1 Concepts of Motion

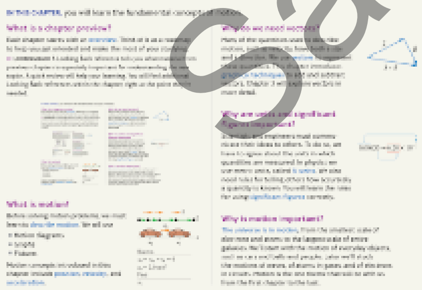
Motion takes many forms. The cyclists seen here are an example of translational motion.

IN THIS CHAPTER, you will learn the fundamental concepts of motion.

What is a chapter preview?

Each chapter starts with an **overview**. Think of it as a roadmap to help you get oriented and make the most of your studying.

« LOOKING BACK A Looking Back reference tells you what material from previous chapters is especially important for understanding the new topics. A quick review will help your learning. You will find additional Looking Back references within the chapter, right at the point they're needed.

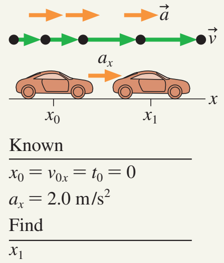


What is motion?

Before solving motion problems, we must learn to ***describe* motion**. We will use

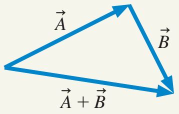
- Motion diagrams
- Graphs
- Pictures

Motion concepts introduced in this chapter include **position**, **velocity**, and **acceleration**.



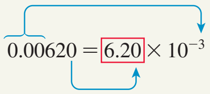
Why do we need vectors?

Many of the quantities used to describe motion, such as velocity, have both a size and a direction. We use **vectors** to represent these quantities. This chapter introduces **graphical techniques** to add and subtract vectors. Chapter 3 will explore vectors in more detail.



Why are units and significant figures important?

Scientists and engineers must communicate their ideas to others. To do so, we have to agree about the *units* in which quantities are measured. In physics we use metric units, called **SI units**. We also need rules for telling others how accurately a quantity is known. You will learn the rules for using **significant figures** correctly.



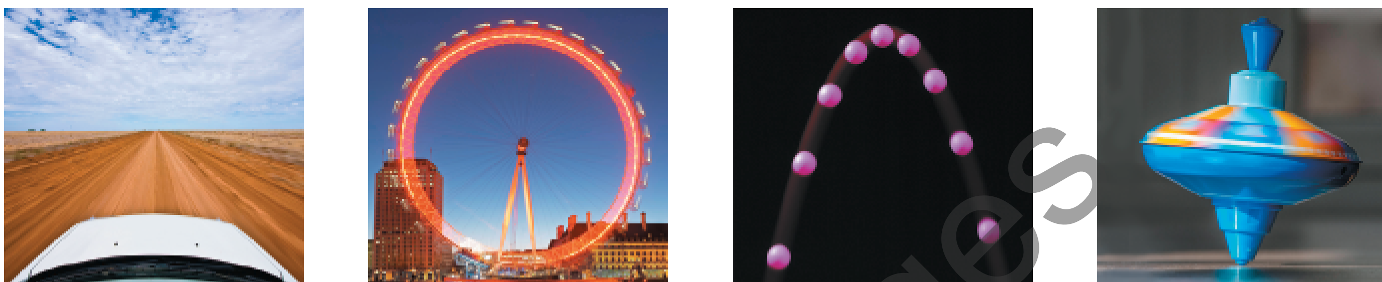
Why is motion important?

The universe is in motion, from the smallest scale of electrons and atoms to the largest scale of entire galaxies. We'll start with the motion of everyday objects, such as cars and balls and people. Later we'll study the motions of waves, of atoms in gases, and of electrons in circuits. Motion is the one theme that will be with us from the first chapter to the last.

1.1 Motion Diagrams

Motion is a theme that will appear in one form or another throughout this entire book. Although we all have intuition about motion, based on our experiences, some of the important aspects of motion turn out to be rather subtle. So rather than jumping immediately into a lot of mathematics and calculations, this first chapter focuses on *visualizing* motion and becoming familiar with the *concepts* needed to describe a moving object. Our goal is to lay the foundations for understanding motion.

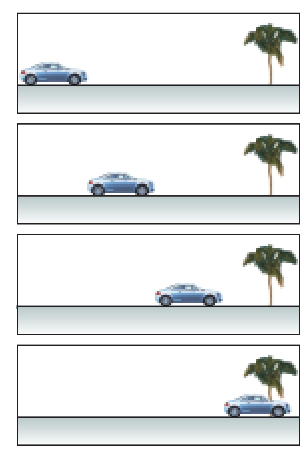
FIGURE 1.1 Four basic types of motion.



Linear motion **Circular motion** **Projectile motion** **Rotational motion**

To begin, let's define **motion** as the change of an object's position with time. **FIGURE 1.1** shows four basic types of motion that we will study in this book. The first three—linear, circular, and projectile motion—in which the object moves through space are called **translational motion.** The path along which the object moves, whether straight or curved, is called the object's **trajectory.** Rotational motion is somewhat different because there's movement but the object as a whole doesn't change position. We'll defer rotational motion until later and, for now, focus on translational motion.

FIGURE 1.2 Four frames from a video.

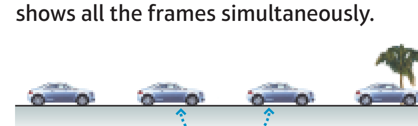


Making a Motion Diagram

An easy way to study motion is to make a video of a moving object. A video camera, as you probably know, takes images at a fixed rate, typically 30 every second. Each separate image is called a *frame*. As an example, **FIGURE 1.2** shows four frames from a video of a car going past. Not surprisingly, the car is in a somewhat different position in each frame.

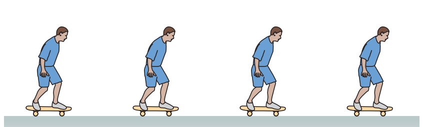
Suppose we edit the video by layering the frames on top of each other, creating the composite image shown in **FIGURE 1.3**. This edited image, showing an object's position at several *equally spaced instants of time,* is called a **motion diagram.** As the examples below show, we can define concepts such as constant speed, speeding up, and slowing down in terms of how an object appears in a motion diagram.

FIGURE 1.3 A motion diagram of the car shows all the frames simultaneously.

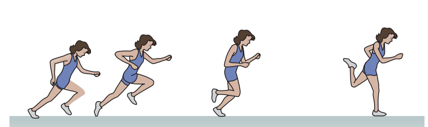


NOTE It's important to keep the camera in a *fixed position* as the object moves by. Don't "pan" it to track the moving object.

