1

Whole numbers

1

It's a Santa stampede! Another Guinness World Record, this time for 'the largest gathering of Santa Claus'. (18 112 of them!)

For decades, *Guinness World Records* has been recording extreme and bizarre feats, including 'the most cockroaches eaten in one minute' (36), and 'the most consecutive skateboard frontside ollies off a half-pipe ramp' (348). The book itself holds the record for being the best-selling copyrighted series of all time. People are fascinated to know the most, the biggest, the fastest and the tallest.

To keep records, you need a system of numbers to count, order, measure and calculate with. People also need numbers for our personal lives: how much money we spend, save and earn; how far and how often we travel; and how much of the Earth's resources we use. Every day, people need to find totals, work out differences, divide quantities, and make estimates. To live in this world, you need to be able to work with numbers!

Forum

Is it important to keep 'world records'? If so, then what kinds of of achievements should records be kept for?

What other kinds of records are good to keep?

Why learn this?

You are buying two $18 T-shirts and a $65 pair of jeans. Will $100 be enough? If you swim 12 laps of a 50 metre pool 4 times a week, how many metres per week is that? Working with numbers means more than just being able to add, subtract, multiply and divide. It means being able to choose which skill to use, to estimate and round numbers, and to have a range of mental skills so that you can work things out efficiently and accurately.

After completing this chapter you will be able to:

- choose and use a range of mental strategies for calculations
- understand how the properties of numbers can be used to calculate efficiently
- interpret and work with numbers in index form
- estimate answers to problems using estimating and rounding strategies
- apply the order of operations
- solve problems involving whole numbers.

Curriculum links

Australian CURRICULUM | VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Number and Algebra

Number and place value

ACMNA149/VCMNA238
Investigate index notation and represent whole numbers as products of powers of prime numbers

- defining and comparing prime and composite numbers and explaining the difference between them
- applying knowledge of factors to strategies for expressing whole numbers as products of powers of prime factors, such as repeated division by prime factors or creating factor trees
- solving problems involving lowest common multiples and greatest common divisors (highest common facors) for pairs of whole numbers by comparing their prime factorisation

ACMNA150/VCMNA239
Investigate and use square roots of perfect square numbers

- investigating square numbers such as 25 and 36 and developing square-root notation
- investigating between which two whole numbers a square root lies

ACMNA151
Apply the associative, commutative and distributive laws to aid mental and written computation

- understanding that arithmetic laws are powerful ways of describing and simplifying calculations

VCMNA240
Apply the associative, commutative and distributive laws to aid mental and written computation and make estimates for these computations

- understanding that arithmetic laws are powerful ways of describing and simplifying calculations
- simplifying calculations
- forming simple estimates for calculations involving multiple and/or combined operations

ACMNA280
Compare, order, add and subtract integers

VCMNA241
Compare, order, add and subtract integers

- using a variety of models to represent, add and subtract integers

Real number

ACMNA156/VCMNA246
Round decimals to a specified number of decimal places

- using rounding to estimate the results of calculations with whole numbers and decimals, and understanding the conventions for rounding

General capabilities

- Literacy
- Numeracy
- Critical and creative thinking
- Ethical understanding
- Personal and social capability
- Intercultural understanding

Big ideas

- Order of operations are an agreed upon set of rules that produce a correct result from a set of calculations.
- Strategies that apply the laws of arithmetic provide efficient methods of performing mental and written calculations.
- Estimation allows us to check the reasonableness of our answers.

notes:

Recall 1

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 What is the place value of the red digit shown?

(a) 45 783

A eight **B** tens **C** eighty-three **D** hundreds

(b) 1 264 184

A two **B** hundreds **C** ten-thousands **D** hundred-thousands

2 (a) Rearrange the following numbers in ascending order (from smallest to largest).
567, 4500, 0, 74, 11 100, 6008, 12, 602

(b) Rearrange the following numbers in descending order (from largest to smallest).
1200, 204, 987, 2196, 240, 95, 2400, 1010

3 Find:

(a) 50 000 + 6000 + 800 + 90 + 5 **(b)** 7 000 000 + 20 000 + 5000 + 70 + 3

4 (a) Round 1245 to the nearest:

