air resistance, the two objects fell side by side as they accelerated towards the Moon's surface at exactly the same rate.

Popular physicist, musician and TV presenter Professor Brian Cox repeated a version of this experiment in the world's biggest vacuum chamber, the Space Simulation Chamber at NASA's Space Power Facility in Ohio. Professor Cox set up a mechanist that would drop a bowling ball and a feather at exactly the same time. When the air was removed from the chamber, they filmed the two different masses accelerating for over 9 metres at exactly the same rate.

FIGURE 3.5.4 Astronaut David Scott dropping a feather and a hammer on the Moon. Professor Brian Cox dropping some feathers and a bowling ball in the world's largest vacuum chamber.

PHYSICS IN ACTION

Theories of motion: Aristotle and Galileo

Aristotle was a Greek philosopher who lived in the fourth century BCE. He was such an influential individual that his ideas on motion were generally accepted for nearly 2000 years. Aristotle didn't do experiments as we know them today, but simply *thought* about different bodies in motion to arrive at a plausible explanation for the way in which they moved.

Aristotle spent a lot of time classifying different animals and adopted a similar approach to his study of motion. His theory gave inanimate objects, such as rocks and rain, characteristics that were similar to living things. Aristotle then organised these objects into four terrestrial groups or elements: earth, water, air and fire (see Figure 3.5.5). He said that any object was a mixture of these elements in different proportions.

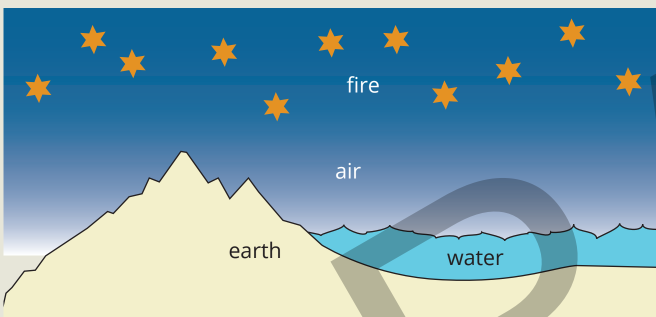


FIGURE 3.5.5 Aristotle's four elements of the universe: earth, water, air and fire.

According to Aristotle, a body would move because of a tendency that could come from inside or outside of the body. An internal tendency would cause 'natural' motion and result in a body returning to its proper place. For example, if a rock, which is an earth substance, is held in the air and released, then its natural tendency would be to return to the Earth, i.e. it falls to the ground. Similarly, fire was thought to head upwards, to return to its proper place in the universe, i.e. the Sun.

An external push that acts when something is thrown or hit was the cause of 'violent' motion according to the Aristotelian model. In other words, an external push acted to take a body away from its proper place. For example, when an apple is thrown into the air, a violent motion carries the apple away from the Earth, but then the natural tendency of the apple takes over and it returns to the ground.

Aristotle's theory worked quite well and could be used to explain the motion of many objects. However, there were also many examples that it could not successfully explain, such as why some solids floated while other solids sunk.

Aristotle explained the behaviour of a falling body by saying that its velocity depended on how much earth element it contained. This suggested that a 2.00 kg cat would fall twice as fast and in half the time as a 1.00 kg cat dropped from the same height. Many centuries later, Galileo Galilei (pictured in Figure 3.5.6) noticed that, at the start of a hailstorm, small hailstones arrived at the same time as large hailstones. This caused Galileo to doubt Aristotle's theory, and so he set about finding a better explanation for the motion of freely falling bodies.

FIGURE 3.5.6 Galileo Galilei.

A famous story in science is that of Galileo dropping different masses from the Leaning Tower of Pisa in Italy. This story may or may not be true, but Galileo did perform a very detailed analysis of falling bodies. Galileo used inclined planes because freely falling bodies moved too fast to analyse. He completed detailed experiments that showed conclusively that Aristotle was in fact incorrect.

By using a water clock to time balls as they rolled from rest down different inclines, he was able to show that the balls were accelerating, and that the distance they travelled was proportional to the square of the period of time, i.e. $d \propto \Delta t^2$.

Galileo found that this relationship also held true when he inclined the plane at larger and larger angles, allowing him to conclude that freely falling bodies actually fall with a uniform acceleration.

Since the acceleration of a free-falling body is constant, it is appropriate to use the equations that were studied in the previous section, under 'Equations for uniform acceleration'. It is necessary to specify whether up or down is positive when doing these problems, though you can simply follow the mathematical convention of regarding up as positive, which would mean the acceleration due to gravity would always be $-9.80\,m\,s^{-2}$. The variable for uniform acceleration, a, in these equations can be replaced by the variable for gravitational acceleration, g, in calculations involving vertical motion.

Worked example 3.5.1

VERTICAL MOTION

A construction worker accidentally knocks a brick from a building so that it falls vertically a distance of 47.0 m to the ground. Use $g = -9.80\,m\,s^{-2}$ and ignore air resistance when answering these questions.

a How long does the brick take to fall halfway, to 23.5 m?

Thinking	Working
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) Apply the sign convention that up is positive and down is negative.	The brick starts at rest, so: $s = -23.5\,m$ $v_i = 0.00\,m\,s^{-1}$ $v_f = ?\,m\,s^{-1}$ $g = -9.80\,m\,s^{-2}$ $\Delta t = ?$
Identify the correct equation for uniform acceleration to use, but substitute the a for g.	$s = v_i\Delta t + \frac{1}{2}g\Delta t^2$
Substitute known values into the equation and solve for Δt. Think about whether the value seems reasonable.	$(-23.5) = (0.00)\Delta t + \frac{1}{2}(-9.80)\Delta t^2$ $(-23.5) = (-4.90)\Delta t^2$ $\Delta t = \sqrt{\frac{(-23.5)}{(-4.90)}}$ $\Delta t = 2.18996$ $\Delta t = 2.19\text{ s}$

b How long does the brick take to fall all the way to the ground?

Thinking	Working
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) Apply the sign convention that up is positive and down is negative.	$s = -47.0\,m$ $v_i = 0.00\,m\,s^{-1}$ $v_f = ?\,m\,s^{-1}$ $g = -9.80\,m\,s^{-2}$ $\Delta t = ?$
Identify the correct equation for uniform acceleration to use, but substitute the a for g.	$s = v_i\Delta t + \frac{1}{2}g\Delta t^2$
Substitute known values into the equation and solve for t. Think about whether the value seems reasonable. Notice that the brick takes 2.19 s to travel the first 23.5 m and only 0.91 s more to travel the final 23.5 m. This is because it is accelerating.	$(-47.0) = (0.00)\Delta t + \frac{1}{2}(-9.80)\Delta t^2$ $(-47.0) = (-4.90)\Delta t^2$ $\Delta t = \sqrt{\frac{(-47.0)}{(-4.90)}}$ $\Delta t = 3.09707$ $\Delta t = 3.10\,s$

PHYSICSFILE

Strength of gravity

The acceleration due to gravity, g, on Earth varies slightly from the accepted value of $-9.80\,m\,s^{-2}$ depending on the location. The reasons for this will be studied in Unit 3 Physics. On the Moon, the strength of gravity, g, is much weaker than on Earth, which causes falling objects to accelerate at the much lower rate of $-1.60\,m\,s^{-2}$. Other planets and bodies in the Solar System have different values of g depending on their mass and radius. The value of g at various locations in the Solar System is provided in Table 3.5.1.

TABLE 3.5.1 Acceleration due to gravity at different locations on Earth, and on other bodies in the Solar System.

Location	Acceleration due to gravity ($m\,s^{-2}$)
Perth	−9.794
South Pole	−9.832
Equator	−9.780
Moon	−1.600
Mars	−3.600
Jupiter	−24.600
Pluto	−0.670

In Unit 2 Physics you can assume that acceleration due to gravity is always $-9.80\,m\,s^{-2}$.

c What is the final velocity of the brick as it hits the ground?	
Thinking	**Working**
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) Apply the sign convention that up is positive and down is negative.	$s = -47.0\,\text{m}$ $v_i = 0.00\,\text{m}\,\text{s}^{-1}$ $v_f = ?\ \text{m}\,\text{s}^{-1}$ $g = -9.80\,\text{m}\,\text{s}^{-2}$ $\Delta t = 3.09707\,\text{s}$
Identify the correct equation to use. Since you now know four values, any equation involving *v* will work, but substitute the *a* for *g*.	$v_f = v_i + g\Delta t$
Substitute the known values into the equation and solve for v_f. Think about whether the value seems reasonable.	$v_f = (0.00) + (-9.80)(3.09707)$ $v_f = -30.3513$ $v_f = 30.4\,\text{m}\,\text{s}^{-1}$
Use the sign and direction convention to describe the direction of the final velocity.	$v = 30.4\,\text{m}\,\text{s}^{-1}$ downwards

Worked example: Try yourself 3.5.1

VERTICAL MOTION

A construction worker accidentally knocks a hammer from a building so that it falls vertically a distance of 60.0 m to the ground. Use $g = -9.80\,\text{m}\,\text{s}^{-2}$ and ignore air resistance when answering these questions.

a How long does the hammer take to fall halfway, to 30.0 m?

b How long does it take the hammer to fall all the way to the ground?

c What is the velocity of the hammer as it hits the ground?

Remember that when an object is thrown vertically up into the air, it will eventually reach a point where its velocity is zero for an instant in time, but not for any period of time, before returning back down. So, the vertical velocity of the object decreases as the object rises, is zero at the instant it achieves its maximum height, and then increases vertically downwards as the object falls. Throughout this motion, however, the object is still in the same gravitational field, so *g* remains at $-9.80\,\text{m}\,\text{s}^{-2}$. Knowing that the velocity of an object thrown into the air is zero at the top of its flight allows you to calculate the maximum height reached.

Worked example 3.5.2

MAXIMUM HEIGHT PROBLEMS

On winning a tennis match the victorious player, Corey, smashes the ball vertically into the air at $27.5\,\text{m}\,\text{s}^{-1}$. In the following questions ignore air resistance and use $g = -9.80\,\text{m}\,\text{s}^{-2}$.

a Determine the maximum height reached by the ball.	
Thinking	**Working**
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) At the maximum height the velocity is zero. Apply the sign convention that up is positive and down is negative.	$s = ?\text{m}$ $v_i = +27.5\,\text{m s}^{-1}$ $v_f = 0.00\,\text{m s}^{-1}$ $g = -9.80\,\text{m s}^{-2}$ $\Delta t = ?$
Identify the correct equation to use, but substitute the a for *g*.	$v_f^2 = v_i^2 + 2gs$
Substitute known values into the equation and solve for *s*.	$s = \frac{v_f^2 - v_i^2}{2g}$ $s = \frac{(0.00)^2 - (+27.5)^2}{2(-9.80)}$ $s = +38.5841$ $s = +38.6\,\text{m}$ i.e. the ball reaches a height of 38.6 m above the racquet.

b Calculate the time that the ball takes to return to its starting position.	
Thinking	**Working**
To work out the time the ball is in the air, first calculate the time it takes to reach its maximum height. Write down the known quantities and the quantity that you need to find.	$v_i = +27.5\,\text{m s}^{-1}$ $v_f = 0.00\,\text{m s}^{-1}$ $g = -9.80\,\text{m s}^{-2}$ $s = 38.5481\,\text{m}$ $\Delta t = ?$
Identify the correct equation to use, but substitute the *a* for *g*.	$v_f = v_i + g\Delta t$
Substitute known values into the equation and solve for Δt.	$\Delta t = \frac{v_f - v_i}{g}$ $\Delta t = \frac{(0.00) - (+27.5)}{(-9.80)}$ $\Delta t = 2.80612$ to maximum height $\Delta t = 2.80612$ to return to the racquet total $\Delta t = 5.61224$ total $\Delta t = 5.61$ s

Worked example: Try yourself 3.5.2

MAXIMUM HEIGHT PROBLEMS

On winning a cricket match, a fielder throws a cricket ball vertically into the air at 15.5 m s^{-1}. In the following questions, ignore air resistance and use $g = -9.80$ m s^{-2}.

a Determine the maximum height reached by the ball.

b Calculate the time that the ball takes to return to its starting position.

3.5 Review

SUMMARY

- If air resistance can be ignored, all bodies falling freely near the Earth will move with the same constant acceleration.
- The acceleration due to gravity is represented by *g* and is equal to $-9.80\,m\,s^{-2}$ if the direction towards the centre of the Earth is considered to be negative.
- The equations for uniform acceleration can be used to solve vertical motion problems by substituting the variable *a*, for the constant *g*. It is necessary to specify and use a sign convention, such as up is positive and down is negative.

KEY QUESTIONS

For these questions, ignore the effects of air resistance and assume that the acceleration due to gravity is $-9.80\,m\,s^{-2}$ unless instructed otherwise.

1 A ball is thrown into the air. Describe how the velocity of the ball changes when it leaves the hand up until the instant before it hits the hand again.

2 Angus inadvertently drops an egg while baking a cake, and the egg falls vertically towards the ground. Which one of the following statements correctly describes how the egg falls?

A The egg's acceleration increases.
B The egg's acceleration is constant.
C The egg's velocity is constant.
D The egg's acceleration decreases.