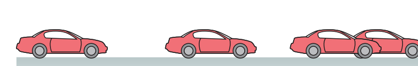
Examples of motion diagrams



Images that are *equally spaced* indicate an object moving with *constant speed.*



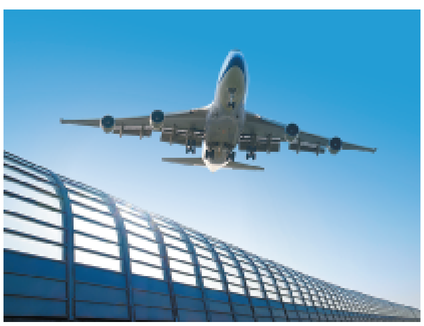
An *increasing distance* between the images shows that the object is *speeding up.*



A *decreasing distance* between the images shows that the object is *slowing down.*

STOP TO THINK 1.1 Which car is going faster, A or B? Assume there are equal intervals of time between the frames of both videos.

NOTE Each chapter will have several *Stop to Think* questions. These questions are designed to see if you've understood the basic ideas that have been presented. The answers are given at the end of the book, but you should make a serious effort to think about these questions before turning to the answers.



We can model an airplane's takeoff as a particle (a descriptive model) undergoing constant acceleration (a descriptive model) in response to constant forces (an explanatory model).

1.2 Models and Modeling

The real world is messy and complicated. Our goal in physics is to brush aside many of the real-world details in order to discern patterns that occur over and over. For example, a swinging pendulum, a vibrating guitar string, a sound wave, and jiggling atoms in a crystal are all very different—yet perhaps not so different. Each is an example of a system moving back and forth around an equilibrium position. If we focus on understanding a very simple oscillating system, such as a mass on a spring, we'll automatically understand quite a bit about the many real-world manifestations of oscillations.

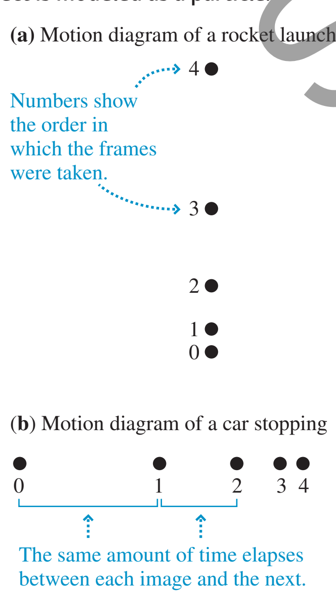
Stripping away the details to focus on essential features is a process called *modeling*. A **model** is a highly simplified picture of reality, but one that still captures the essence of what we want to study. Thus "mass on a spring" is a simple but realistic model of almost all oscillating systems.

Models allow us to make sense of complex situations by providing a framework for thinking about them. One could go so far as to say that developing and testing models is at the heart of the scientific process. Albert Einstein once said, "Physics should be as simple as possible—but not simpler." We want to find the simplest model that allows us to understand the phenomenon we're studying, but we can't make the model so simple that key aspects of the phenomenon get lost.

We'll develop and use many models throughout this textbook; they'll be one of our most important thinking tools. These models will be of two types:

- *Descriptive models:* What are the essential characteristics and properties of a phenomenon? How do we describe it in the simplest possible terms? For example, the mass-on-a-spring model of an oscillating system is a descriptive model.
- *Explanatory models:* Why do things happen as they do? Explanatory models, based on the laws of physics, have predictive power, allowing us to test—against experimental data—whether a model provides an adequate explanation of our observations.

FIGURE 1.4 Motion diagrams in which the object is modeled as a particle.



The Particle Model

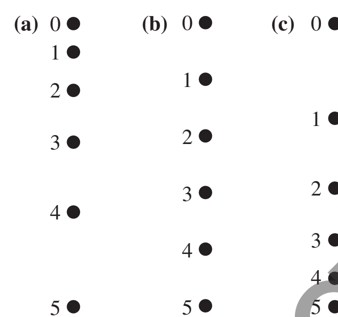
For many types of motion, such as that of balls, cars, and rockets, the motion of the object *as a whole* is not influenced by the details of the object's size and shape. All we really need to keep track of is the motion of a single point on the object, so we can treat the object *as if* all its mass were concentrated into this single point. An object that can be represented as a mass at a single point in space is called a **particle.** A particle has no size, no shape, and no distinction between top and bottom or between front and back.

If we model an object as a particle, we can represent the object in each frame of a motion diagram as a simple dot rather than having to draw a full picture. **FIGURE 1.4** shows how much simpler motion diagrams appear when the object is represented as a particle. Note that the dots have been numbered 0, 1, 2, . . . to tell the sequence in which the frames were taken.

Treating an object as a particle is, of course, a simplification of reality—but that's what modeling is all about. The **particle model** of motion is a simplification in which we treat a moving object as if all of its mass were concentrated at a single point. The particle model is an excellent approximation of reality for the translational motion of cars, planes, rockets, and similar objects.

Of course, not everything can be modeled as a particle; models have their limits. Consider, for example, a rotating gear. The center doesn't move at all while each tooth is moving in a different direction. We'll need to develop new models when we get to new types of motion, but the particle model will serve us well throughout Part I of this book.

STOP TO THINK 1.2 Three motion diagrams are shown. Which is a dust particle settling to the floor at constant speed, which is a ball dropped from the roof of a building, and which is a descending rocket slowing to make a soft landing on Mars?



1.3 Position, Time, and Displacement

To use a motion diagram, you would like to know *where* the object is (i.e., its *position*) and *when* the object was at that position (i.e., the *time*). Position measurements can be made by laying a coordinate-system grid over a motion diagram. You can then measure the (x, y) coordinates of each point in the motion diagram. Of course, the world does not come with a coordinate system attached. A coordinate system is an artificial grid that *you* place over a problem in order to analyze the motion. You place the origin of your coordinate system wherever you wish, and different observers of a moving object might all choose to use different origins.

Time, in a sense, is also a coordinate system, although you may never have thought of time this way. You can pick an arbitrary point in the motion and label it "$t = 0$ seconds." This is simply the instant you decide to start your clock or stopwatch, so it is the origin of your time coordinate. Different observers might choose to start their clocks at different moments. A video frame labeled "$t = 4$ seconds" was taken 4 seconds after you started your clock.