(i) 10 **(ii)** 100 **(iii)** 1000

(b) Round 8983 to the nearest:

(i) 10 **(ii)** 100 **(iii)** 1000

5 Find:

(a) $3 \times 2 \times 3$ **(b)** $5 \times 3 \times 3 \times 2$ **(c)** $2 \times 2 \times 2$ **(d)** $10 \times 10 \times 10 \times 10$

6 Set out these calculations in your preferred way and work out the answers.

(a) 456 + 56 **(b)** 16 + 2047 **(c)** 90 + 1267 + 341

7 Set out these calculations in your preferred way and work out the answers.

(a) 298 – 123 **(b)** 854 – 227 **(c)** 1406 – 249

8 Set out these calculations in your preferred way and work out the answers.

(a) 45×7 **(b)** 134×5 **(c)** 34×95

9 Set out these calculations in your preferred way and work out the answers.

(a) 844 ÷ 4 **(b)** 3708 ÷ 9 **(c)** 897 ÷ 7

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

The root of the problem

In this activity, you will explore the relationship between square roots and the geometric properties of squares.

Mental strategies 1.1

This section looks at some strategies that are useful for doing calculations in your head. Try writing them out first, before you do them only in your head. You might find some methods easier than others, or you might have different strategies of your own. That's okay—people are different in the way they like to approach things.

Strategy 1—Make easy numbers

This strategy uses these two properties of numbers:

1 The order in which you add or multiply any two numbers does not change the result.

For example: $2 \times 3 = 6$ and $3 \times 2 = 6$
$4 + 5 = 9$ and $5 + 4 = 9$

This is known as the **commutative law**.

2 The order in which three or more numbers are added, or the order in which they are multiplied, also does not change the result.

In the following examples, brackets are used to show which pair of numbers is added or multiplied first.

Multiplication:

$(2 \times 3) \times 5$ and $2 \times (3 \times 5)$
$= 6 \times 5$ $= 2 \times 15$
$= 30$ $= 30$

Addition:

$(6 + 7) + 8$ and $6 + (7 + 8)$
$= 13 + 8$ $= 6 + 15$
$= 21$ $= 21$

This is known as the **associative law**.

It is important to understand that the commutative and associative laws do *not* apply to subtraction and division, which both need to be worked from left to right.

For example: $(9 - 5) - 4$ but $9 - (5 - 4)$
$= 4 - 4$ $= 9 - 1$
$= 0$ $= 8$

$(40 \div 10) \div 2$ but $40 \div (10 \div 2)$
$= 4 \div 2$ $= 40 \div 5$
$= 2$ $= 8$

The order in which you add numbers and the order in which you multiply numbers does not change the result. The order only changes the result for subtraction or division.

You can use the commutative and associative laws to 'shuffle' or rearrange the calculation to create multiples of 10, which are easier to add and multiply.

For example:

$7 + 9 + 3$ is the same as: $3 + 7 + 9$
$= 10 + 9$

$5 \times 37 \times 2$ is the same as: $5 \times 2 \times 37$
$= 10 \times 37$

You can also split one number into two parts, then use the associative law to add one part to the other number to create a multiple of 10.

Resources

Recall Worksheets

- R1.1: Place value
- R1.2: Rearranging numbers in order
- R1.3: Addition with place value
- R1.4: Rounding to the nearest 10, 100 and 1000
- R1.5: Multiplying with more than two numbers
- R1.6: Simple addition
- R1.7: Simple subtraction
- R1.8: Simple multiplication
- R1.9: Simple division

Exploration Task

- The root of the problem

Videos

- Numbers 101

Lightbook Starter

- Before you begin 1

Answers

Recall 1

1 (a) B 8 is in the tens position, hence $8 \times 10 = 80$.

(b) D 2 is in the hundred thousands position, hence $2 \times 100\,000 = 200\,000$.