3 Yvette is an Olympic trampolinist and is practising some routines. Which one or more of the following statements correctly describes Yvette's motion at the instant she is at the highest point of the bounce? Assume that her motion is vertical.

A She has zero velocity.
B Her acceleration is zero.
C Her acceleration is upwards and downwards.
D Her acceleration is always downwards.

4 A window cleaner working on the Bell tower accidently drops her mobile phone. The phone falls vertically towards the ground with an acceleration of $-9.80\,m\,s^{-2}$.

a Determine the velocity of the phone after 3.04 s.
b How fast is the phone moving after it has fallen 30.0 m?
c What is the average velocity of the phone during a fall of 30.0 m?

5 A person tosses a marble straight up into the air at $5.18\,m\,s^{-1}$ and then catches it at the same height from which it was thrown. Ignore air resistance.

a Is the acceleration of the marble on the way up the same as, less than, or greater than, its acceleration on the way down? Justify your answer.
b Is the magnitude of the launch velocity of the marble the same as, less than or greater than the magnitude of its landing velocity? Justify your answer.

6 A rubber ball is bounced off a concrete floor so that it travels straight up into the air, reaching its highest point after 1.58 s.

a What is the initial velocity of the rubber ball just as it leaves the ground?
b What is the maximum height reached by the ball?

7 A book is knocked off a bench and falls vertically to the floor. If the book takes 0.400 s to fall to the floor, calculate the following descriptions of its motion.

a What is the book's velocity the instant before it lands?
b From what height did the book fall?
c How far did the book fall during the first 0.200 s?
d How far did the book fall during the final 0.200 s?

8 Jet the labrador is playing with a new toy. When Jet drops a tennis ball into a launcher, it shoots the ball into the air for him to catch. The ball travels vertically upwards into the air. Being a very clever dog, Jet notices that the ball takes 4.08 s to return to its starting position.

a How long does the tennis ball take to reach its maximum height?
b Calculate the velocity of the ball the instant it left the launcher.
c What was the maximum height reached by the tennis ball?
d What was the velocity of the ball as it returned to its starting point?

3.5 Review *continued*

9 Two physics students conduct the following experiment from a very high bridge. Asuka drops a 1.57 kg shot-put from a vertical height of 60.0 m and, at exactly the same time, Jordan throws a 109 g mass with an initial downwards velocity of 10.0 m s^{-1} from a point 10.0 m above Asuka.

a How long does it take Asuka's shot-put to reach the ground?

b How long does it take Jordan's 109 g mass to reach the ground?

10 At the start of a football match, the umpire bounces the ball off a rubber plate so that it travels vertically upwards and reaches a height of 15.0 m.

a How long does the ball take to reach this maximum height?

b One of the players is able to leap and reach to a height of 4.00 m with their hand. How long after the bounce should this player try to make contact with the ball as it is on its way down?

11 A stone is held out over the edge of a seaside cliff and thrown vertically upwards at t_i = 0.00 s with an initial velocity of 8.00 m s^{-1}. It lands in the sea below at t_f = 3.00 s. Calculate:

a the maximum height above the top of cliff reached by the stone if it left the thrower's hand 1.90 m above the level of the cliff

b the time taken by the stone to reach its maximum height

c the height of the cliff above the sea.

Chapter review

03

KEY TERMS

acceleration
air resistance
centre of mass
dimensional analysis
displacement
distance travelled
free fall
magnitude
position
speeds

For the following questions, ignore air resistance and use $g = -9.80\,\text{m}\,\text{s}^{-2}$ unless indicated otherwise.

1 A car travels at $95.0\,\text{km}\,\text{h}^{-1}$ along a freeway. What is its speed in $\text{m}\,\text{s}^{-1}$?

2 A cyclist travels at $15.3\,\text{m}\,\text{s}^{-1}$ during a sprint finish. What is this speed in $\text{km}\,\text{h}^{-1}$?

The following information relates to questions 3 and 4.

An athlete in training for a marathon runs 15.4 km north along a straight road before realising that they have dropped their drink bottle. The athlete turns around and runs back 5.7 km to find the bottle, then resumes running in the original direction. After running for 3.00 hours, the athlete reaches 20.2 km from their original starting position and stops.

3 Calculate the average speed of the athlete in $\text{km}\,\text{h}^{-1}$.

4 Calculate the average velocity in:

a $\text{km}\,\text{h}^{-1}$

b $\text{m}\,\text{s}^{-1}$.

5 A ping pong ball is falling vertically at $-6.00\,\text{m}\,\text{s}^{-1}$ as it hits the floor. It rebounds at $+4.50\,\text{m}\,\text{s}^{-1}$ up. What is its change in speed during the bounce?

6 A car is moving in a positive direction. It approaches a red light and slows down. Which of the following statements correctly describes its acceleration and velocity as it slows down?

A The car has positive acceleration and negative velocity.

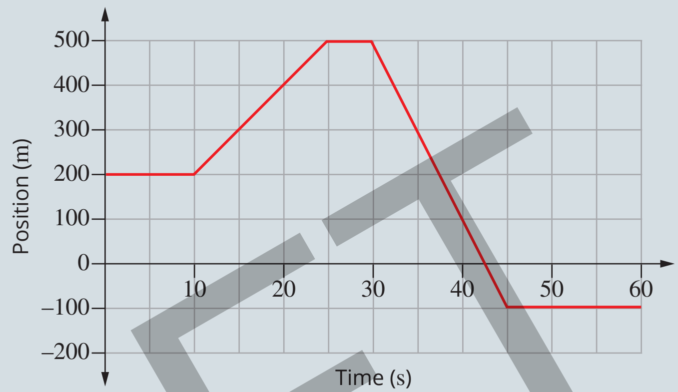
B The car has negative acceleration and positive velocity.

C Both the velocity and acceleration of the car are positive.

D Both the velocity and acceleration of the car are negative.

7 A skier is travelling along a horizontal ski run at a speed of $15.6\,\text{m}\,\text{s}^{-1}$. After falling over, the skier takes 2.55 s to come to rest. Calculate the average acceleration of the skier as they stop.

8 The following graph shows the position of a motorbike along a straight stretch of road as a function of time. The motorcyclist starts 200.0 m north of an intersection.



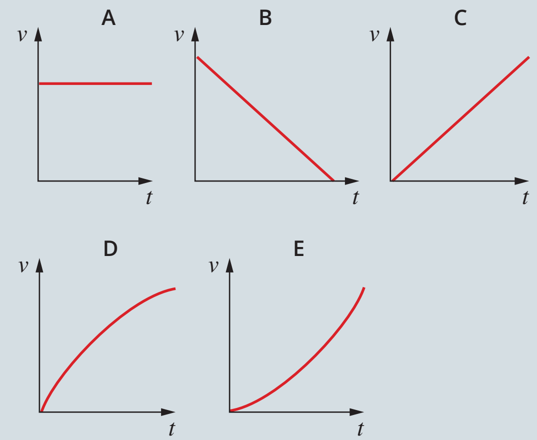
a At what time interval is the motorcyclist travelling in a northerly direction?

b At what time interval is the motorcyclist travelling in a southerly direction?

c At what time intervals is the motorcyclist stationary?

d At what time is the motorcyclist passing back through the intersection?

9 For each of the activities below, indicate which of the following velocity–time graphs best represents the motion involved.

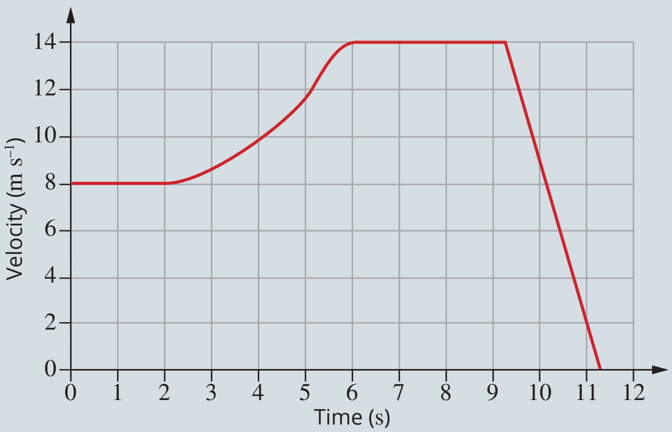


a A car comes to a stop at a red light.

b A swimmer is travelling at a constant speed.

c A motorbike starts from rest with uniform acceleration.

10 This velocity–time graph is for an Olympic road cyclist as they travel, initially north, along a straight section of the track.



a Estimate the displacement of the cyclist during the journey.

b Calculate the magnitude of the average velocity of the cyclist during this 11.3 s interval.

c Determine the acceleration of the cyclist at $t = 1.0$ s.

d Calculate the acceleration of the cyclist at $t = 10.0$ s.

e Which one or more of the following statements correctly describes the motion of the cyclist?

A They are always travelling north.

B They travel south during the final 2.0 s.

C They are stationary at $t = 8.0$ s.

D They return to the starting point after 11.0 s.

11 A car starts from rest and has a constant acceleration of 3.59 m s^{-2} west for 4.51 s. What is its final velocity?

12 A jet-ski starts from rest and accelerates uniformly east. If it travels 2.80 m in its first second of motion, calculate:

a its acceleration

b its velocity at the end of the first second

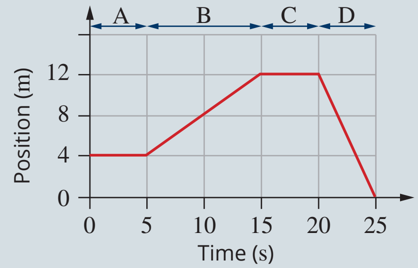
c the displacement of the jet-ski as it travels in its next one-second period of time from $t_i = 1.00$ s to $t_f = 2.00$ s.

13 A skater is travelling south along a horizontal skate rink at a speed of 10.3 m s^{-1}. After falling over, the skater travels in a straight line for 10.6 m before coming to rest. Calculate the answers to the following questions about the skater's movement.

a What is the average acceleration of the skater?

b How long does it take the skater to come to a stop?

14 The graph shows the position of Candice, who is dancing across a stage.



a What is Candice's starting position?

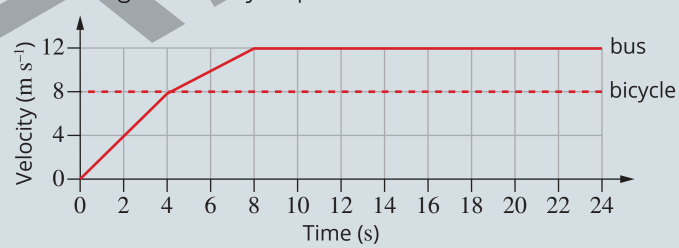
b In which of the sections (A–D) is Candice at rest?

c In which of the sections (A–D) is Candice moving in a positive direction. Determine the velocity during the section by looking at the graph, without using calculations. Explain how you arrived at the answer.

d In which of the sections (A–D) is Candice moving with a negative velocity and what is the magnitude of this velocity?

e Calculate Candice's average speed during the 25.0 s of motion.

15 The velocity–time graphs for a bus and a bicycle travelling along the same straight stretch of road are shown below. The bus is initially at rest and starts moving as the bicycle passes it.



a What is the magnitude of the initial acceleration of the bus?

b At what time does the bus overtake the bicycle? Determine the time by looking at the graph, without using calculations.

c How far has the bicycle travelled before the bus catches it?

d What is the magnitude of the average velocity of the bus during the first 8.0 s?

16 **a** Draw an acceleration–time graph for the bus discussed in Question 15.

b Use your acceleration–time graph to determine the change in velocity of the bus over the first 8.0 s.

17 A slingshot is used to launch a marble vertically into the air at 39.2 m s^{-1}. Discuss the velocity and acceleration of the marble as it travels to its maximum height. Indicate the time that it takes to reach the top. Consider up as positive.

18 A golfer mis-hits a golf ball straight up into the air. Which one of the following statements best describes the acceleration of the ball while it is in the air?

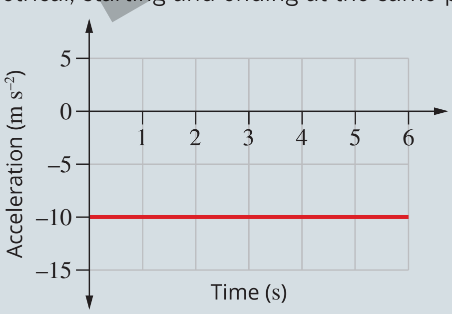
A The acceleration of the ball decreases as it travels upwards, becoming zero at the point in time it reaches its highest point.
B The acceleration is constant as the ball travels upwards, then reverses direction as the ball falls down again.
C The acceleration of the ball is greatest when the ball is at the highest point.
D The acceleration is constant for the entire time the ball is in the air.

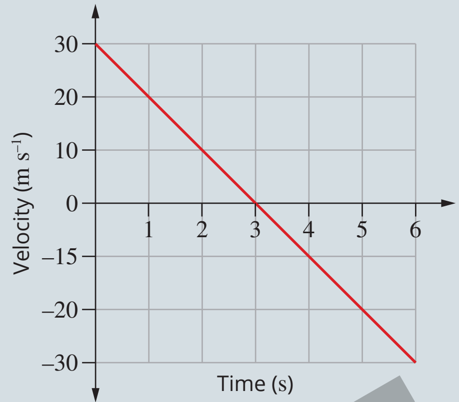
19 Steph tosses a rock vertically into the air. Which of the options below correctly fills the blanks of the following statement about the rock's motion?