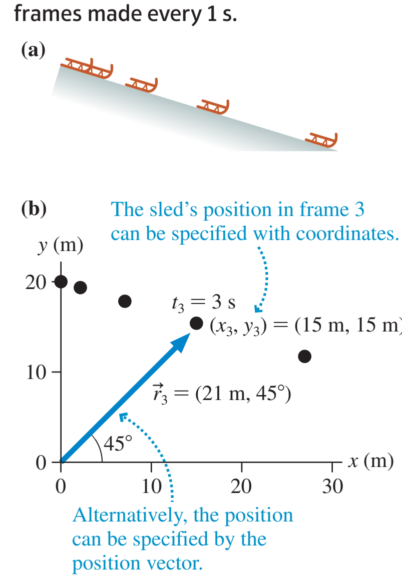
We typically choose $t = 0$ to represent the "beginning" of a problem, but the object may have been moving before then. Those earlier instants would be measured as negative times, just as objects on the x-axis to the left of the origin have negative values of position. Negative numbers are not to be avoided; they simply locate an event in space or time *relative to an origin*.

To illustrate, **FIGURE 1.5a** shows a sled sliding down a snow-covered hill. **FIGURE 1.5b** is a motion diagram for the sled, over which we've drawn an xy-coordinate system. You can see that the sled's position is $(x_3, y_3) = (15 \text{ m}, 15 \text{ m})$ at time $t_3 = 3$ s. Notice how we've used subscripts to indicate the time and the object's position in a specific frame of the motion diagram.

NOTE The frame at $t = 0$ s is frame 0. That is why the fourth frame is labeled 3.

Another way to locate the sled is to draw its **position vector:** an arrow from the origin to the point representing the sled. The position vector is given the symbol $\vec{r}$. Figure 1.5b shows the position vector $\vec{r}_3 = (21 \text{ m}, 45°)$. The position vector $\vec{r}$ does not tell us anything different than the coordinates (x, y). It simply provides the information in an alternative form.

FIGURE 1.5 Motion diagram of a sled with frames made every 1 s.



Scalars and Vectors

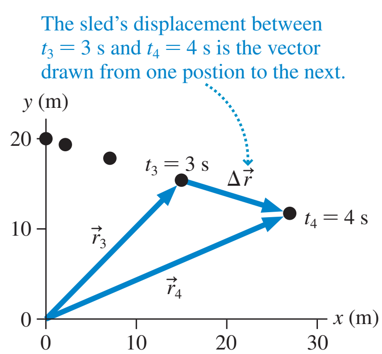
Some physical quantities, such as time, mass, and temperature, can be described completely by a single number with a unit. For example, the mass of an object is 6 kg and its temperature is 30°C. A single number (with a unit) that describes a physical quantity is called a **scalar.** A scalar can be positive, negative, or zero.

Many other quantities, however, have a directional aspect and cannot be described by a single number. To describe the motion of a car, for example, you must specify not only how fast it is moving, but also the *direction* in which it is moving. A quantity having both a *size* (the "How far?" or "How fast?") and a *direction* (the "Which way?") is called a **vector.** The size or length of a vector is called its *magnitude*. Vectors will be studied thoroughly in Chapter 3, so all we need for now is a little basic information.

We indicate a vector by drawing an arrow over the letter that represents the quantity. Thus $\vec{r}$ and $\vec{A}$ are symbols for vectors, whereas r and A, without the arrows, are symbols for scalars. In handwritten work you must draw arrows over all symbols that represent vectors. This may seem strange until you get used to it, but it is very important because we will often use both r and $\vec{r}$, or both A and $\vec{A}$, in the same problem, and they mean different things! Note that the arrow over the symbol always points to the right, regardless of which direction the actual vector points. Thus we write $\vec{r}$ or $\vec{A}$, never $\overleftarrow{r}$ or $\overleftarrow{A}$.

Displacement

FIGURE 1.6 The sled undergoes a displacement $\Delta\vec{r}$ from position $\vec{r}_3$ to position $\vec{r}_4$.



We said that motion is the change in an object's position with time, but how do we show a change of position? A motion diagram is the perfect tool. FIGURE 1.6 is the motion diagram of a sled sliding down a snow-covered hill. To show how the sled's position changes between, say, $t_3 = 3$ s and $t_4 = 4$ s, we draw a vector arrow between the two dots of the motion diagram. This vector is the sled's **displacement,** which is given the symbol $\Delta\vec{r}$. The Greek letter delta (Δ) is used in math and science to indicate the *change* in a quantity. In this case, as we'll show, the displacement $\Delta\vec{r}$ is the change in an object's position.

NOTE $\Delta\vec{r}$ is a *single* symbol. It shows "from here to there." You cannot cancel out or remove the Δ.

Notice how the sled's position vector $\vec{r}_4$ is a combination of its early position $\vec{r}_3$ with the displacement vector $\Delta\vec{r}$. In fact, $\vec{r}_4$ is the *vector sum* of the vectors $\vec{r}_3$ and $\Delta\vec{r}$. This is written

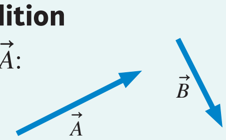
$$\vec{r}_4 = \vec{r}_3 + \Delta\vec{r} \qquad (1.1)$$

Here we're adding vector quantities, not numbers, and vector addition differs from "regular" addition. We'll explore vector addition more thoroughly in Chapter 3, but for now you can add two vectors $\vec{A}$ and $\vec{B}$ with the three-step procedure of « TACTICS BOX 1.1.

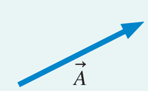
TACTICS BOX 1.1

Vector addition

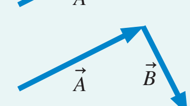
To add $\vec{B}$ to $\vec{A}$:



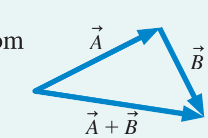
1. Draw $\vec{A}$.



2. Place the tail of $\vec{B}$ at the tip of $\vec{A}$.



3. Draw an arrow from the tail of $\vec{A}$ to the tip of $\vec{B}$. This is vector $\vec{A} + \vec{B}$.



If you examine Figure 1.6, you'll see that the steps of Tactics Box 1.1 are exactly how $\vec{r}_3$ and $\Delta\vec{r}$ are added to give $\vec{r}_4$.

NOTE A vector is not tied to a particular location on the page. You can move a vector around as long as you don't change its length or the direction it points. Vector $\vec{B}$ is not changed by sliding it to where its tail is at the tip of $\vec{A}$.