2 (a) 0, 12, 74, 567, 602, 4500, 6008, 11 100

(b) 2400, 2196, 1200, 1010, 987, 240, 204, 95

3 (a) 56 895

(b) 7 025 073

4 (a) (i) 1250 (ii) 1200

(iii) 1000

(b) (i) 8980 (ii) 9000

(iii) 9000

5 (a) 18 (b) 90

(c) 8 (d) 10 000

6 (a) 512 (b) 2063

(c) 1698

7 (a) 175 (b) 627

(c) 1157

8 (a) 315 (b) 670

(c) 3230

9 (a) 211 (b) 412

(c) 128 rem 1

1.1

Resources

eWorked examples

- Mental maths strategies
- Using the distributive property

Interactives

- The distributive law

Tutorials and quizzes

- Mental methods

Lightbook Starter

- Check-in 1.1

Appendices

- 1A 12 × 12 Multiplication grid

Recap

Question	Answer
1 What is the place value of the digit 3 in the number 3456?	3 thousands
2 $7 \times \square = 56$.	8
3 Find $4000 + 600 + 1$.	4601
4 Calculate $345 + 129$.	474
5 Calculate $966 \div 3$.	322

Suggested examples

1 **Calculate the following using the 'make easy numbers' strategy:**

(a) $8 + 23 + 22$

(b) $5 \times 17 \times 20$

(c) $657 + 36$

Answer:

(a) $8 + 23 + 22$
$= 30 + 23$
$= 53$

(b) $5 \times 17 \times 20$
$= 5 \times 20 \times 17$
$= 100 \times 17$
$= 1700$

(c) $657 + 36$
$= 657 + 3 + 33$
$= 660 + 33$
$= 693$

2 **Calculate the following using the distributive property.**

(a) 3×53

(b) 17×9

Answer:

(a) 3×53
$= 3 \times (50 + 3)$
$= 3 \times 50 + 3 \times 3$
$= 150 + 9$
$= 159$

(b) 17×9
$= 17 \times (10 - 1)$
$= 17 \times 10 - 17 \times 1$
$= 170 - 17$
$= 153$

For example: 135 + 46
= 135 + 5 + 41 (splitting 46 into 5 and 41)
= 140 + 41 (adding 5 to 135)

Worked example 1 W.E.1

Calculate the following using the 'make easy numbers' strategy.

(a) 7 + 32 + 13 **(b)** 2 × 13 × 5 **(c)** 293 + 568

Thinking		Working
(a) 1	Rearrange the addition to form 'easy' numbers, such as multiples of 10.	*(a)* 7 + 32 + 13 = 7 + 13 + 32
2	Perform these calculations first.	= 20 + 32
3	Complete the question.	= 52
(b) 1	Rearrange the multiplication to form 'easy' numbers, such as multiples of 10.	*(b)* 2 × 13 × 5 = 2 × 5 × 13
2	Perform these calculations first.	= 10 × 13
3	Complete the question.	= 130
(c) 1	Split one number into two parts, then add one part to the other number to create an 'easy' number. (Here, we have split 7 away from 568 and added it to 293.)	*(c)* 293 + 568 = 293 + 7 + 561 = 300 + 561
2	Perform the calculation to complete the question.	= 861

Strategy 2—Use the distributive law

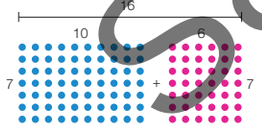
This strategy uses a property of numbers called the **distributive law**. The distributive law lets you multiply a large number by splitting it up into 10s and 1s (or 100s, 10s and 1s), multiplying each part separately, then adding or subtracting each of the products.

(When two numbers are multiplied together, the result is called the **product**.)

For example, you can split 7 × 16 into 7 lots of 10 plus 7 lots of 6.

You can represent the multiplication in an array diagram:

7 × 16
= 7 × (10 + 6) (because 16 = 10 + 6)
= 7 × 10 + 7 × 6
= 70 + 42
= 112

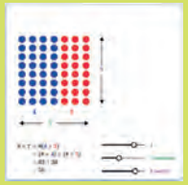


Interactive

The distributive law

Explore visually how the distributive law works with changing numbers.

Go to the eBook or the Pearson Places website to access this interactive.

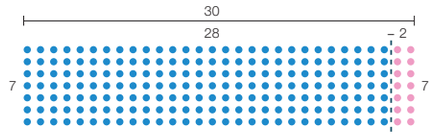


The distributive law means that 7 × 16 = 7 × 10 + 7 × 6.