On its way upwards, the rock has ______________________ velocity and ____________________ acceleration. At the highest point, the rock has ___________________ velocity and ___________________ acceleration. On its way downwards, the rock has __________________ velocity and ____________________ acceleration.

A upwards; upwards; zero; downwards; downwards; downwards
B upwards; downwards; zero; downwards; downwards; downwards
C upwards; upwards; zero; zero; downwards; downwards
D upwards; downwards; zero; zero; downwards; downwards

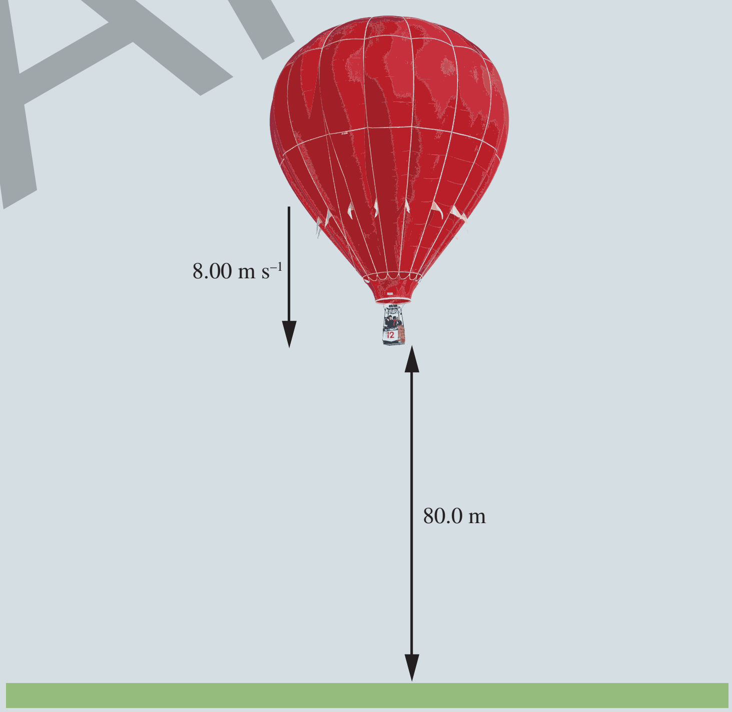
20 After winning a tennis match, Claire hits a tennis ball vertically into the air at $30.0\,m\,s^{-1}$. The *v*–*t* and *a*–*t* graphs for the tennis ball are shown below. Use the graphs or the equations for uniform acceleration to answer the following questions. Use $g = -10.0\,m\,s^{-2}$ for these questions. Assume the motion in question is symmetrical, starting and ending at the same point.





a What is the maximum height reached by the ball?
b What is the time that the ball takes to return to its starting position?
c What is the velocity of the ball 5.0 s after Claire hits it?
d What is the acceleration of the ball at its maximum height?

21 A hot-air balloon is 80.0 m above the ground and travelling vertically downwards at a constant $-8.00\,m\,s^{-1}$ when one of the passengers, Tom, accidentally drops a coin over the side.



a. How long does the balloon take to reach the ground?
b. What is the velocity of the coin as it reaches the ground?
c. How long after the coin reaches the ground does the balloon touch down?

The following information relates to questions 22 and 23.

During a game of minigolf, Renee putts a ball so that it hits an obstacle and rebounds vertically up into the air, reaching its highest point after 1.50 s.

22 What was the initial velocity of the ball the instant it is launched into the air?

23 Calculate the maximum height reached by the ball.

Chapter 3 Linear motion

Section 3.1 Displacement, speed and velocity

Worked example: Try yourself 3.1.1

CALCULATING VELOCITY AND CONVERTING UNITS

Sally is an athlete performing a training routine by running back and forth along a straight stretch of running track. Sally jogs 108.0 m west in a time of 20.0 s, then turns and walks 165.0 m east in a further 45.0 s before stopping.

a Calculate Sally's velocity in $m\,s^{-1}$.	
Thinking	**Working**
Calculate the displacement, remembering that total displacement is the sum of individual displacements. Sally's total journey consists of two displacements: 108.0 m west and 165.0 m east. Take east to be the positive direction.	s = sum of displacements s = 108.0 m west + 165.0 m east $s = (-108.0) + (165.0)$ $s = +57.0$ m or 57.0 m east
Work out the total time taken for the journey.	$\Delta t = (20.0) + (45.0) = 65.0$ s
Substitute the values into the velocity equation.	Displacement, s, is 57.0 m east. Time taken, Δt, is 65.0 s. $v = \frac{s}{\Delta t}$ $v = \frac{(57.0)}{(65.0)}$ $v = 0.87692$ $v = 0.877\,m\,s^{-1}$
Velocity is a vector, so a direction must be given.	$v = 0.877\,m\,s^{-1}$ east

b Calculate the magnitude of Sally's velocity in $km\,h^{-1}$.	
Thinking	**Working**
Convert from $m\,s^{-1}$ to $km\,h^{-1}$ by multiplying by 3.60.	$km\,h^{-1} = m\,s^{-1} \times 3.60$ $v = (0.87692)(3.60)$ $v = 3.1569$ $v = 3.16\,km\,h^{-1}$ east
As the magnitude of the velocity is needed, the direction is not required in this answer.	magnitude of $v = 3.16\,km\,h^{-1}$

c What is Sally's speed in m s^{-1}?	
Thinking	**Working**
Calculate the distance, remembering that distance is the length of the path covered over the entire journey. The direction does not matter. Sally travels 108.0 m in one direction and then 165.0 m in the other direction.	$d = (108.0) + (165.0)$ $d = 273.0\text{ m}$
Work out the total time taken for the journey.	$\Delta t = (20.0) + (45.0) = 65.0\text{ s}$
Substitute the values into the speed equation.	Distance, d, is 273.0 m. Time taken, Δt, is 65.0 s. $v = \frac{d}{\Delta t}$ $v = \frac{(273.0)}{(65.0)}$ $v = 4.2000$ $v = 4.20\text{ m s}^{-1}$

d What is Sally's speed in km h^{-1}?	
Thinking	**Working**
Convert from m s^{-1} to km h^{-1} by multiplying by 3.60.	$\text{km h}^{-1} = \text{m s}^{-1} \times 3.60$ $v = (4.2000)(3.60)$ $v = 15.120$ $v = 15.1\text{ km h}^{-1}$

Section 3.1 **Review**

KEY QUESTIONS SOLUTIONS

1 **a** average speed, $v_{av} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{d}{\Delta t} = \frac{(400.0)}{(2.00)(60)} = \frac{(400.0)}{(120)}$

$v_{av} = 3.3333$

$v_{av} = 3.33\text{ m s}^{-1}$

b average velocity, $v_{av} = \frac{\text{displacement}}{\text{time taken}} = \frac{s}{\Delta t} = \frac{(0.0)}{(120)}$

$v_{av} = 0.00\text{ m s}^{-1}$

Her displacement is zero because the initial (or starting) and the final (or finishing) positions are the same.

2 B and C. The distance travelled is (25.0)(10) = 250.0 m, but the displacement is zero because the swimmer's initial and final positions are the same.

3 **a** displacement = final position – initial position

$s = X_f - X_i$

$s = (40.0) - (0.0)$

$s = +40.0\text{ cm}$

distance travelled, $d = 40.0\text{ cm}$

b displacement = final position − initial position

$s = X_f - X_i$

$s = (40.0) - (50.0)$

$s = -10.0\text{ cm}$

distance travelled, $d = 10.0\text{ cm}$

c displacement = final position – initial position

$s = X_f - X_i$

$s = (70.0) - (50.0)$

$s = +20.0\text{ cm}$

distance travelled, $d = 20\text{ cm}$

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d displacement = final position – initial position

$s = X_f - X_i$

$s = (70.0) - (50.0)$

$s = +20.0\,\text{cm}$

distance covered, $d = (50.0) + (30.0)$

$d = 80.0\,\text{cm}$

4 **a** $d = (50.0) + (30.0) = 80.0\,\text{km}$

b $S = S_1 - S_2$

$s = 50.0\,\text{km north} + 30.0\,\text{km south}$

$s = (50.0) + (-30.0)$

$s = +20.0\,\text{km}$ or $20.0\,\text{km}$ north

5 **a** The basement is 10.0 m downwards or –10.0 m from the ground floor starting position. The displacement can be calculated using the following equation:

$s = X_f - X_i$

$s = (-10.0) - (0.0)$

$s = -10.0\,\text{m}$ or $10.0\,\text{m}$ downwards

b The total displacement from the basement to the top floor is 60.0 m upwards. This can be calculated using the following equation:

$s = X_f - X_i$

$s = (+50.0) - (-10.0)$

$s = +60.0\,\text{m}$ or $60.0\,\text{m}$ upwards

c The total distance travelled is 70.0 m.

$d = (10.0) + (10.0) + (50.0) = 70.0\,\text{m}$

d The top floor is 50.0 m upwards from the starting position on the ground floor. This can be calculated using the following equation:

$s = X_f - X_i$

$s = (50.0) - (0.0)$

$s = +50.0\,\text{m}$ or $50.0\,\text{m}$ upwards

6 **a** average speed, $v_{av} = \dfrac{d}{\Delta t}$

$v_{av} = \dfrac{(400.0)}{(12.0)}$

$v_{av} = 33.333$

$v_{av} = 33.3\,\text{m}\,\text{s}^{-1}$

b The car travelled a distance of 25.0 m. This can be calculated using the following method:

average speed, $v_{av} = \dfrac{d}{\Delta t}$

$d = v_{av}\Delta t$

$d = (33.3)(0.750)$

$d = 25.000$

$d = 25.0\,\text{m}$

7 **a** $90.0\ \text{min} = \dfrac{90.0}{60.0}$

$= 1.5000$

$= 1.50\,\text{h}$

average speed, $v_{av} = \dfrac{d}{\Delta t}$

$v_{av} = \dfrac{(25.0)}{(1.50)}$

$v_{av} = 16.666$

$v_{av} = 16.7\,\text{km}\,\text{h}^{-1}$

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b To convert from $km\,h^{-1}$ to $m\,s^{-1}$, you need to divide by 3.60, so:

$$m\,s^{-1} = \frac{km\,h^{-1}}{3.60}$$

$$v_{av} = \frac{(16.666)}{(3.60)}$$

$$v_{av} = 4.6296$$

$$v_{av} = 4.63\,m\,s^{-1}$$

8 **a** average speed, $v_{av} = \dfrac{d}{\Delta t}$

$$v_{av} = \frac{(9.00)}{(10.0)}$$

$$v_{av} = 0.90000$$

$$v_{av} = 0.900\,m\,s^{-1}$$

b displacement, $S = S_1 - S_2$

$S = (+5.00) + (-4.00)$

$s = +1.00\,m$, or 1.00 m east

average velocity, $v_{av} = \dfrac{s}{\Delta t}$

$$v_{av} = \frac{(+1.00)}{(10.0)}$$

$$v_{av} = +0.10000$$

$v_{av} = +0.100\,m\,s^{-1}$, or $0.100\,m\,s^{-1}$ east

9 **a** average speed, $v_{av} = \dfrac{d}{\Delta t}$, with 15.0 minutes equal to 0.250 hours

$$v_{av} = \frac{(2.50)}{(0.250)}$$

$$v_{av} = 10.000$$

$$v_{av} = 10.0\,km\,h^{-1}$$

b average velocity, $m\,s^{-1} = \dfrac{km\,h^{-1}}{3.60}$

$$v_{av} = \frac{(10.000)}{(3.60)}$$

$$v_{av} = 2.7777$$

$v_{av} = 2.78\,m\,s^{-1}$ south

10 **a** distance travelled, $d = (10.0) + (3.0) + (8.0$ to finish 15.0 km north of the start)

$d = 21.0\,km$

b displacement, $S = S_1 + S_2 + S_3$

$s = (+10.0) + (-3.0) + (+8.0)$

$s = +15.0\,km$, or 15.0 km north

c average speed, $v_{av} = \dfrac{d}{\Delta t}$

$$v_{av} = \frac{(21.0)}{(1.50)}$$

$$v_{av} = 14.000$$

$$v_{av} = 14.0\,km\,h^{-1}$$

d average velocity, $v_{av} = \dfrac{s}{\Delta t}$

$$v_{av} = \frac{(+15.0)}{(1.50)}$$

$$v_{av} = +10.000$$

$v_{av} = +10.0\,km\,h^{-1}$, or $10.0\,km\,h^{-1}$ north

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Section 3.2 Acceleration

Worked example: Try yourself 3.2.1

CHANGE IN SPEED AND VELOCITY 1

A golf ball is dropped onto a wooden floor and strikes the floor at $9.00\,\text{m}\,\text{s}^{-1}$. It then rebounds at $7.00\,\text{m}\,\text{s}^{-1}$.