Equation 1.1 told us that $\vec{r}_4 = \vec{r}_3 + \Delta\vec{r}$. This is easily rearranged to give a more precise definition of displacement: **The displacement $\Delta\vec{r}$ of an object as it moves from one position $\vec{r}_a$ to a different position $\vec{r}_b$ is**

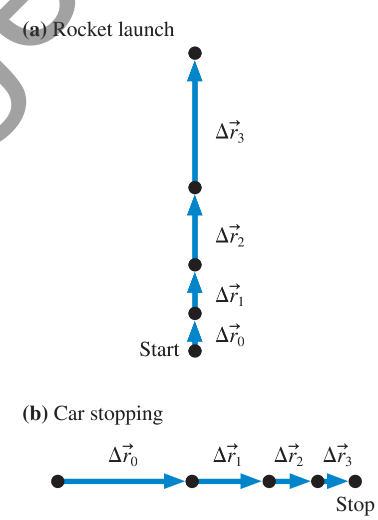
$$\Delta\vec{r} = \vec{r}_b - \vec{r}_a \tag{1.2}$$

That is, displacement is the change (i.e., the difference) in position. **Graphically, $\Delta\vec{r}$ is a vector arrow drawn from position $\vec{r}_a$ to position $\vec{r}_b$.**

Motion Diagrams with Displacement Vectors

The first step in analyzing a motion diagram is to determine all of the displacement vectors, which are simply the arrows connecting each dot to the next. Label each arrow with a *vector* symbol $\Delta\vec{r}_n$, starting with $n = 0$. FIGURE 1.7 shows the motion diagrams of Figure 1.4 redrawn to include the displacement vectors.

FIGURE 1.7 Motion diagrams with the displacement vectors.



NOTE When an object either starts from rest or ends at rest, the initial or final dots are *as close together* as you can draw the displacement vector arrow connecting them. In addition, just to be clear, you should write "Start" or "Stop" beside the initial or final dot. It is important to distinguish stopping from merely slowing down.

Now we can conclude, more precisely than before, that, as time proceeds:

- An object is speeding up if its displacement vectors are increasing in length.
- An object is slowing down if its displacement vectors are decreasing in length.

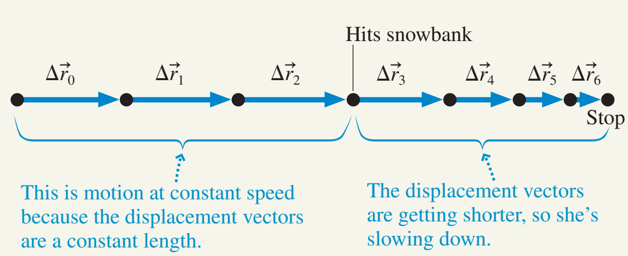
EXAMPLE 1.1 ■ Headfirst into the snow

Alice is sliding along a smooth, icy road on her sled when she suddenly runs headfirst into a large, very soft snowbank that gradually brings her to a halt. Draw a motion diagram for Alice. Show and label all displacement vectors.

MODEL The details of Alice and the sled—their size, shape, color, and so on—are not relevant to understanding their overall motion. So we can model Alice and the sled as one particle.

VISUALIZE FIGURE 1.8 shows a motion diagram. The problem statement suggests that the sled's speed is very nearly constant until it hits the snowbank. Thus the displacement vectors are of equal length as Alice slides along the icy road. She begins slowing when she hits the snowbank, so the displacement vectors then get shorter until the sled stops. We're told that her stop is gradual, so we want the vector lengths to get shorter gradually rather than suddenly.

FIGURE 1.8 The motion diagram of Alice and the sled.



A stopwatch is used to measure a time interval.

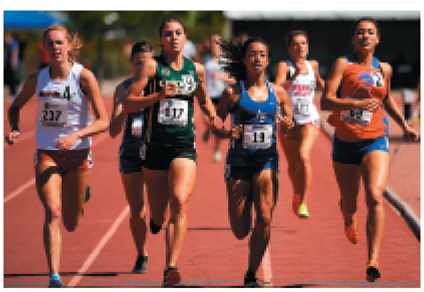
Time Interval

It's also useful to consider a *change* in time. For example, the clock readings of two frames of a video might be t_1 and t_2. The specific values are arbitrary because they are timed relative to an arbitrary instant that you chose to call $t = 0$. But the **time interval** $\Delta t = t_2 - t_1$ is *not* arbitrary. It represents the elapsed time for the object to move from one position to the next.

The time interval $\Delta t = t_b - t_a$ measures the elapsed time as an object moves from position $\vec{r}_a$ at time t_a to position $\vec{r}_b$ at time t_b. The value of Δt is independent of the specific clock used to measure the times.

To summarize the main idea of this section, we have added coordinate systems and clocks to our motion diagrams in order to measure *when* each frame was exposed and *where* the object was located at that time. Different observers of the motion may choose different coordinate systems and different clocks. However, all observers find the *same* values for the displacements $\Delta\vec{r}$ and the time intervals Δt because these are independent of the specific coordinate system used to measure them.

1.4 Velocity



The victory goes to the runner with the highest average speed.

It's no surprise that, during a given time interval, a speeding bullet travels farther than a speeding snail. To extend our study of motion so that we can compare the bullet to the snail, we need a way to measure how fast or how slowly an object moves.

One quantity that measures an object's fastness or slowness is its **average speed,** defined as the ratio

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time interval spent traveling}} = \frac{d}{\Delta t} \tag{1.3}$$

If you drive 15 miles (mi) in 30 minutes ($\frac{1}{2}$ h), your average speed is

$$\text{average speed} = \frac{15\text{ mi}}{\frac{1}{2}\text{ h}} = 30\text{ mph} \tag{1.4}$$

Although the concept of speed is widely used in our day-to-day lives, it is not a sufficient basis for a science of motion. To see why, imagine you're trying to land a jet plane on an aircraft carrier. It matters a great deal to you whether the aircraft carrier is moving at 20 mph (miles per hour) to the north or 20 mph to the east. Simply knowing that the ship's speed is 20 mph is not enough information!

It's the displacement $\Delta\vec{r}$, a vector quantity, that tells us not only the distance traveled by a moving object, but also the *direction* of motion. Consequently, a more useful ratio than $d/\Delta t$ is the ratio $\Delta\vec{r}/\Delta t$. In addition to measuring how fast an object moves, this ratio is a vector that points in the direction of motion.

It is convenient to give this ratio a name. We call it the **average velocity,** and it has the symbol $\vec{v}_{avg}$. **The average velocity of an object during the time interval Δt, in which the object undergoes a displacement $\Delta\vec{r}$, is the vector**

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} \tag{1.5}$$

An object's average velocity vector points in the same direction as the displacement vector $\Delta\vec{r}$. This is the direction of motion.

> **NOTE** In everyday language we do not make a distinction between speed and velocity, but in physics *the distinction is very important.* In particular, speed is simply "How fast?" whereas velocity is "How fast, and in which direction?" As we go along we will be giving other words more precise meanings in physics than they have in everyday language.