In a similar way, you can write 19 as 20 – 1, or 28 as 30 – 2. For example, you can split 7 × 28 into 7 lots of 30 minus 7 lots of 2.

You can represent this multiplication as an array diagram (the number subtracted is on the right of the dotted line).

7 × 28
= 7 × (30 – 2) (because 28 = 30 – 2)
= 7 × 30 – 7 × 2
= 210 – 14
= 196



To multiply a large number, split it up into 10s and 1s (or 100s, 10s and 1s). Multiply by these separately, then add or subtract each of the products.

Worked example 2 W.E.2

Evaluate the following using the distributive law.

(a) 7 × 22 **(b)** 15 × 9

Thinking		Working
(a) 1	Split the number you are multiplying by into 10s and 1s, writing it in brackets.	*(a)* 7 × 22 = 7 × (20 + 2)
2	Multiply the 10s and the 1s by the number in front of the brackets.	= 7 × 20 + 7 × 2 = 140 + 14
3	Add the two products together.	= 154
(b) 1	Round one of the numbers to the nearest multiple of 10, then write it as a subtraction in brackets. Here, you write 9 as (10 – 1).	*(b)* 15 × 9 = 15 × (10 – 1)
2	Multiply the numbers inside the brackets by the number in front.	= 15 × 10 – 15 × 1 = 150 – 15
3	Complete the subtraction.	= 135

Teaching strategies

Rusty tables
Students who have not yet mastered their times tables may find it helpful to use a multiplication grid (**Appendix 1A**). Alternatively, students can be given empty times tables grids and given 4 minutes to see how many they can fill in. Teachers should allow students to share their methods of filling in; for example, do the 'easy' tables first. By doing this activity regularly students will soon see the patterns in the multiplication tables.

Class activities

Strategy discussion
An effective way for students to discover different strategies is to hear their peers share them. Students naturally use a range of different arithmetic strategies such as 'counting up' for subtraction. To further develop students' mathematical thinking, they should be encouraged to explain their strategy using diagrams that use different colours, arrows and other descriptive symbols. Give students some calculations to do individually and afterwards have a group discussion about the stategies used by each student. Some interesting and useful methods should arise.

notes:

1.1 Mental strategies

Navigator

Answers p. 646

1 (columns 1–2), 2 (columns 1–2), 3 (columns 1–2), 4 (a–g), 5, 6, 11, 12, 13, 15	1 (columns 2–3), 2 (columns 2–3), 3 (columns 2–3), 4 (column 1), 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17	1 (column 3), 2 (column 3), 3 (column 1), 4 (column 2), 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17

Fluency

W.E. 1

1 Calculate the following using the 'make easy numbers' strategy.

(a) $5 + 32 + 5$	**(b)** $1 + 28 + 9$	**(c)** $7 + 24 + 33$
(d) $2 \times 9 \times 5$	**(e)** $5 \times 7 \times 2$	**(f)** $5 \times 18 \times 2$
(g) $8 + 23 + 42$	**(h)** $15 + 57 + 35$	**(i)** $64 + 79 + 56$
(j) $5 \times 6 \times 2$	**(k)** $4 \times 6 \times 5$	**(l)** $2 \times 42 \times 5$
(m) $5 \times 7 \times 6$	**(n)** $5 \times 3 \times 8$	**(o)** $5 \times 14 \times 4$
(p) $47 + 73$	**(q)** $124 + 56$	**(r)** $211 + 169$
(s) $37 + 128 + 63$	**(t)** $77 + 78 + 23$	**(u)** $89 + 116 + 11$

W.E. 2

2 Evaluate the following using the distributive law.

(a) 2×13	**(b)** 4×12	**(c)** 2×43
(d) 3×13	**(e)** 3×42	**(f)** 3×24
(g) 17×9	**(h)** 19×8	**(i)** 49×6
(j) 6×31	**(k)** 7×52	**(l)** 5×43
(m) 99×9	**(n)** 77×3	**(o)** 57×8
(p) 14×11	**(q)** 15×13	**(r)** 16×12
(s) 101×8	**(t)** 113×5	**(u)** 124×11