a Calculate the change in speed of the ball.		
Thinking	**Working**	
Find the values for the initial speed and the final speed of the ball.	$v_i = 9.00\,\text{m}\,\text{s}^{-1}$ $v_f = 7.00\,\text{m}\,\text{s}^{-1}$	
Substitute the values into the change in speed equation: $\Delta v = v_f - v_i$	$\Delta v = v_f - v_i$ $\Delta v = 7.00 - 9.00$ $\Delta v = -2.00\,\text{m}\,\text{s}^{-1}$	Note that speed is a scalar quantity so the negative value indicates a decrease in magnitude, as opposed to a negative direction.

b What is the change in velocity of the ball?	
Thinking	**Working**
Apply the sign convention to replace the directions.	$v_i = 9.00\,\text{m}\,\text{s}^{-1}$ down $v_i = -9.00\,\text{m}\,\text{s}^{-1}$ $v_f = 7.00\,\text{m}\,\text{s}^{-1}$ up $v_f = +7.00\,\text{m}\,\text{s}^{-1}$
As the change in velocity equation is a vector subtraction equation, reverse the direction of v_i to get $-v_i$, then add the two vectors.	$v_i = -9.00\,\text{m}\,\text{s}^{-1}$ $-v_i = +9.00\,\text{m}\,\text{s}^{-1}$
Substitute the values into the vector addition equation: $\Delta v = v_f + (-v_i)$	$\Delta v = v_f + (-v_i)$ $= (+7.00) + (+9.00)$ $= +16.0\,\text{m}\,\text{s}^{-1}$
Apply the sign convention to describe the direction.	$\Delta v = 16.0\,\text{m}\,\text{s}^{-1}$ up

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Worked example: Try yourself 3.2.2

CHANGE IN SPEED AND VELOCITY 2

<table>
<tr><td colspan="2">A netball is dropped vertically onto a court and strikes the surface at 9.00 m s^{-1}. It then rebounds upwards at 7.00 m s^{-1}. The contact time with the court is 35.0 milliseconds.
Calculate the average acceleration of the ball during its contact with the court.</td></tr>
<tr><td>Thinking</td><td>Working</td></tr>
<tr><td>Note the values you will need to find in order to calculate the average acceleration are initial velocity, final velocity and period of time.
Convert 35.0 ms into s by multiplying by 10^{-3}, as the symbol m, for milli, represents 10^{-3}.</td><td>$v_i = -9.00\ \text{m s}^{-1}$
$-v_i = +9.00\ \text{m s}^{-1}$
$v_f = +7.00\ \text{m s}^{-1}$
$\Delta v = v_f - v_i$
$\Delta v = (+7.00) - (+9.00)$
$\Delta v = +16.00\ \text{m s}^{-1}$
$\Delta t = 35.0\ \text{ms}$
$\Delta t = 35.0 \times 10^{-3}$
$\Delta t = 3.50 \times 10^{-2}\ \text{s}$</td></tr>
<tr><td>Substitute the values into the average acceleration equation.</td><td>$a = \frac{\text{change in velocity}}{\text{time taken}}$
$a = \frac{\Delta v}{\Delta t}$
$a = \frac{(+16.00)}{(3.50 \times 10^{-2})}$
$a = +457.143$
$a = +457\ \text{m s}^{-2}$</td></tr>
<tr><td>Acceleration is a vector, so you must include a direction in your answer.</td><td>$a = 457\ \text{m s}^{-2}$ up</td></tr>
</table>

Section 3.2 Review

KEY QUESTIONS SOLUTIONS

1 $\Delta v = v_f - v_i$

$\Delta v = (3.00) - (10.0)$

$\Delta v = -7.00$

So the change in speed is –7.00 km h^{-1}.

Note that speed is a scalar, so the negative value indicates a decrease in magnitude rather than a negative direction.

2 Down is negative, so the initial velocity is –5.00 m s^{-1}.

$\Delta v = v_f - v_i$

$\Delta v = v_f + (-v_i)$

$\Delta v = (0) + (+5.00)$

$\Delta v = +5.00\ \text{m s}^{-1}$ or $5.00\ \text{m s}^{-1}$ up

Note that velocity is a vector, so the vector subtraction equation becomes a vector addition of the opposite of the initial velocity to the final velocity.

3 Down is negative, so the initial velocity is –6.00 m s^{-1}.

$\Delta v = v_f - v_i$

$\Delta v = v_f + (-v_i)$

$\Delta v = (+3.00) + (+6.00)$

$\Delta v = +9.00\ \text{m s}^{-1}$

$\Delta v = 9.00\ \text{m s}^{-1}$ up

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4 $a = \dfrac{\Delta v}{\Delta t}$

$a = \dfrac{v_f - v_i}{t_f - t_i}$

$a = \dfrac{(0) - (7.50)}{(90:00:00.00) - (89:59:58.50)}$

$a = \dfrac{(-7.50)}{(1.50)}$

$a = -5.0000$

$a = 5.00\,\text{m}\,\text{s}^{-2}$ south

5 $a = \dfrac{\text{change in velocity}}{\text{time taken}}$

$a = \dfrac{\Delta v}{\Delta t}$

$a = \dfrac{v_f - v_i}{t_f - t_i}$

$a = \dfrac{(+155) - (0.00)}{(3.50) - (0.00)}$

$a = +44.2857$

$a = 44.3\,\text{m}\,\text{s}^{-2}$ up

6 **a** $\Delta v = v_f - v_i$

$\Delta v = (15.2) - (25.7)$

$\Delta v = -10.500$

$\Delta v = -10.5\,\text{m}\,\text{s}^{-1}$

Note that speed is a scalar, so the negative value indicates a decrease in magnitude as opposed to a negative direction.

b East is positive and west is negative, so the final velocity is $-15.2\,\text{m}\,\text{s}^{-1}$, and the opposite of the initial velocity is $-25.7\,\text{m}\,\text{s}^{-1}$.

$\Delta v = v_f - v_i$

$\Delta v = v_f + (-v_i)$

$\Delta v = (-15.2) + (-25.7)$

$\Delta v = -40.900$

$\Delta v = 40.9\,\text{m}\,\text{s}^{-1}$ west

c $a = \dfrac{\text{change in velocity}}{\text{time taken}}$

$a = \dfrac{\Delta v}{\Delta t}$

$a = \dfrac{(-40.900)}{(0.0535)}$

$a = -764.49$

$a = 764\,\text{m}\,\text{s}^{-2}$ west

7 **a** $\Delta v = v_f - v_i$

$\Delta v = (8.08) - (0.00)$

$\Delta v = 8.08\,\text{m}\,\text{s}^{-1}$

b $\Delta v = v_f - v_i$

$\Delta v = v_f + (-v_i)$

$\Delta v = (-8.08) + (0.00)$

$\Delta v = -8.08\,\text{m}\,\text{s}^{-1}$

$\Delta v = 8.08\,\text{m}\,\text{s}^{-1}$ south

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c $a = \frac{\text{change in velocity}}{\text{time taken}}$

$a = \frac{\Delta v}{t_f - t_i}$

$a = \frac{(-8.08)}{(1.25) - (0.00)}$

$a = -6.46400$

$a = 6.46\,\text{m}\,\text{s}^{-2}$ south

8 $a = \frac{v_f - v_i}{\Delta t}$

$\Delta t = \frac{v_f - v_i}{a}$

$\Delta t = \frac{(30.0) - (10.0)}{(3.00)}$

$\Delta t = 6.6666$

$\Delta t = 6.67\,\text{s}$

9 $a = \frac{v_f - v_i}{\Delta t}$

$\Delta t = \frac{v_f - v_i}{a}$

$\Delta t = \frac{(0.00) - (20.0)}{(-2.50)}$

$\Delta t = 8.0000$

$\Delta t = 8.00\,\text{s}$

10 $a = \frac{v_f - v_i}{\Delta t}$

$v_i = v_f - a\Delta t$

$v_i = (0.00) - (-3.00)(4.00)$

$v_i = +12.0$

$v_i = 12.0\,\text{m}\,\text{s}^{-1}$

Section 3.3 Graphing position, velocity and acceleration over time

Worked example: Try yourself 3.3.1

ANALYSING A POSITION–TIME GRAPH

Use the graph shown in Worked example 3.3.1 to answer the following questions.

a What is the velocity of the cyclist between E and F?	
Thinking	**Working**
Determine the change in position (displacement) of the cyclist between E and F using: s = final position – initial position $s = x_f - x_i$	At E, $x_i = 300.0\,\text{m}$ At F, $x_f = 0.0\,\text{m}$ $s = (0.0) - (300.0)$ $s = -300.0\,\text{m}$ or $300.0\,\text{m}$ backwards (towards the starting point)
Determine the time taken to travel from E to F. $\Delta t = t_f - t_i$	At E, $t_i = 80.0\,\text{s}$ At F, $t_f = 100.0\,\text{s}$ $\Delta t = (100.0) - (80.0)$ $\Delta t = 20.0\,\text{s}$

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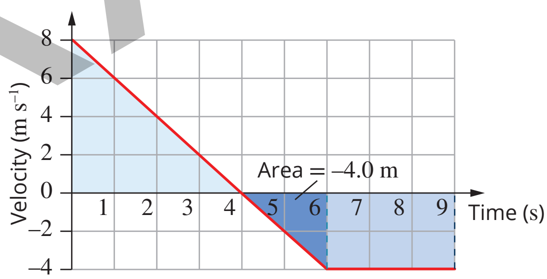
Calculate the gradient of the graph between E and F using: gradient of x–t graph $= \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$ Remember that $\Delta x = s$.	gradient $= \frac{\Delta x}{\Delta t}$ gradient $= \frac{(-300.0)}{(20.0)}$ gradient $= -15.0$
State the velocity, using: gradient of x–t graph = velocity Velocity is a vector quantity, so a direction must be given.	Since the gradient is –15.0, the velocity is $-15.0\,\text{m}\,\text{s}^{-1}$ or $15.0\,\text{m}\,\text{s}^{-1}$ backwards (towards the starting point).

b Describe the motion of the cyclist between D and E.	
Thinking	**Working**
Interpret the shape of the graph between D and E.	The graph is flat between D and E, indicating that the cyclist's position is not changing during this time. So, the cyclist is not moving. If the cyclist is not moving, the velocity is $0.0\,\text{m}\,\text{s}^{-1}$.
You may confirm the result by calculating the gradient of the graph between D and E using: gradient of x–t graph $= \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$ Remember that $\Delta x = s$.	gradient $= \frac{(0.00)}{(20.0)}$ gradient $= 0.00$ Since the gradient is 0.00, the velocity is $0.00\,\text{m}\,\text{s}^{-1}$.

Worked example: Try yourself 3.3.2

ANALYSING A VELOCITY–TIME GRAPH

Use the graph shown in Worked example 3.3.2 to answer the following questions.

a What is the displacement of the car from 4.0 to 6.0 s?	
Thinking	**Working**
Displacement is the area under the graph. You must therefore find the area under the graph for the period of time for which you want to calculate the displacement. As $s = v\Delta t$ or $s = \Delta t \times v$, the base, b, is Δt and height, h, is v. Use $s = b \times h$ for squares and rectangles. Use $s = \frac{1}{2}(b \times h)$ for triangles.	 The area from 4.0 to 6.0 s is a triangle, so: $s = \frac{1}{2}(b \times h)$ $s = \frac{1}{2}(2.0)(-4.0)$ $s = -4.0000$
Displacement is a vector quantity, so a direction is needed.	displacement = 4.0 m west

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b What is the average velocity of the car from 4.0 to 6.0 s?	
Thinking	**Working**
Identify the equation and variables, and apply the sign convention.	$v = \frac{s}{\Delta t}$ $s = -4.0\,\text{m}$ $\Delta t = 2.0\,\text{s}$
Substitute values into the equation: $v = \frac{s}{\Delta t}$	$v = \frac{s}{\Delta t}$ $v = \frac{(-4.0)}{(2.0)}$ $v = -2.0000$
Velocity is a vector quantity, so a direction is needed.	$v = 2.0\,\text{m s}^{-1}$ west

Worked example: Try yourself 3.3.3

FINDING ACCELERATION USING A VELOCITY–TIME GRAPH

Use the graph shown in Worked example 3.3.3 to answer the following question. What is the acceleration of the car during the period from 4.0 to 6.0 s?	
Thinking	**Working**
Acceleration is the gradient of a v–t graph. Calculate the gradient using: $\text{gradient} = \frac{\text{rise}}{\text{run}}$ $a = \text{gradient} = \frac{\Delta v}{\Delta t}$	gradient from 4.0 to 6.0 s = $\frac{\text{rise}}{\text{run}}$ $a = \frac{\Delta v}{\Delta t}$ $a = \frac{v_f - v_i}{t_f - t_i}$ $a = \frac{(-4.0)-(0.0)}{(6.0)-(4.0)}$ $a = -2.0000$ $a = -2.0\,\text{m s}^{-1}$
Acceleration is a vector quantity, so a direction is needed. Note: In this case, the car is moving in the negative direction and speeding up.	$a = 2.0\,\text{m s}^{-2}$ west

Section 3.3 Review

KEY QUESTIONS SOLUTIONS

1 D. The gradient is the displacement over the period of time taken, hence velocity.

2 The car initially moves in a positive direction and travels 8.0 m in 2.0 s. It then stops for 2.0 s. The car then reverses direction for 5.0 s, passing back through its starting point after 8.0 s. It travels a further 2.0 m in a negative direction before stopping after 9.0 s.