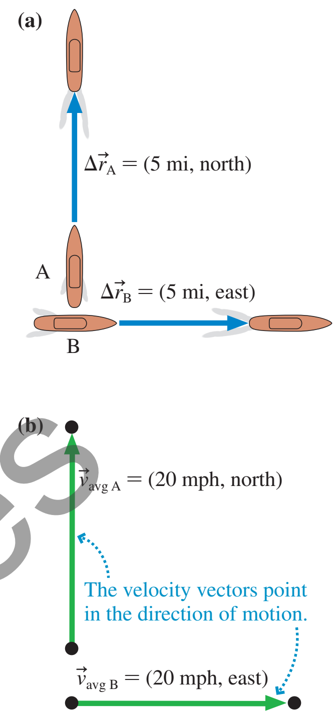
As an example, FIGURE 1.9a shows two ships that move 5 miles in 15 minutes. Using Equation 1.5 with $\Delta t = 0.25$ h, we find

$$\vec{v}_{\text{avg A}} = (20 \text{ mph, north})$$
$$\vec{v}_{\text{avg B}} = (20 \text{ mph, east}) \tag{1.6}$$

Both ships have a speed of 20 mph, but their velocities differ. Notice how the velocity *vectors* in FIGURE 1.9b point in the direction of motion.

NOTE Our goal in this chapter is to *visualize* motion with motion diagrams. Strictly speaking, the vector we have defined in Equation 1.5, and the vector we will show on motion diagrams, is the *average* velocity $\vec{v}_{\text{avg}}$. But to allow the motion diagram to be a useful tool, we will drop the subscript and refer to the average velocity as simply $\vec{v}$. Our definitions and symbols, which somewhat blur the distinction between average and instantaneous quantities, are adequate for visualization purposes, but they're not the final word. We will refine these definitions in Chapter 2, where our goal will be to develop the mathematics of motion.

FIGURE 1.9 The displacement vectors and velocities of ships A and B.



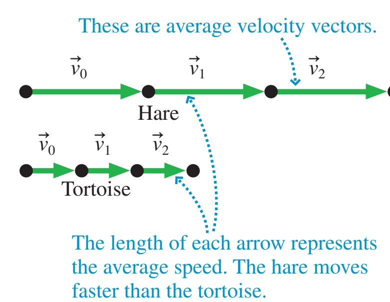
Motion Diagrams with Velocity Vectors

The velocity vector points in the same direction as the displacement $\Delta\vec{r}$, and the length of $\vec{v}$ is directly proportional to the length of $\Delta\vec{r}$. Consequently, the vectors connecting each dot of a motion diagram to the next, which we previously labeled as displacements, could equally well be identified as velocity vectors.

This idea is illustrated in FIGURE 1.10, which shows four frames from the motion diagram of a tortoise racing a hare. The vectors connecting the dots are now labeled as velocity vectors $\vec{v}$. **The length of a velocity vector represents the average speed with which the object moves between the two points.** Longer velocity vectors indicate faster motion. You can see that the hare moves faster than the tortoise.

Notice that the hare's velocity vectors do not change; each has the same length and direction. We say the hare is moving with *constant velocity*. The tortoise is also moving with its own constant velocity.

FIGURE 1.10 Motion diagram of the tortoise racing the hare.



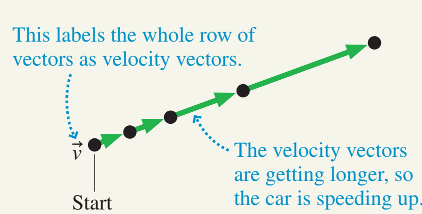
EXAMPLE 1.2 ■ Accelerating up a hill

The light turns green and a car accelerates, starting from rest, up a 20° hill. Draw a motion diagram showing the car's velocity.

MODEL Use the particle model to represent the car as a dot.

VISUALIZE The car's motion takes place along a straight line, but the line is neither horizontal nor vertical. A motion diagram should show the object moving with the correct orientation—in this case, at an angle of 20°. FIGURE 1.11 shows several frames of the motion diagram, where we see the car speeding up. The car starts from rest, so the first arrow is drawn as short as possible and the first dot is labeled "Start." The displacement vectors have been drawn from each dot to the next, but then they are identified and labeled as average velocity vectors $\vec{v}$.

FIGURE 1.11 Motion diagram of a car accelerating up a hill.



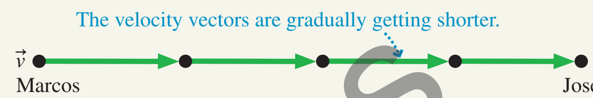
EXAMPLE 1.3 ■ **A rolling soccer ball**

Marcos kicks a soccer ball. It rolls along the ground until stopped by Jose. Draw a motion diagram of the ball.

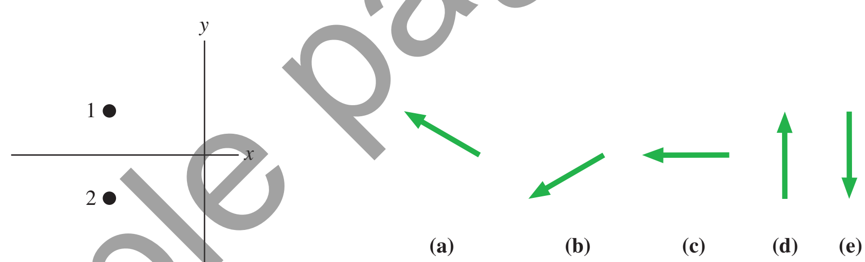
MODEL This example is typical of how many problems in science and engineering are worded. The problem does not give a clear statement of where the motion begins or ends. Are we interested in the motion of the ball just during the time it is rolling between Marcos and Jose? What about the motion *as* Marcos kicks it (ball rapidly speeding up) or *as* Jose stops it (ball rapidly slowing down)? The point is that *you* will often be called on to make a *reasonable interpretation* of a problem statement. In this problem, the details of kicking and stopping the ball are complex. The motion of the ball across the ground is easier to describe, and it's a motion you might expect to learn about in a physics class. So our *interpretation* is that the motion diagram should start as the ball leaves Marcos's foot (ball already moving) and should end the instant it touches Jose's foot (ball still moving). In between, the ball will slow down a little. We will model the ball as a particle.

VISUALIZE With this interpretation in mind, **FIGURE 1.12** shows the motion diagram of the ball. Notice how, in contrast to the car of Figure 1.11, the ball is already moving as the motion diagram video begins. As before, the average velocity vectors are found by connecting the dots. You can see that the average velocity vectors get shorter as the ball slows. Each $\vec{v}$ is different, so this is *not* constant-velocity motion.