3 Use any appropriate mental strategy to work out the following.

(a) $33 + 4 + 7$	**(b)** $135 + 45$	**(c)** $2 \times 24 \times 5$
(d) $4 \times 7 \times 5$	**(e)** 3×85	**(f)** 5×199
(g) $23 + 41 + 57$	**(h)** $347 + 156$	**(i)** $335 - 170$
(j) $8 \times 9 \times 5$	**(k)** 14×7	**(l)** 21×9
(m) 103×6	**(n)** 22×11	**(o)** 3×194
(p) $147 + 213$	**(q)** 19×14	**(r)** $4 \times 7 \times 15$

Understanding

4 Use your calculator to find whether each equation is true (T) or false (F). If it is true, then state which law makes it true: commutative law, associative law or distributive law.

(a) $435 \times 543 = 543 \times 435$	**(b)** $24 \div 6 = 6 \div 24$
(c) $24 + (45 + 76) = (24 + 45) + 76$	**(d)** $325 - (85 - 75) = (325 - 85) - 75$
(e) $45 \times (53 \times 26) = (45 \times 53) \times 26$	**(f)** $(48 \div 6) \div 2 = 48 \div (6 \div 2)$
(g) $45 \times (53 + 26) = 45 \times 53 + 45 \times 26$	**(h)** $45 \times (53 - 26) = 45 \times 53 - 45 \times 26$
(i) $45 + (53 \times 26) = (45 + 53) \times (45 + 26)$	

5 Choose the correct answer to the following question.

23×7 could be calculated by:

A multiplying 3 and 7, then adding 20

B multiplying 20 and 7, then adding 3

C multiplying 20 and 7, multiplying 3 and 7, then adding the products together

D multiplying 20, 3 and 7 all together.

6 Bilal has completed the first 3 stages of a bike rally. He rode 87 km in Stage 1, 95 km in Stage 2, and 63 km in Stage 3. Use mental strategies to calculate:

(a) the total distance that Bilal has ridden so far

(b) how far Bilal still has to ride, if the total rally distance is 480 km.

7 Year 7 students at Mountain View Secondary College are doing a project to improve their environment. Each student will plant 5 seedlings of a native plant. Use a mental strategy to calculate how many seedlings will be needed for 8 classes of 25 students.

8 Jason is saving $8 every week for some new cricket gear. Use mental strategies to calculate:

(a) how much Jason has saved after 17 weeks

(b) how much he still has to save if the cricket gear he wants costs $189.

9 Carlos is monitoring traffic on a busy road. Twelve cars go past him in 1 minute.

(a) Use a mental strategy to calculate how many cars Carlos can expect to go past in 1 hour, based on his 1 minute count.

(b) List two reasons why the actual number of cars might be more or less than your answer to **(a)**.

10 Jessica earns $5 every time she walks her neighbour's dog. If she walks the dog 3 times a week, how much will she earn in 6 months?

11 Alicia is ordering stationery for her office cupboard. Use mental strategies to calculate the cost of each of the following, in dollars.

(a) 8 notepads at 98 cents each

(b) 3 gluesticks at 77 cents each

(c) 12 pens at 59 cents each

(d) 5 boxes of paperclips at 82 cents each

Reasoning

12 Below are some mistakes made by students on a test, and their explanation of the method they used. Write what each student has done incorrectly and what the answer should be.

(a) $21 \times 7 = 161$. Kate: 'I multiplied 7 by 20, and this gave me one less lot of 21 than I needed. So, then I added 21.'

(b) $35 \times 3 = 140$. Sam: 'I doubled 35, then doubled my answer to get 140.'

(c) $256 - 65 = 209$. Leah: 'I first subtracted 56 to get back to 200, and then added the remaining 9.'