3 Reading from the graph:
- **a** +8.0 m
- **b** +8.0 m
- **c** +4.0 m
- **d** −2.0 m

4 The car returns to its starting point when the position is zero again, which occurs at $t = 8.0\,\text{s}$.

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5 **a** The velocity during the first 2.0 s is equal to the gradient of the graph during this interval.

$v = \frac{\text{rise}}{\text{run}} = \frac{x_f - x_i}{\Delta t}$

$v = \frac{(8.0) - (0.0)}{(2.0)}$

$v = +4.0\,\text{m}\,\text{s}^{-1}$

b After 3.0 s the velocity, $v = 0.0\,\text{m}\,\text{s}^{-1}$, since the gradient of the graph = 0.0.

c $v = \frac{\text{rise}}{\text{run}} = \frac{x_f - x_i}{\Delta t}$

$v = \frac{(0.0) - (8.0)}{(4.0)}$

$v = -2.0\,\text{m}\,\text{s}^{-1}$

d The velocity at 8.0 s is $-2.0\,\text{m}\,\text{s}^{-1}$ since the car is travelling at a constant velocity of $-2.0\,\text{m}\,\text{s}^{-1}$ between $t_i = 4.0$ s and $t_f = 9.0$ s.

e The velocity from $t_i = 8.0$ s to $t_f = 9.0\,\text{s} = -2.0\,\text{m}\,\text{s}^{-1}$ since the car is travelling at a constant velocity of $-2.0\,\text{m}\,\text{s}^{-1}$ between 4.0 s and 9.0 s.

6 **a** distance, $d = d_1 + d_2 + d_3$

$d = (8.0) + (8.0) + (2.0) = 18.0\,\text{m}$

b displacement, $s = \Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4$

$s = ((8.0) - (0.0)) + ((8.0) - (8.0)) + ((-2.0) - (8.0)) + ((-2.0) - (-2.0))$

$s = (8.0) + (0.0) + (-10.0) + (0.0)$

$s = -2.0\,\text{m}$

7 **a** average speed = gradient of the line segment

$v = \frac{\Delta x}{\Delta t}$

$v = \frac{(150.0) - (0.0)}{(30.0) - (0.0)}$

$v = +5.00\,\text{m}\,\text{s}^{-1}$

b average velocity = gradient of the line segment plus direction

$v = \frac{\Delta x}{\Delta t}$

$v = \frac{(500.0) - (300.0)}{(50.0) - (40.0)}$

$v = +20.000$

$v = 20.0\,\text{m}\,\text{s}^{-1}$ north

The velocity is positive so the direction of the cyclist is north.

c average velocity = gradient over the whole period of time plus direction

$v = \frac{\Delta x}{\Delta t}$

$v = \frac{(500.0) - (0.0)}{(50.0) - (0.0)}$

$v = +10.000$

$v = 10.0\,\text{m}\,\text{s}^{-1}$ north

8 **a** acceleration at 1.0 s = gradient of section from $t_i = 0.0$ s to $t_f = 2.0$ s

$a = \frac{\Delta v}{\Delta t}$

$a = \frac{(1.0) - (1.0)}{(2.0) - (0.0)}$

$a = 0.0\,\text{m}\,\text{s}^{-2}$

b acceleration at 5.0 s = gradient of section from $t_i = 4.0\,\text{s}$ to $t_f = 7.0\,\text{s}$

$a = \frac{\Delta v}{\Delta t}$

$a = \frac{(0.0) - (3.0)}{(7.0) - (4.0)}$

$a = -1.0000$

$a = -1.0\,\text{m}\,\text{s}^{-2}$

This is magnitude only, so direction is not required; however, the negative acceleration indicates the dog was slowing down.

c Split the area up into shapes and add the values together to get the full area under the graph.

displacement = area under the graph

$s = A_1 + A_2 + A_3$

$s = (b \times h) + \left(\frac{1}{2}(b \times h)\right) + \left(\frac{1}{2}(b \times h)\right)$

$s = (4.0 \times 1.0) + \left(\frac{1}{2}(2.0 \times 2.0)\right) + \left(\frac{1}{2}(3.0 \times 3.0)\right)$

$s = (4.0) + (2.0) + (4.5)$

$s = 10.5\,\text{m}$

This is magnitude only, so direction is not required.

d average velocity, $v = \frac{s}{\Delta t}$

$v = \frac{(10.5)}{(7.0) - (0.0)}$

$v = 1.5000$

$v = 1.5\,\text{m}\,\text{s}^{-1}$

9 **a** instantaneous velocity at 15.0 s = gradient of section from $t_i = 10.0\,\text{s}$ to $t_f = 25.0\,\text{s}$

$v = \frac{\Delta x}{\Delta t}$

$v = \frac{(500.0) - (200.0)}{(25.0) - (10.0)}$

$v = \frac{(300.0)}{(15.0)}$

$v = 20.000$

$v = 20.0\,\text{m}\,\text{s}^{-1}$ north

b instantaneous velocity at 35.0 s = gradient of section from $t_i = 30.0\,\text{s}$ to $t_f = 45.0\,\text{s}$

$v = \frac{\Delta x}{\Delta t}$

$v = \frac{(-100.0) - (500.0)}{(45.0) - (30.0)}$

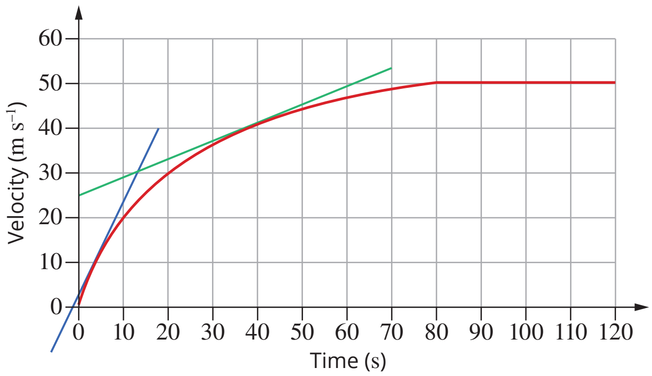
$v = \frac{(-600.0)}{(15.0)}$

$v = -40.000$

$v = 40.0\,\text{m}\,\text{s}^{-1}$ south

10 a Reading from the graph, the gradient of the curve is zero from $t = 80.0\,s$, which means that there is no further increase in velocity.

b Draw a tangent to the graph at $t = 10.0\,s$ and determine the gradient of the tangent.



$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$

$a = \frac{(40.0) - (3.0)}{(18.0) - (0.0)}$

$a = \frac{(37.0)}{(18.0)}$

$a = +2.0555$

$a = +2.0\,m\,s^{-2}$ (answers may vary slightly)

c Draw a tangent to the graph at $t = 40.0\,s$ and determine the gradient of the tangent.

$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$

$a = \frac{(53.0) - (25.0)}{(70.0) - (0.0)}$

$a = \frac{(28.0)}{(70.0)}$

$a = +0.40000$

$a = +0.40\,m\,s^{-2}$ (answers may vary slightly)

d displacement, s = area under the graph

Counting squares gives 49 squares, each of area $10.0\,m\,s^{-1} \times 10.0\,s$.

Therefore each square = $1.00 \times 10^2\,m$

$s = (49)(1.00 \times 10^2)$

$s = 4.9 \times 10^3\,m$

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Section 3.4 Equations for uniform acceleration

Worked example: Try yourself 3.4.1

USING THE EQUATIONS OF MOTION

A snowboarder in a race is travelling 15.5 m s^{-1} east as she crosses the finishing line. She then decelerates uniformly until coming to a stop over a distance of 30.0 m.

a Calculate her acceleration as she comes to a stop.

Thinking	Working
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) Apply the sign convention that east is positive and west is negative.	$s = +30.0$ m $v_i = +15.5$ m s^{-1} $v_f = 0.00$ m s^{-1} $a = ?$ $\Delta t = ?$
Identify the correct equation to use.	$v_f^2 = v_i^2 + 2as$
Substitute known values into the equation and solve for *a*.	$v_f^2 = v_i^2 + 2as$ $a = \frac{v_f^2 - v_i^2}{2s}$ $a = \frac{(0.00)^2 - (+15.5)^2}{2(+30.0)}$ $a = \frac{(-240.25)}{(+60.0)}$ $a = -4.0042$
Use the sign convention to state the answer with its direction, units and the correct number of significant figures.	$a = 4.00$ m s^{-2} west

b How long does she take to come to a stop?

Thinking	Working
Write down the known quantities and the quantity you need to find. (The term 'sifat' may help you to recall them.) Apply the sign convention that east is positive and west is negative.	$s = 30$ m $v_i = 15.5$ m s^{-1} $v_f = 0.00$ m s^{-1} $a = -4.0042$ m s^{-2} $\Delta t = ?$
Identify the correct equation to use. Since you now know four values, any equation involving Δt will work.	$v_f = v_i + a\Delta t$
Substitute known values into the equation and solve for Δt.	$\Delta t = \frac{v_f - v_i}{a}$ $\Delta t = \frac{(0.00) - (+15.5)}{(-4.0042)}$ $\Delta t = 3.87096$ s
State the answer with its units and the correct number of significant figures.	$\Delta t = 3.87$ s

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c What is the average velocity of the snowboarder as she comes to a stop?	
Thinking	**Working**
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) Apply the sign convention that east is positive and west is negative.	$v_i = +15.5\,\text{m s}^{-1}$ $v_f = 0.00\,\text{m s}^{-1}$ $v_{av} = ?$
Identify the correct equation to use.	$v_{av} = \frac{1}{2}(v_f + v_i)$
Substitute known quantities into the equation and solve for v_{av}.	$v_{av} = \frac{1}{2}(v_f + v_i)$ $v_{av} = \frac{1}{2}(0.00 + 15.5)$ $v_{av} = 7.7500$
Use the sign convention to state the answer with its direction, units and the correct number of significant figures.	$v_{av} = 7.75\,\text{m s}^{-1}$ east

Section 3.4 **Review**

KEY QUESTIONS SOLUTIONS

1 E. The chosen equation must contain s, v_i, v_f, and a.

2 **a** $s = 445\,\text{m}$, $v_i = 0.00\,\text{m s}^{-1}$, $v_f = ?$, $a = ?$, $\Delta t = 16.0\,\text{s}$

$s = v_i\Delta t + \frac{1}{2}a\Delta t^2$

$a = \frac{2(s - v_i\Delta t)}{\Delta t^2}$

$a = \frac{2(445 - (0.00)(16.0))}{(16.0)^2}$

$a = \frac{(890)}{(256)}$

$a = 3.47656$

$a = 3.48\,\text{m s}^{-2}$; no direction is required.

b $s = 445\,\text{m}$, $v_i = 0.00\,\text{m s}^{-1}$, $v_f = ?$, $a = 3.47656\,\text{m s}^{-2}$, $\Delta t = 16.0\,\text{s}$

$v_f = v_i = a\Delta t$

$v_f = (0.00) + (3.47656)(16.0)$

$v_f = 55.6250$

$v_f = 55.6\,\text{m s}^{-1}$

c $\text{km h}^{-1} = \text{m s}^{-1} \times 3.60$

$v_f = (55.6250)(3.60)$

$v_f = 200.250$

$v_f = 2.00 \times 10^2\,\text{km h}^{-1}$

3 **a** $s = ?$, $v_i = 0\,\text{m s}^{-1}$, $v_f = 19.9\,\text{m s}^{-1}$, $a = ?$, $\Delta t = 3.10\,\text{s}$

$v_f = v_i = a\Delta t$

$a = \frac{v_f - v_i}{\Delta t}$

$a = \frac{(19.9) - (0.00)}{(3.10)}$

$a = 6.41935$

$a = 6.42\,\text{m s}^{-2}$

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b $v_{av} = \frac{v_i + v_f}{2}$

$v_{av} = \frac{(0.00) + (19.9)}{2}$

$v_{av} = 9.9500$

$v_{av} = 9.95\,\text{m s}^{-1}$

c $s = ?$, $v_i = 0.00\,\text{m s}^{-1}$, $v_f = 19.9\,\text{m s}^{-1}$, $a = 6.41935\,\text{m s}^{-2}$, $\Delta t = 3.10\,\text{s}$

$s = v_i\Delta t + \frac{1}{2}a\Delta t^2$

$s = (0.00)(3.10) + \frac{1}{2}(6.41935)(3.10)^2$

$s = 30.8450$

$s = 30.8\,\text{m}$

4 **a** $v_i = 0.00\,\text{m s}^{-1}$, $v_f = 167\,\text{m s}^{-1}$, $\Delta t = 4.02\,\text{s}$, $a = ?$

$v_f = v_i + a\Delta t$

$a = \frac{v_f - v_i}{\Delta t}$

$a = \frac{(167) - (0.00)}{(4.02)}$

$a = 41.5423$

$a = 41.5\,\text{m s}^{-2}$

b In the first phase: $s = ?$, $v_i = 0.00\,\text{m s}^{-1}$, $v_f = 167\,\text{m s}^{-1}$, $a = 41.5423\,\text{m s}^{-2}$, $\Delta t = 4.02\,\text{s}$