FIGURE 1.12 Motion diagram of a soccer ball rolling from Marcos to Jose.



STOP TO THINK 1.3 A particle moves from position 1 to position 2 during the time interval Δt. Which vector shows the particle's average velocity?



1.5 Linear Acceleration

Position, time, and velocity are important concepts, and at first glance they might appear to be sufficient to describe motion. But that is not the case. Sometimes an object's velocity is constant, as it was in Figure 1.10. More often, an object's velocity changes as it moves, as in Figures 1.11 and 1.12. We need one more motion concept to describe a *change* in the velocity.

Because velocity is a vector, it can change in two possible ways:

1. The magnitude can change, indicating a change in speed; or
2. The direction can change, indicating that the object has changed direction.

We will concentrate for now on the first case, a change in speed. The car accelerating up a hill in Figure 1.11 was an example in which the magnitude of the velocity vector changed but not the direction. We'll return to the second case in Chapter 4.

When we wanted to measure changes in position, the ratio $\Delta\vec{r}/\Delta t$ was useful. This ratio is the *rate of change of position*. By analogy, consider an object whose velocity changes from $\vec{v}_a$ to $\vec{v}_b$ during the time interval Δt. Just as $\Delta\vec{r} = \vec{r}_b - \vec{r}_a$ is the change of position, the quantity $\Delta\vec{v} = \vec{v}_b - \vec{v}_a$ is the change of velocity. The ratio $\Delta\vec{v}/\Delta t$ is then the *rate of change of velocity*. It has a large magnitude for objects that speed up quickly and a small magnitude for objects that speed up slowly.

The ratio $\Delta\vec{v}/\Delta t$ is called the **average acceleration,** and its symbol is $\vec{a}_{avg}$. **The average acceleration of an object during the time interval Δt, in which the object's velocity changes by $\Delta\vec{v}$, is the vector**

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} \tag{1.7}$$

The average acceleration vector points in the same direction as the vector $\Delta\vec{v}$.

Acceleration is a fairly abstract concept. Yet it is essential to develop a good intuition about acceleration because it will be a key concept for understanding why objects move as they do. Motion diagrams will be an important tool for developing that intuition.

The Audi TT accelerates from 0 to 60 mph in 6 s.

NOTE As we did with velocity, we will drop the subscript and refer to the average acceleration as simply $\vec{a}$. This is adequate for visualization purposes, but not the final word. We will refine the definition of acceleration in Chapter 2.

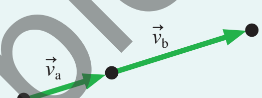
Finding the Acceleration Vectors on a Motion Diagram

Perhaps the most important use of a motion diagram is to determine the acceleration vector $\vec{a}$ at each point in the motion. From its definition in Equation 1.7, we see that **$\vec{a}$ points in the same direction as $\Delta\vec{v}$,** the change of velocity, so we need to find the direction of $\Delta\vec{v}$. To do so, we rewrite the definition $\Delta\vec{v} = \vec{v}_b - \vec{v}_a$ as $\vec{v}_b = \vec{v}_a + \Delta\vec{v}$. This is now a vector addition problem: What vector must be added to $\vec{v}_a$ to turn it into $\vec{v}_b$? Tactics Box 1.2 shows how to do this.

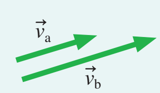
TACTICS BOX 1.2

Finding the acceleration vector

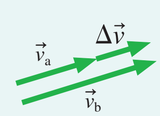
To find the acceleration as the velocity changes from $\vec{v}_a$ to $\vec{v}_b$, we must determine the *change* of velocity $\Delta\vec{v} = \vec{v}_b - \vec{v}_a$.



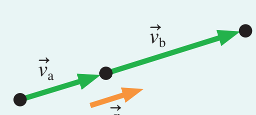
❶ Draw velocity vectors $\vec{v}_a$ and $\vec{v}_b$ with their tails together.



❷ Draw the vector from the tip of $\vec{v}_a$ to the tip of $\vec{v}_b$. This is $\Delta\vec{v}$ because $\vec{v}_b = \vec{v}_a + \Delta\vec{v}$.



❸ Return to the original motion diagram. Draw a vector at the middle dot in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration at the midpoint between $\vec{v}_a$ and $\vec{v}_b$.



Exercises 21–24

Many Tactics Boxes will refer you to exercises in the *Student Workbook* where you can practice the new skill.

Notice that the acceleration vector goes beside the middle dot, not beside the velocity vectors. This is because each acceleration vector is determined by the *difference* between the *two* velocity vectors on either side of a dot. The length of $\vec{a}$ does not have to be the exact length of $\Delta\vec{v}$; it is the direction of $\vec{a}$ that is most important.

The procedure of « **TACTICS BOX 1.2** can be repeated to find $\vec{a}$ at each point in the motion diagram. Note that we cannot determine $\vec{a}$ at the first and last points because we have only one velocity vector and can't find $\Delta\vec{v}$.

The Complete Motion Diagram

You've now seen two *Tactics Boxes*. Tactics Boxes to help you accomplish specific tasks will appear in nearly every chapter in this book. We'll also, where appropriate, provide *Problem-Solving Strategies*.

PROBLEM-SOLVING STRATEGY 1.1

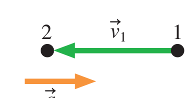
Motion diagrams

MODEL Determine whether it is appropriate to model the moving object as a particle. Make simplifying assumptions when interpreting the problem statement.

VISUALIZE A complete motion diagram consists of:

- The position of the object in each frame of the video, shown as a dot. Use five or six dots to make the motion clear but without overcrowding the picture. The motion should change gradually from one dot to the next, not drastically. More complex motions will need more dots.
- The average velocity vectors, found by connecting each dot in the motion diagram to the next with a vector arrow. There is *one* velocity vector linking each *two* position dots. Label the row of velocity vectors $\vec{v}$.
- The average acceleration vectors, found using Tactics Box 1.2. There is *one* acceleration vector linking each *two* velocity vectors. Each acceleration vector is drawn at the dot between the two velocity vectors it links. Use $\vec{0}$ to indicate a point at which the acceleration is zero. Label the row of acceleration vectors $\vec{a}$.

STOP TO THINK 1.4 A particle undergoes acceleration $\vec{a}$ while moving from point 1 to point 2. Which of the choices shows the most likely velocity vector $\vec{v}_2$ as the particle leaves point 2?



Examples of Motion Diagrams

Let's look at some examples of the full strategy for drawing motion diagrams.