Answers

Exercise 1.1

1 **(a)** 42 **(b)** 38 **(c)** 64
(d) 90 **(e)** 70 **(f)** 180
(g) 73 **(h)** 107 **(i)** 199
(j) 60 **(k)** 120 **(l)** 420
(m) 210 **(n)** 120 **(o)** 280
(p) 120 **(q)** 180 **(r)** 380
(s) 228 **(t)** 178 **(u)** 216

2 **(a)** 26 **(b)** 48 **(c)** 86
(d) 39 **(e)** 126 **(f)** 72
(g) 153 **(h)** 152 **(i)** 294
(j) 186 **(k)** 364 **(l)** 215
(m) 891 **(n)** 231 **(o)** 456
(p) 154 **(q)** 195 **(r)** 192
(s) 808 **(t)** 565 **(u)** 1364

3 **(a)** 44 **(b)** 180 **(c)** 240
(d) 140 **(e)** 255 **(f)** 995
(g) 121 **(h)** 503 **(i)** 165
(j) 360 **(k)** 98 **(l)** 189
(m) 618 **(n)** 242 **(o)** 582
(p) 360 **(q)** 266 **(r)** 420

4 **(a)** $435 \times 543 = 236\,205$
and
$543 \times 435 = 236\,205$
The equation is true.
The commutative law applies.

(b) $24 \div 6 = 4$
and
$6 \div 24 = 0.25$
The equation is false.

(c) $24 + (45 + 76) = 24 + 121$
$= 145$
and
$(24 + 45) + 76 = 69 + 76$
$= 145$
The equation is true.
The associative law applies.

(d) $325 - (85 - 75) = 325 - 10$
$= 315$
and
$(325 - 85) - 75 = 240 - 75$
$= 165$
The equation is false.

(e) $(45 \times 53) \times 26 = 45 \times 1378$
$= 62\,010$
and
$(45 \times 53) \times 26 = 2385 \times 26$
$= 62\,010$
The equation is true.
The associative law applies.

(f) $(48 \div 6) \div 2 = 8 \div 2$
$= 4$
and
$48 \div (6 \div 2) = 48 \div 3$
$= 16$
The equation is false.

(g) $45 \times (53 + 26) = 45 \times 79$
$= 3555$
and
$45 \times 53 + 45 \times 26 = 2385 + 1170$
$= 3555$
The equation is true.
The distributive law applies.

(h) $45 \times (53 - 26) = 45 \times 27$
$= 1215$
and
$45 \times 53 - 45 \times 26 = 2385 - 1170$
$= 1215$
The equation is true.
The distributive law applies.

(i) $45 + (53 \times 26) = 45 + 1378$
$= 62\,010$
and
$(45 + 53) \times (45 + 26) = 98 \times 71$
$= 6958$
The equation is false.

5 C Need to multiply each of 20 and 3 by 7, then add the products.

6 **(a)** $87 + 95 + 63 = 90 + 60 + 100 - 5$
$= 245$
Bilal has ridden 245 km so far.

(b) $480 - 245 = 480 - 240 - 5$
$= 235$
Bilal has 235 km to ride.

7 $5 \times 8 \times 25 = 5 \times 200$
$= 1000$
The Year 7s will need 1000 seedlings.

8 **(a)** $8 \times 17 = 80 + 56$
$= 136$
Jason has saved $136 in 17 weeks.

(b) $189 - 136 = 50 + 3$
$= 53$
Jason still has to save $53.

9 **(a)** $12 \times 60 = 72 \times 10$
$= 720$
Carlos would expect 720 cars to go past in 1 hour.

(b) May be about to begin or end a busy time (e.g. peak hour). A bottleneck, accident or other traffic hold-up may occur.

10 3×26 weeks $\times$ \$5 $= 260 + 130$
$= 390$
Jessica earns \$390 in 6 months.

11 **(a)** $8 \times 98 = 800 - 16$
$= 784$
8 notepads cost \$7.84.

(b) $3 \times 77 = 240 - 9$
$= 231$
3 gluesticks cost \$2.31.

(c) $12 \times 59 = 720 - 12$
$= 708$
12 pens cost \$7.08.

(d) $5 \times 82 = 41 \times 10$
$= 410$
5 boxes of paperclips cost \$4.10.

12 **(a)** Multiplying 7 by 20 gives one lot less of 7 than needed, not one lot less of 21. Need to multiply 7 by 20, then add 7 to get 147.

(b) Doubling twice is the same as multiplying by 4, not by 3. You can double 35 to get 70, then add another 35 to get 105.

(c) The remaining 9 should have been subtracted to get 191, not added.