$s = v_i\Delta t + \frac{1}{2}a\Delta t^2$

$s = (0.00)(4.02) + \frac{1}{2}(41.5423)(4.02)^2$

$s = 335.670\,\text{m}$

$s = 0.335670\,\text{km}$

In the last phase: $s = ?$, $v = 167\,\text{m s}^{-1}$, $\Delta t = 5.40\,\text{s}$

$s = v\Delta t$

$s = (167)(5.40)$

$s = 901.800\,\text{m}$

$s = 0.901800\,\text{km}$

Total distance in 9.42 s is:

$s = (0.335670) + (0.901800)$

$s = 1.23747\,\text{km}$

$s = 1.24\,\text{km}$

c $167\,\text{m s}^{-1} \times 3.60 = 601.200 = 601\,\text{km h}^{-1}$

d $v_i = 0.00\,\text{m s}^{-1}$, $v_f = 167\,\text{m s}^{-1}$

$v_{av} = \frac{v_i + v_f}{2}$

$v_{av} = \frac{(0.00) + (167)}{2}$

$v_{av} = 83.500$

$v_{av} = 83.5\,\text{m s}^{-1}$

e $v_{av} = \frac{s}{\Delta t}$

$v_{av} = \frac{(1.23747 \times 10^3)}{(9.42)}$

$v_{av} = 131.366$

$v_{av} = 131\,\text{m s}^{-1}$

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5 **a** $s = ?$, $v_i = 4.12\,\text{m s}^{-1}$, $v_f = 6.07\,\text{m s}^{-1}$, $a = ?\,\text{m s}^{-2}$, $\Delta t = 0.508\,\text{s}$

$a = \frac{v_f - v_i}{\Delta t}$

$a = \frac{(6.07) - (4.12)}{(0.508)}$

$a = 3.83858$

$a = 3.84\,\text{m s}^{-2}$

b $s = ?$, $v_i = 4.12\,\text{m s}^{-1}$, $v_f = 6.07\,\text{m s}^{-1}$, $a = 3.83858\,\text{m s}^{-2}$, $\Delta t = 0.508\,\text{s}$

$s = v_i\Delta t + \frac{1}{2}a\Delta t^2$

$s = (4.12)(0.508) + \frac{1}{2}(3.83858)(0.508)^2$

$s = 2.58826$

$s = 2.59\,\text{m}$

c $v_{av} = \frac{v_i + v_f}{2}$

$v_{av} = \frac{(4.12) + (6.07)}{2}$

$v_{av} = 5.09500$

$v_{av} = 5.10\,\text{m s}^{-1}$

6 **a** $s = -4.06\,\text{m}$, $v_i = -18.0\,\text{m s}^{-1}$, $v_f = 0.00\,\text{m s}^{-1}$, $a = ?\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$v_f^2 = v_i^2 + 2as$

$a = \frac{v_f^2 - v_i^2}{2s}$

$a = \frac{(0.00)^2 - (-18.0)^2}{2(-4.06)}$

$a = +39.9015$

$a = 39.9\,\text{m s}^{-2}$ up

b $s = -4.06\,\text{m}$, $v_i = -18.0\,\text{m s}^{-1}$, $v_f = 0.00\,\text{m s}^{-1}$, $a = +39.9015\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$v_f = v_i + a\Delta t$

$\Delta t = \frac{v_f - v_i}{a}$

$\Delta t = \frac{(0.00) - (-18.0)}{(39.9015)}$

$\Delta t = 0.45111$

$\Delta t = 0.451\,\text{s}$

c $s = -2.00\,\text{m}$, $v_i = -18.0\,\text{m s}^{-1}$, $v_f = ?\,\text{m s}^{-1}$, $a = +39.9015\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$v_f^2 = v_i^2 + 2as$

$v_f = \sqrt{v_i^2 + 2as}$

$v_f = \sqrt{(-18.0)^2 + 2(+39.9015)(-2.00)}$

$v_f = -21.9910$

$v_f = 22.0\,\text{m s}^{-1}$ down

7 **a** $\text{ms}^{-1} = \frac{\text{kmh}^{-1}}{3.60}$

$v_i = \frac{(75.0)}{(3.60)}$

$v_i = 20.8333$

$v_i = 20.8\,\text{m s}^{-1}$

b $s = ?$, $v_i = 20.8333\,\text{m s}^{-1}$, $\Delta t = 0.254\,\text{s}$,

$v_{av} = \frac{s}{\Delta t}$

$s = v_{av}\Delta t$

$s = (20.8333)(0.254)$

$s = 5.29167$

$s = 5.29\,\text{m}$ forwards

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c $s = ?\,\text{m}$, $v_i = 20.8333\,\text{m s}^{-1}$, $v_f = 0.00\,\text{m s}^{-1}$, $a = -6.70\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$v_f^2 = v_i^2 + 2as$

$s = \dfrac{v_f^2 - v_i^2}{2a}$

$s = \dfrac{(0.00)^2 - (20.8333)^2}{2(-6.70)}$

$s = 32.3901$

$s = 32.4\,\text{m}$ forwards

d $s = (5.29167) + (32.3901)$

$s = 37.6818$

$s = 37.7\,\text{m}$

8 **a** $s = 4.00\,\text{m}$, $v_i = 0.00\,\text{m s}^{-1}$, $v_f = ?\,\text{m s}^{-1}$, $a = 2.60\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$v_f^2 = v_i^2 + 2as$

$v_f = \sqrt{v_i^2 + 2as}$

$v_f = \sqrt{(0.00)^2 + 2(2.60)(4.00)}$

$v_f = 4.56070$

$v_f = 4.56\,\text{m s}^{-1}$ down the ramp

b $s = 8.00\,\text{m}$, $v_i = 0.00\,\text{m s}^{-1}$, $v_f = ?\,\text{m s}^{-1}$, $a = 2.60\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$v_f^2 = v_i^2 + 2as$

$v_f = \sqrt{v_i^2 + 2as}$

$v_f = \sqrt{(0.00)^2 + 2(2.60)(8.00)}$

$v_f = 6.44981$

$v_f = 6.45\,\text{m s}^{-1}$ down the ramp

c $s = 4.00\,\text{m}$, $v_i = 0.00\,\text{m s}^{-1}$, $v_f = 4.56070\,\text{m s}^{-1}$, $a = 2.60\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$a = \dfrac{v_f - v_i}{\Delta t}$

$\Delta t = \dfrac{v_f - v_i}{a}$

$\Delta t = \dfrac{(4.56070) - (0.00)}{(2.60)}$

$\Delta t = 1.75412$

$\Delta t = 1.75\,\text{s}$

d $s = 4.00\,\text{m}$, $v_i = 4.56070\,\text{m s}^{-1}$, $v_f = 6.44981\,\text{m s}^{-1}$, $a = 2.60\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$a = \dfrac{v_f - v_i}{\Delta t}$

$\Delta t = \dfrac{v_f - v_i}{a}$

$\Delta t = \dfrac{(6.44981) - (4.56070)}{(2.60)}$

$\Delta t = 0.72658$

$\Delta t = 0.728\,\text{s}$

9 **a** $s = ?\,\text{m}$, $v_i = 0.00\,\text{m s}^{-1}$, $v_f = 12.2\,\text{m s}^{-1}$, $a = 1.50\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$a = \dfrac{v_f - v_i}{\Delta t}$

$\Delta t = \dfrac{v_f - v_i}{a}$

$\Delta t = \dfrac{(12.2) - (0.00)}{(1.50)}$

$\Delta t = 8.13333$

$\Delta t = 8.13\,\text{s}$

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b The bus will catch up with Nolan when they have each travelled the same distance from the point at which Nolan first passes the bus.

Nolan is at a constant velocity, so:

$s = v_{av}\Delta t$

$s = (12.2)\Delta t$

Bus has uniform acceleration: $s = ?\,\text{m}$, $v_i = 0.00\,\text{m s}^{-1}$, $v_f = ?\,\text{m s}^{-1}$, $a = 1.50\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$s = v_i\Delta t + \frac{1}{2}a\Delta t^2$

$s = (0.00)\Delta t + \frac{1}{2}(1.50)\Delta t^2$

$s = (0.750)\Delta t^2$

When the bus catches up with Nolan, their displacements are equal, so:

$(12.2)\Delta t = (0.750)\Delta t^2$

$\Delta t = \frac{(12.2)}{(0.750)}$

$\Delta t = 16.2667$

$\Delta t = 16.3\,\text{s}$

c $s = v_{av}\Delta t$

$s = (12.2)(16.2667)$

$s = 198.453$

$s = 198\,\text{m}$

Section 3.5 **Vertical motion**

Worked example: Try yourself 3.5.1

VERTICAL MOTION

A construction worker accidentally knocks a hammer from a building so that it falls vertically a distance of 60.0 m to the ground. Use $g = -9.80\,\text{m s}^{-2}$ and ignore air resistance when answering these questions.

a How long does the hammer take to fall halfway, to 30.0 m?	
Thinking	**Working**
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) Apply the sign convention that up is positive and down is negative.	$s = -30.0\,\text{m}$ $v_i = 0.00\,\text{m s}^{-1}$ $v_f = ?\,\text{m s}^{-1}$ $g = -9.80\,\text{m s}^{-2}$ $\Delta t = ?\,\text{s}$
Identify the correct equation for uniform acceleration to use, but substitute the *a* for *g*.	$s = v_i\Delta t + \frac{1}{2}g\Delta t^2$
Substitute known values into the equation and solve for *t*. Think about whether the value seems reasonable.	$(-30.0) = (0.00)\Delta t + \frac{1}{2}(-9.80)\Delta t^2$ $(-30.0) = (-4.90)\Delta t^2$ $\Delta t = \sqrt{\frac{(-30.0)}{(-4.90)}}$ $\Delta t = 2.47436$ $\Delta t = 2.47\,\text{s}$

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b How long does it take the hammer to fall all the way to the ground?	
Thinking	**Working**
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) Apply the sign convention that up is positive and down is negative.	$s = -60.0\,m$ $v_i = 0.00\,m\,s^{-1}$ $v_f = ?\,m\,s^{-1}$ $g = -9.80\,m\,s^{-2}$ $\Delta t = ?\,s$
Identify the correct equation for uniform acceleration to use.	$s = v_i\Delta t + \frac{1}{2}g\Delta t^2$
Substitute known values into the equation and solve for t. Think about whether the value seems reasonable. Notice that the hammer takes 2.47 s to travel the first 30.0 m and only 1.03 s to travel the final 30.0 m. This is because it is accelerating.	$(-60.0) = (0.00)\Delta t + \frac{1}{2}(-9.80)\Delta t^2$ $(-60.0) = (-4.90)\Delta t^2$ $\Delta t = \sqrt{\frac{(-60.0)}{(-4.90)}}$ $\Delta t = 3.49927$ $\Delta t = 3.50\,s$

c What is the velocity of the hammer as it hits the ground?	
Thinking	**Working**
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) Apply the sign convention that up is positive and down is negative.	$s = -60.0\,m$ $v_i = 0.00\,m\,s^{-1}$ $v_f = ?\,m\,s^{-1}$ $g = -9.80\,m\,s^{-2}$ $\Delta t = 3.49927\,s$
Identify the correct equation to use. Since you now know four values, any equation involving v_f will work.	$v_f = v_i + g\Delta t$
Substitute the known values into the equation and solve for v_f. Think about whether the value seems reasonable.	$v_f = (0.00) + (-9.80)(3.49927)$ $v_f = -34.2929$
Use the sign and direction convention to describe the direction of the final velocity.	$v_f = 34.3\,m\,s^{-1}$ downwards

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Worked example: Try yourself 3.5.2

MAXIMUM HEIGHT PROBLEMS

On winning a cricket match, a fielder throws a cricket ball vertically into the air at 15.0 m s^{-1}. In the following questions, ignore air resistance and use $g = 9.80\,\text{m s}^{-2}$.

a Determine the maximum height reached by the ball.

Thinking	Working
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) At the maximum height the velocity is zero. Apply the sign convention that up is positive and down is negative.	$s = ?\text{m}$ $v_i = +15.5\text{ms}^{-1}$ $v_f = 0.00\text{ms}^{-1}$ $g = -9.80\text{ms}^{-2}$ $\Delta t = ?$
Identify the correct equation to use, but substitute the *a* for *g*.	$v_f = v_i + g\Delta t$
Substitute known values into the equation and solve for *s*.	$s = \dfrac{v_f^2 - v_i^2}{2g}$ $s = \dfrac{(0.00)^2 - (+15.5)^2}{2(-9.80)}$ $s = +12.2577$ $s = +12.3\,\text{m}$ i.e. the ball reaches a height of 12.3 m above the fielder's hand.

b Calculate the time that the ball takes to return to its starting position.

Thinking	Working
To work out the time the ball is in the air, first calculate the time it takes to reach its maximum height. Write down the known quantities and the quantity that you need to find.	$s = ?\text{m}$ $v_i = +15.5\text{ms}^{-1}$ $v_f = 0.00\text{ms}^{-1}$ $g = -9.80\text{ms}^{-2}$ $\Delta t = ?\text{s}$
Identify the correct equation to use.	$v_f = v_i + g\Delta t$
Substitute known values into the equation and solve for Δt.	$\Delta t = \dfrac{v_f - v_i}{g}$ $\Delta t = \dfrac{(0.00) - (+15.5)}{(-9.80)}$ Δt = 1.58163 to maximum height Δt = 1.58163 to return to the fielder total Δt = 3.16327 total Δt = 3.16 s

Section 3.5 **Review**

KEY QUESTIONS SOLUTIONS

1 The upwards velocity will decrease by –9.80 m s^{-1} for every second that passes until the ball reaches its highest point. At this point in time the velocity is zero, after which the velocity will increase by –9.80 m s^{-1} for every second of its downwards motion until the instant before it hits the hand.