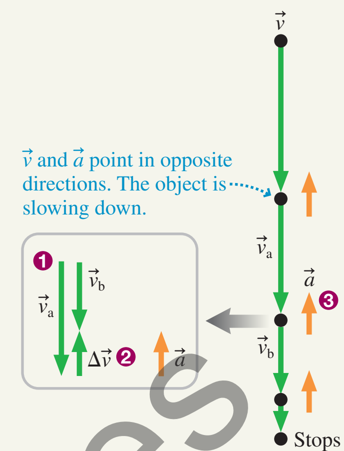
EXAMPLE 1.4 ■ The first astronauts land on Mars

A spaceship carrying the first astronauts to Mars descends safely to the surface. Draw a motion diagram for the last few seconds of the descent.

MODEL The spaceship is small in comparison with the distance traveled, and the spaceship does not change size or shape, so it's reasonable to model the spaceship as a particle. We'll assume that its motion in the last few seconds is straight down. The problem ends as the spacecraft touches the surface.

VISUALIZE **FIGURE 1.13** shows a complete motion diagram as the spaceship descends and slows, using its rockets, until it comes to rest on the surface. Notice how the dots get closer together as it slows. The inset uses the steps of Tactics Box 1.2 (numbered circles) to show how the acceleration vector $\vec{a}$ is determined at one point. All the other acceleration vectors will be similar because for each pair of velocity vectors the earlier one is longer than the later one.

FIGURE 1.13 Motion diagram of a spaceship landing on Mars.



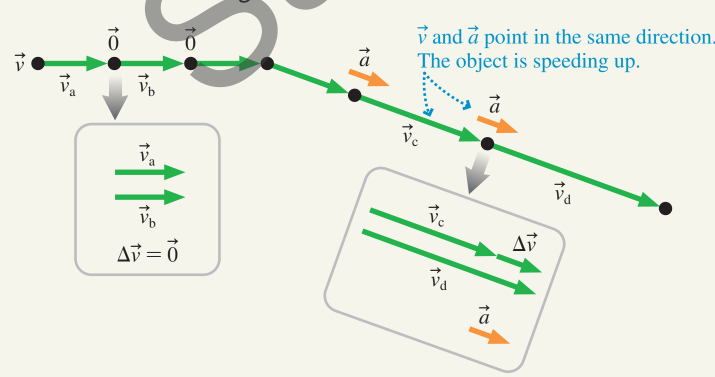
EXAMPLE 1.5 ■ Skiing through the woods

A skier glides along smooth, horizontal snow at constant speed, then speeds up going down a hill. Draw the skier's motion diagram.

MODEL Model the skier as a particle. It's reasonable to assume that the downhill slope is a straight line. Although the motion as a whole is not linear, we can treat the skier's motion as two separate linear motions.

VISUALIZE **FIGURE 1.14** shows a complete motion diagram of the skier. The dots are equally spaced for the horizontal motion, indicating constant speed; then the dots get farther apart as the skier speeds up going down the hill. The insets show how the average acceleration vector $\vec{a}$ is determined for the horizontal motion and along the slope. All the other acceleration vectors along the slope will be similar to the one shown because each velocity vector is longer than the preceding one. Notice that we've explicitly written $\vec{0}$ for the acceleration beside the dots where the velocity is constant. The acceleration at the point where the direction changes will be considered in Chapter 4.

FIGURE 1.14 Motion diagram of a skier.



Notice something interesting in Figures 1.13 and 1.14. Where the object is speeding up, the acceleration and velocity vectors point in the *same direction*. Where the object is slowing down, the acceleration and velocity vectors point in *opposite directions*. These results are always true for motion in a straight line. **For motion along a line:**

- **An object is speeding up if and only if $\vec{v}$ and $\vec{a}$ point in the same direction.**
- **An object is slowing down if and only if $\vec{v}$ and $\vec{a}$ point in opposite directions.**
- **An object's velocity is constant if and only if $\vec{a} = \vec{0}$.**

NOTE In everyday language, we use the word *accelerate* to mean "speed up" and the word *decelerate* to mean "slow down." But speeding up and slowing down are both changes in the velocity and consequently, by our definition, *both* are accelerations. In physics, *acceleration* refers to changing the velocity, no matter what the change is, and not just to speeding up.

EXAMPLE 1.6 ■ Tossing a ball

Draw the motion diagram of a ball tossed straight up in the air.

MODEL This problem calls for some interpretation. Should we include the toss itself, or only the motion after the ball is released? What about catching it? It appears that this problem is really concerned with the ball's motion through the air. Consequently, we begin the motion diagram at the instant that the tosser releases the ball and end the diagram at the instant the ball touches his hand. We will consider neither the toss nor the catch. And, of course, we will model the ball as a particle.

VISUALIZE We have a slight difficulty here because the ball retraces its route as it falls. A literal motion diagram would show the upward motion and downward motion on top of each other, leading to confusion. We can avoid this difficulty by horizontally separating the upward motion and downward motion diagrams. This will not affect our conclusions because it does not change any of the vectors. **FIGURE 1.15** shows the motion diagram drawn this way. Notice that the very top dot is shown twice—as the end point of the upward motion and the beginning point of the downward motion.

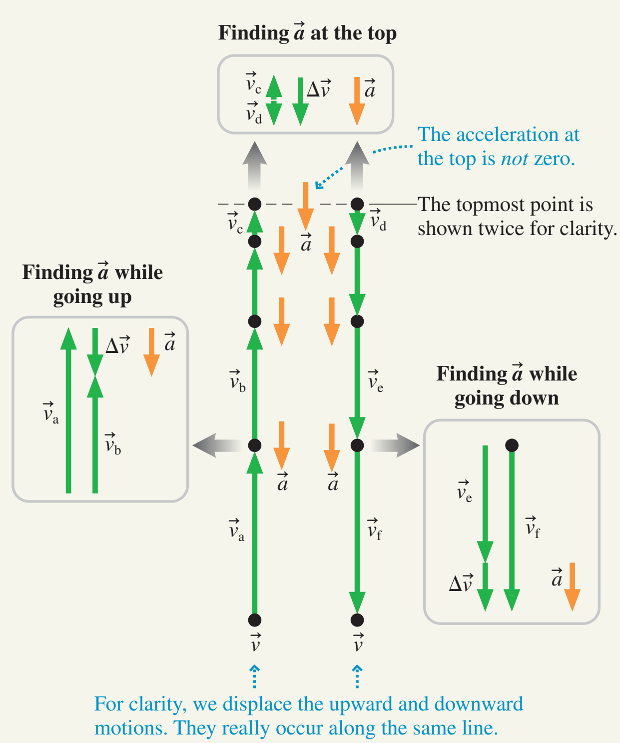
The ball slows down as it rises. You've learned that the acceleration vectors point opposite the velocity vectors for an object that is slowing down along a line, and they are shown accordingly. Similarly, $\vec{a}$ and $\vec{v}$ point in the same direction as the falling ball speeds up. Notice something interesting: The acceleration vectors point downward both while the ball is rising *and* while it is falling. Both "speeding up" and "slowing down" occur with the *same* acceleration vector. This is an important conclusion, one worth pausing to think about.