2 B. The acceleration of a falling object is due to gravity, so it is constant.

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3 A and D. Acceleration due to gravity is constant in the downwards direction; however, Yvette's velocity changes throughout the journey and is zero at the top of the flight.

4 **a** $s = ?\,\text{m}$, $v_i = 0.00\,\text{m s}^{-1}$, $v_f = ?\,\text{m s}^{-1}$, $g = -9.80\,\text{m s}^{-2}$, $\Delta t = 3.04\,\text{s}$

$v_f = v_i = a\Delta t$

$v_f = (0.00) + (-9.80)(3.04)$

$v_f = -29.7920$

$v_f = 29.8\,\text{m s}^{-1}$ downwards

b $s = -30.0\,\text{m}$, $v_i = 0.00\,\text{m s}^{-1}$, $v_f = ?\,\text{m s}^{-1}$, $g = -9.80\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$v_f^2 = v_i^2 + 2as$

$v_f = \sqrt{(0.00)^2 + 2(-9.80)(-30.0)}$

$v_f = -24.2487$

$v_f = 24.2\,\text{m s}^{-1}$ downwards

c $v_i = 0.00\,\text{m s}^{-1}$, $v_f = -24.2487\,\text{m s}^{-1}$

$v_{av} = \dfrac{v_i + v_f}{2}$

$v_{av} = \dfrac{(0.00) + (-24.2487)}{2}$

$v_{av} = -12.1244$

$v_{av} = 12.1\,\text{m s}^{-1}$ downwards

5 **a** The acceleration of the marble on the way up is the same as its acceleration on the way down. The acceleration of a falling object is due to gravity and it is constant, no matter the direction of vertical travel (upwards or downwards).

b The magnitude of the launch velocity of the marble is the same as the magnitude of its landing velocity. The flight is symmetrical, so the magnitudes of the starting and landing velocities are the same, but in opposite directions. This only applies if the marble returns to the same vertical position from which it left.

6 **a** $s = ?\,\text{m}$, $v_i = ?\,\text{m s}^{-1}$, $v_f = 0.00\,\text{m s}^{-1}$, $g = -9.80\,\text{m s}^{-2}$, $\Delta t = -1.58\,\text{s}$

$v_f = v_i = a\Delta t$

$v_i = v_f = a\Delta t$

$v_i = (0.00) - (-9.80)(1.58)$

$v_i = +15.4840$

$v_i = 15.5\,\text{m s}^{-1}$ upwards

b $s = ?\,\text{m}$, $v_i + 15.4840\,\text{m s}^{-1}$, $v_f = 0.00\,\text{m s}^{-1}$, $g = -9.80\,\text{m s}^{-2}$, $\Delta t = -1.58\,\text{s}$

$v_f^2 = v_i^2 + 2gs$

$s = \dfrac{v_f^2 - v_i^2}{2g}$

$s = \dfrac{(0.00)^2 - (15.4840)^2}{2(-9.80)}$

$s = +12.2324$

$s = 12.2\,\text{m}$ upwards

7 **a** $s = ?\,\text{m}$, $v_i - 0.00\,\text{m s}^{-1}$, $v_f = ?\,\text{m s}^{-1}$, $g = -9.80\,\text{m s}^{-2}$, $\Delta t = -0.400\,\text{s}$

$v_f = v_i = g\Delta t$

$v_f = (0.00) + (-9.80)(0.400)$

$v_f = -3.92000$

$v_f = 3.92\,\text{m}$ downwards

b $s = ?\,\text{m}$, $v_i = -3.92000\,\text{m s}^{-1}$, $v_f = 0.00\,\text{m s}^{-1}$, $g = -9.80\,\text{m s}^{-2}$, $\Delta t = -0.400\,\text{s}$

$s = v_i\Delta t + \dfrac{1}{2}g\Delta t^2$

$s = (0.00)(0.400) + \dfrac{1}{2}(-9.80)(0.400)^2$

$s = -0.78400$

$s = 0.784\,\text{m}$ downwards

c $s = ?\,\text{m}, v_i = -0.00\,\text{m s}^{-1}, v_f = ?\,\text{m s}^{-1}, g = -9.80\,\text{m s}^{-2}, \Delta t = -0.200\,\text{s}$

$s = v_i\Delta t + \frac{1}{2}g\Delta t^2$

$s = (0.00)(0.200) + \frac{1}{2}(-9.80)(0.200)^2$

$s = -0.19600$

$s = 0.196\,\text{m}$ downwards

d displacement from $t_i = 0.200\,\text{s}$ to $t_f = 0.400\,\text{s}$

$s = s_{0.400} - s_{0.200}$

$s = (-0.78400) - (-0.19600)$

$s = -0.58800$

$s = 0.588\,\text{m}$ downwards

8 **a** The time to the top is half of the total time, i.e. 2.04 s.

b $s = ?\,\text{m}, v_i = ?\,\text{m s}^{-1}, v_f = 0.00\,\text{m s}^{-1}, g = -9.80\,\text{m s}^{-2}, \Delta t = -2.04\,\text{s}$

$v_f = v_i + g\Delta t$

$v_i = v_f - g\Delta t$

$v_i = (0.00) - (-9.80)(2.04)$

$v_i = +19.9920$

$v_f = 20.0\,\text{m s}^{-1}$ upwards

c $s = ?\,\text{m}, v_i = 0.00\,\text{m s}^{-1}, v_f = +19.9920\,\text{m s}^{-1}, g = -9.80\,\text{m s}^{-2}, \Delta t = 2.04\,\text{s}$

$s = v_i\Delta t + \frac{1}{2}g\Delta t^2$

$s = (+19.9920)(2.04) + \frac{1}{2}(-9.80)(2.04)^2$

$s = +20.3918$

$s = 20.4\,\text{m}$ upwards

d The lid returns to its starting position, so the final velocity will be the same as the launch velocity, but in the opposite direction, i.e.

$v_f = -19.9920$

$v_f = 20.0\,\text{m s}^{-1}$ downwards

9 **a** Shot-put: $s = -60.0\,\text{m}, v_i = 0.00\,\text{m s}^{-1}, v_f = ?\,\text{m s}^{-1}, g = -9.80\,\text{m s}^{-2}, \Delta t = ?\,\text{s}$

Note that mass is not a factor in this problem.

$s = v_i\Delta t + \frac{1}{2}g\Delta t^2$

$(-60.0) = (0.00)\Delta t + \frac{1}{2}(-9.80)\Delta t^2$

$\Delta t = \sqrt{\frac{(-60.0)}{(-4.90)}}$

$\Delta t = 3.49927$

$\Delta t = 3.50\,\text{s}$

b 100 g mass: $s = -70.0\,\text{m}, v_i = ?\,\text{m s}^{-1}, v_f = -10.0\,\text{m s}^{-1}, g = -9.80\,\text{m s}^{-2}, \Delta t = ?\,\text{s}$

Note that mass is not a factor in this problem.

$v_f^2 = v_i^2 + 2gs$

$v_f^2 = (-10.0)^2 + 2(-9.80)(-70.0)$

$v_f = \pm\sqrt{1472.00}$

$v_f = -38.3667$

Because the mass has a downwards final velocity, we choose the negative value for its direction.

$v_f = v_i + g\Delta t$

$\Delta t = \frac{v_f - v_i}{g}$

$\Delta t = \frac{(-38.3667) - (-10.0)}{(-9.80)}$

$\Delta t = 2.89456$

$\Delta t = 2.89\,\text{s}$

You can also solve this using the formula $s = v_i\Delta t + \frac{1}{2}g\Delta t^2$ and the quadratic formula.

10 **a** $s = +15.0\,\text{m}$, $v_i = ?\,\text{m s}^{-1}$, $v_f = 0.00\,\text{m s}^{-1}$, $g = -9.80\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$$s = v_f\Delta t - \frac{1}{2}g\Delta t^2$$

$$(+15.0) = (0.00)\Delta t - \frac{1}{2}(-9.80)\Delta t^2$$

$$\Delta t = \sqrt{\frac{(+15.0)}{-(-4.90)}}$$

$$\Delta t = 1.74964$$

$$\Delta t = 1.75\,\text{s}$$

b The displacement of the ball must be −11.0 m below its maximum height of 15.0 m. You need to find how long it takes to travel to this displacement on its way down.

$s = -11.0\,\text{m}$, $v_i = 0.00\,\text{m s}^{-1}$, $v_f = ?\,\text{m s}^{-1}$, $g = -9.80\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$$s = v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$(-11.0) = (0.00)\Delta t + \frac{1}{2}(-9.80)\Delta t^2$$

$$\Delta t = \sqrt{\frac{(-11.0)}{(-4.90)}}$$

$$\Delta t = 1.49830$$

$$\Delta t_{\text{total}} = t_{\text{up}} + t_{\text{down}}$$

$$\Delta t_{\text{total}} = (1.74964) + (1.49830)$$

$$\Delta t_{\text{total}} = 3.24793$$

$$\Delta t_{\text{total}} = 3.25\,\text{s}$$

11 **a** $s = ?\,\text{m}$, $v_i = +8.00\,\text{m s}^{-1}$, $v_f = 0.00\,\text{m s}^{-1}$, $g = -9.80\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$$v_f^2 = v_i^2 + 2gs$$

$$s = \frac{v_f^2 - v_i^2}{2g}$$

$$s = \frac{(0.00)^2 - (+8.00)^2}{2(-9.80)}$$

$$s = +3.26531\,\text{m}$$

$$s_{\text{total}} = s_{\text{hand}} + s_{\text{throw}}$$

$$s_{\text{total}} = (+1.90) + (+3.26531)$$

$$s_{\text{total}} = +5.16531$$

$$s_{\text{total}} = 5.17\,\text{m upwards}$$

b $s = +3.26531\,\text{m}$, $v_i = +8.00\,\text{m s}^{-1}$, $v_f = 0.00\,\text{m s}^{-1}$, $g = -9.80\,\text{m s}^{-2}$, $\Delta t = ?\,\text{s}$

$$v_f = v_i + g\Delta t$$

$$\Delta t = \frac{v_f - v_i}{g}$$

$$\Delta t = \frac{(0.00) - (+8.00)}{(-9.80)}$$

$$\Delta t = 0.81633$$

$$\Delta t = 0.816\,\text{s}$$

c $s = ?\,\text{m}$, $v_i = +8.00\,\text{m s}^{-1}$, $v_f = ?\,\text{m s}^{-1}$, $g = -9.80\,\text{m s}^{-2}$, $\Delta t = 3.00\,\text{s}$

$$s = (+8.00)(3.00) + \frac{1}{2}(-9.80)(3.00)^2$$

$$s_{\text{hand-sea}} = -20.1000$$

$$s_{\text{cliff-sea}} = s_{\text{hand-sea}} - s_{\text{hand-cliff}}$$

$$s_{\text{cliff-sea}} = (-20.1000) - (-1.90)$$

$$s_{\text{cliff-sea}} = -18.2000$$

$$s_{\text{sea-cliff}} = +18.200$$

The height of cliff is 18.2 m above the sea.

CHAPTER 3 REVIEW

1 $\text{m s}^{-1} = \dfrac{\text{km h}^{-1}}{3.60}$

$v = \dfrac{(95.0)}{(3.60)} = 26.3888$

$v = 26.4\,\text{m s}^{-1}$

2 $\text{km h}^{-1} = \text{m s}^{-1} \times 3.60$

$v = (15.3)(3.60) = 55.0800$

$v = 55.1\,\text{km h}^{-1}$

3 $v_{av} = \dfrac{d}{\Delta t}$

$v_{av} = \dfrac{(15.4) + (5.7) + (10.5)}{(3.00)}$

$v_{av} = 10.53333$

$v_{av} = 10.5\,\text{km h}^{-1}$

4 **a** $v_{av} = \dfrac{s}{\Delta t}$

$v_{av} = \dfrac{(20.2)}{(3.00)}$

$v_{av} = 6.73333$

$v_{av} = 6.73\,\text{km h}^{-1}$ north

b $\text{m s}^{-1} = \dfrac{\text{km h}^{-1}}{3.60}$

$v = \dfrac{(6.73333)}{(3.60)} = 1.87037$

$v = 1.87\,\text{m s}^{-1}$ north

5 $\Delta v = v_f - v_i$

$\Delta v = (4.50) - (6.00)$

$\Delta v = -1.50\,\text{m s}^{-1}$

The change in speed is $-1.50\,\text{m s}^{-1}$. That is, it has decreased by $1.50\,\text{m s}^{-1}$. Speed is a scalar quantity and has no direction.

6 B. The car is moving in a positive direction, so its velocity is positive. The car is slowing down so its acceleration is negative.

7 $s = ?\,\text{m}$, $v_i = 15.6\,\text{m s}^{-1}$, $v_f = 0.00\,\text{m s}^{-1}$, $g = ?\,\text{m s}^{-2}$, $\Delta t = 2.55\,\text{s}$

$a = \dfrac{v_f - v_i}{\Delta t}$

$a = \dfrac{(0.00) - (15.6)}{(2.55)}$

$a = -6.11765$

$a = -6.12\,\text{m s}^{-2}$

8 **a** The only positive gradient section is from $t_i = 10.0$ to $t_f = 25.0\,\text{s}$.

b The only negative gradient section is from $t_i = 30.0$ to $t_f = 45\,\text{s}$.

c The motorbike is stationary when the sections on the position–time graph are horizontal. The horizontal sections are from $t_i = 0.00$ to $t_f = 10.0\,\text{s}$, from $t_i = 25.0$ to $t_f = 30.0\,\text{s}$, and from $t_i = 45.0$ to $t_f = 60.0\,\text{s}$.

d The zero position is at $t = 42.5\,\text{s}$.

9 **a** Graph **B** is the correct answer as it shows speed decreasing to zero to show the car stopping.

b Graph **A** is the correct graph because it shows a constant value for speed. This is indicated by a straight horizontal line on a velocity–time graph.

c Graph **C** is the correct graph because it shows velocity increasing from zero in a straight line, indicating uniform acceleration.

10 a Displacement is the area under a velocity–time graph. Area can be estimated by counting squares under the graph, then multiplying by the area of each square. This gives approximately:

$s = 57 \text{ squares} \times (2.0\,\text{m s}^{-1} \times 1.0\,\text{s})$

$s = 114.000$

$s = 1.1 \times 10^2\,\text{m north}$

Note: Estimations should be expressed to either one or two significant figures.

b $v = \dfrac{s}{\Delta t}$

$v = \dfrac{(114.000)}{(11.3)}$

$v = 10.0885$

$v = 1.0 \times 10^1\,\text{m s}^{-1}$ north

c Acceleration is the gradient of the graph. At $t = 1.0\,\text{s}$, the gradient is flat and therefore zero.

d Acceleration at $t = 10.0\,\text{s}$ is:

$a = \text{gradient} = \dfrac{\text{rise}}{\text{run}} = \dfrac{v_f - v_i}{t_f - t_i}$

$a = \dfrac{(0.0) - (14.0)}{(11.3) - (9.3)}$

$a = -7.0000$

$a = 7.0\,\text{m s}^{-2}$ south

e A. The velocity is always positive (or zero), indicating that the cyclist only travelled in the positive direction or was stationary.

11 $s = ?\ \text{m},\ v_i = 0.00\,\text{m s}^{-1},\ v_f = ?\ \text{m s}^{-1},\ a = -3.59\,\text{m s}^{-2},\ \Delta t = 4.51\,\text{s}$

$v_f = v_i + a\Delta t$

$v_f = (0.00) + (-3.59)(4.51)$

$v_f = -16.1909$

$v_f = 16.2\,\text{m s}^{-1}$ west

12 a $s = +2.80\,\text{m},\ v_i = 0.00\,\text{m s}^{-1},\ v_f = ?\ \text{m s}^{-1},\ a = ?\ \text{m s}^{-2},\ \Delta t = 1.00\,\text{s}$

$s = v_i\Delta t + \dfrac{1}{2}a\Delta t^2$

$(+2.80) = (0.00)(1.00) + \dfrac{1}{2}a(1.00)^2$

$a = 2(+2.80)$

$a = +5.6000$

$a = 5.60\,\text{m s}^{-2}$ east

b $s = +2.80\,\text{m},\ v_i = 0.00\,\text{m s}^{-1},\ v_f = ?\,\text{m s}^{-1},\ a = +5.6000\,\text{m s}^{-2},\ \Delta t = 1.00\,\text{s}$

$v_f = (0.00) + (+5.6000)(1.00)$

$v_f = +5.6000$

$v_f = 5.60\,\text{m s}^{-1}$ east

c $s = ?\,m,\ v_i = +5.6000\,\text{m s}^{-1},\ v_f = ?\,\text{m s}^{-1},\ a = +5.6000\,\text{m s}^{-2},\ \Delta t = 1.00\,\text{s}$

$s = v_i\Delta t + \dfrac{1}{2}a\Delta t^2$

$s = (+5.6000)(1.00) + \dfrac{1}{2}(5.6000)(1.00)^2$

$s = 8.4000$

$s = 8.40$ m east

13 a $s = -10.6\,\text{m},\ v_i = -10.3\,\text{m s}^{-1},\ v_f = 0.00\,\text{m s}^{-1},\ a = ?\,\text{m s}^{-2},\ \Delta t = ?\,\text{s}$

$v_f^2 = v_i^2 + 2as$

$a = \dfrac{v_f^2 - v_i^2}{2s}$

$a = \dfrac{(0.00)^2 - (-10.3)^2}{2(-10.6)}$

$a = +5.00425$

$a = 5.00\,\text{m s}^{-2}$ north

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b $s = -10.6\text{ m},\ v_i = -10.3\text{ m s}^{-1},\ v_f = 0.00\text{ m s}^{-1},\ a = 5.00425\text{ m s}^{-2},\ \Delta t = ?\text{ s}$

$$v_f = v_i + a\Delta t$$

$$\Delta t = \frac{v_f - v_i}{a}$$

$$\Delta t = \frac{(0.00) - (-10.6)}{(5.00425)}$$

$$\Delta t = 3.19729$$

$$\Delta t = 3.20\text{ s}$$

14 **a** She starts at $x_i = 4.0\text{ m}$.

b She is at rest during section **A** and **C**.

c She is moving in a positive direction during section **B** with a velocity of 0.80 m s^{-1}. In the 10.0 second time interval, her position changed by +8.0 m. This means a +0.80 metre increase in position per second or 0.80 m s^{-1}.

d She is moving in the negative direction during section **D** and is travelling at 2.4 m s^{-1}.

e $d = (8.0 + 12.0)\text{ m},\ v = ?\text{ m s}^{-1},\ \Delta t = 25.0\text{ s}$

$$v = \frac{d}{\Delta t}$$

$$v = \frac{(20.0)}{(25.0)}$$

$$v = 0.80000$$

$$v = 0.80\text{ m s}^{-1}$$

15 **a** $a = \text{gradient} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$

$$a = \frac{(8.0) - (0.0)}{(4.0) - (0.0)}$$

$$a = 2.0000$$

$$a = 2.0\text{ m s}^{-2}$$

b The bus will overtake the bike when they have both achieved the same displacement, given by the areas under the two graphs. The bike gains a displacement advantage of two squares while the bus is accelerating. The bus makes up for that two-square advantage by gaining back one square between $t = 4.0$ and $t = 8.0$ seconds, then another square between $t = 8.0$ and $t = 10.0$ seconds. After 10.0 s, the bus and the bike have the same number of squares and so they have the same displacement from the origin.

c $s = \text{area} = (b \times h)$

$$s = (10.0)(8.0)$$

$$s = 80.0000$$

$$s = 8.0 \times 10^1\text{ m}$$

d Determine the total displacement by dividing the area under the graph into rectangles and triangles.

$$s = \frac{1}{2}(b \times h) + (b \times h) + \frac{1}{2}(b \times h)$$

$$s = \frac{1}{2}(4.0 \times 8.0) + (4.0 \times 8.0) + \frac{1}{2}(4.0 \times 4.0)$$

$$s = 56.0000\text{ m}$$

$$v = \frac{s}{\Delta t}$$

$$v = \frac{(56.0000)}{(8.0)}$$

$$v = 7.0000$$

$$v = 7.0\text{ m s}^{-1}$$

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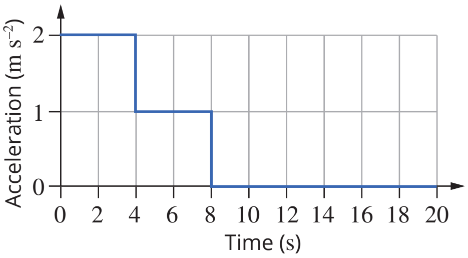
16 a



b The change in velocity of the bus over the first 8.0 s is determined by calculating the area under the acceleration–time graph from $t = 0.0$ to $t = 8.0$ s.

$\Delta v = (b \times h) + (b \times h)$

$\Delta v = (4.0 \times 2.0) + (4.0 \times 1.0)$

$\Delta v = 12.0000$

$\Delta v = 12\,\text{m s}^{-1}$

17 The marble has a positive initial velocity that changes to a final velocity of zero at the highest point. It slows down by $-9.80\,\text{m s}^{-1}$ each second, so it will take 4.00 s to reach $0.00\,\text{m s}^{-1}$ at the instant in time it reaches the top of its journey. Its acceleration is constant at $-9.80\,\text{m s}^{-2}$ due to gravity.

18 D. The acceleration of a falling object is due to gravity, so it is constant.

19 B. Initial velocity is upwards, zero at the highest point and downwards on the way back down. Acceleration due to gravity is constant, never zero and is always downwards.

20 a The area under the v–t graph up to 3.0 s gives:

$s = \frac{1}{2}(b \times h)$

$s = \frac{1}{2}(3.0)(30.0)$

$s = 45.0000$

$s = 45\,\text{m}$

or

$s = \frac{(v_i + v_f)}{2}\Delta t$

$s = \frac{(30.0 + 0.0)}{2}(3.0)$

$s = 45.0000$

$s = 45\,\text{m}$

b From the graph, the ball goes up for 3.0 s, then down for 3.0 s, giving a total period of time, $\Delta t = 6.0$ s or:

$s = ?\text{m},\ v_i = +30.0\,\text{m s}^{-1},\ v_f = -30.0\,\text{m s}^{-1},\ g = -10.0\,\text{m s}^{-2},\ \Delta t = 6.0\,\text{s}$

$v_f = v_i + g\Delta t$

$\Delta t = \frac{v_f - v_i}{g}$

$\Delta t = \frac{(-30.0) - (+30.0)}{(-10.0)}$

$\Delta t = 6.0000$

$\Delta t = 6.0\,\text{s}$

c From the v–t graph, the velocity at $t = 5.0\,\text{s}$ is –20.0 or $20.0\,\text{m s}^{-1}$ down

or:

$s = ?\,\text{m},\ v_i = +30.0\,\text{m s}^{-1},\ v_f = ?\,\text{m s}^{-1},\ g = -10.0\,\text{m s}^{-2},\ \Delta t = 5.0\,\text{s}$

$v_f = v_i + g\Delta t$

$v_f = (+30.0) + (-10.0)(5.0)$

$v_f = -20.0000$

$v_f = 20.0\,\text{m s}^{-1}$ down

d Acceleration is always $-10.0\,\text{m s}^{-2}$ or $10.0\,\text{m s}^{-2}$ down.

21 a Balloon: $d = 80.0\,\text{m},\ v = -8.00\,\text{m s}^{-1},\ \Delta t = ?\,\text{s}$

$v = \dfrac{d}{\Delta t}$

$v = \dfrac{(80.0)}{(8.00)}$

$v = 10.0000$

$v = 10.0\,\text{s}$

b Coin: $s = -80.0\,\text{m},\ v_i = -8.00\,\text{m s}^{-1},\ v_f = ?\,\text{m s}^{-1},\ g = -9.80\,\text{m s}^{-2},\ \Delta t = ?\,\text{s}$

$v_f^2 = v_i^2 + 2gs$

$v_f = \pm\sqrt{(-8.00)^2 + 2(-9.80)(-80.0)}$

$v_f = -38.7814$

$v_f = 38.8\,\text{m s}^{-1}$ down

c Coin: $s = -80.0\,\text{m},\ v_i = -8.00\,\text{m s}^{-1},\ v_f = -38.7814\,\text{m s}^{-1},\ g = -9.80\,\text{m s}^{-2},\ \Delta t = ?\,\text{s}$

$v_f = v_i + g\Delta t$

$\Delta t = \dfrac{v_f - v_i}{g}$

$\Delta t = \dfrac{(-38.7814) - (-8.00)}{(-9.80)}$

$\Delta t = 3.14096$

$\Delta t = 3.14\,\text{s}$

Difference in time $\Delta t = t_{\text{balloon}} - t_{\text{coin}}$

$\Delta t = (10.0000) - (3.14096)$

$\Delta t = 6.85904$

$\Delta t = 6.86\,\text{s}$

22 $s = ?\,\text{m},\ v_i = ?\,\text{m s}^{-1},\ v_f = 0.00\,\text{m s}^{-1},\ g = -9.80\,\text{m s}^{-2},\ \Delta t = 1.50\,\text{s}$

$v_f = v_i + g\Delta t$

$v_i = v_f - g\Delta t$

$v_i = (0.00) - (-9.80)(1.50)$

$v_f = +14.7000$

$v_f = 14.7\,\text{m s}^{-1}$ up

23 $s = ?\,\text{m},\ v_i = +14.7000\,\text{m s}^{-1},\ v_f = 0.00\,\text{m s}^{-1},\ g = -9.80\,\text{m s}^{-2},\ \Delta t = 1.50\,\text{s}$

$v_f^2 = v_i^2 + 2gs$

$s = \dfrac{v_f^2 - v_i^2}{2g}$

$s = \dfrac{(0.00)^2 - (+14.7000)^2}{2(-9.80)}$

$s = +11.0250$

$s = 11.0\,\text{m}$ up