Now look at the top point on the ball's trajectory. The velocity vectors point upward but are getting shorter as the ball approaches the top. As the ball starts to fall, the velocity vectors point downward and are getting longer. There must be a moment—just an instant as $\vec{v}$ switches from pointing up to pointing down—when the velocity is zero. Indeed, the ball's velocity *is* zero for an instant at the precise top of the motion!

But what about the acceleration at the top? The inset shows how the average acceleration is determined from the last upward velocity before the top point and the first downward velocity. We find that the acceleration at the top is pointing downward, just as it does elsewhere in the motion.

Many people expect the acceleration to be zero at the highest point. But the velocity at the top point *is* changing—from up to down. If the velocity is changing, there *must* be an acceleration. A downward-pointing acceleration vector is needed to turn the velocity vector from up to down. Another way to think about this is to note that zero acceleration would mean no change of velocity. When the ball reached zero velocity at the top, it would hang there and not fall if the acceleration were also zero!

FIGURE 1.15 Motion diagram of a ball tossed straight up in the air.



1.6 Motion in One Dimension

An object's motion can be described in terms of three fundamental quantities: its position $\vec{r}$, velocity $\vec{v}$, and acceleration $\vec{a}$. These are vectors, but for motion in one dimension, the vectors are restricted to point only "forward" or "backward." Consequently, we can describe one-dimensional motion with the simpler quantities x, v_x, and a_x (or y, v_y, and a_y). However, we need to give each of these quantities an explicit *sign*, positive or negative, to indicate whether the position, velocity, or acceleration vector points forward or backward.

Determining the Signs of Position, Velocity, and Acceleration

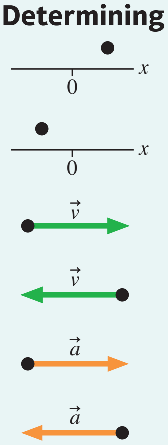
Position, velocity, and acceleration are measured with respect to a coordinate system, a grid or axis that *you* impose on a problem to analyze the motion. We will find it convenient to use an x-axis to describe both horizontal motion and motion along an inclined plane. A y-axis will be used for vertical motion. A coordinate axis has two essential features:

1. An origin to define zero; and
2. An x or y label (with units) at the positive end of the axis.

NOTE In this textbook, we will follow the convention that **the positive end of an x-axis is to the right and the positive end of a y-axis is up.** The signs of position, velocity, and acceleration are based on this convention.

TACTICS BOX 1.3

Determining the sign of the position, velocity, and acceleration



$x > 0$ Position to right of origin.

$x < 0$ Position to left of origin.

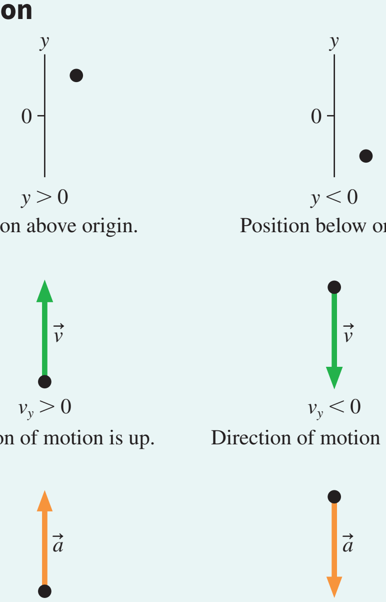
$v_x > 0$ Direction of motion is to the right.

$v_x < 0$ Direction of motion is to the left.

$a_x > 0$ Acceleration vector points to the right.

$a_x < 0$ Acceleration vector points to the left.

- The sign of position (x or y) tells us *where* an object is.
- The sign of velocity (v_x or v_y) tells us *which direction* the object is moving.
- The sign of acceleration (a_x or a_y) tells us which way the acceleration vector points, *not* whether the object is speeding up or slowing down.



$y > 0$ Position above origin.

$y < 0$ Position below origin.

$v_y > 0$ Direction of motion is up.

$v_y < 0$ Direction of motion is down.

$a_y > 0$ Acceleration vector points up.

$a_y < 0$ Acceleration vector points down.

Exercises 30–31

Acceleration is where things get a bit tricky. A natural tendency is to think that a positive value of a_x or a_y describes an object that is speeding up while a negative value describes an object that is slowing down (decelerating). However, this interpretation *does not work*.

Acceleration is defined as $\vec{a}_{avg} = \Delta\vec{v}/\Delta t$. The direction of $\vec{a}$ can be determined by using a motion diagram to find the direction of $\Delta\vec{v}$. The one-dimensional acceleration a_x (or a_y) is then positive if the vector $\vec{a}$ points to the right (or up), negative if $\vec{a}$ points to the left (or down).

FIGURE 1.16 One of these objects is speeding up, the other slowing down, but they both have a positive acceleration a_x.

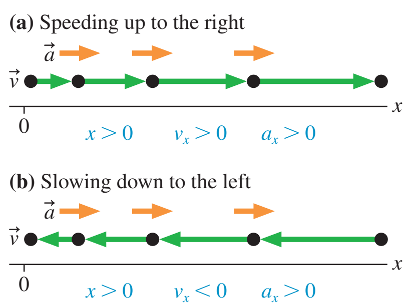


FIGURE 1.16 shows that this method for determining the sign of a does not conform to the simple idea of speeding up and slowing down. The object in Figure 1.16a has a positive acceleration $(a_x > 0)$ not because it is speeding up but because the vector $\vec{a}$ points in the positive direction. Compare this with the motion diagram of Figure 1.16b. Here the object is slowing down, but it still has a positive acceleration $(a_x > 0)$ because $\vec{a}$ points to the right.

In the previous section, we found that an object is speeding up if $\vec{v}$ and $\vec{a}$ point in the same direction, slowing down if they point in opposite directions. For one-dimensional motion this rule becomes:

- An object is speeding up if and only if v_x and a_x have the same sign.
- An object is slowing down if and only if v_x and a_x have opposite signs.
- An object's velocity is constant if and only if $a_x = 0$.

Notice how the first two of these rules are at work in Figure 1.16.

Position-versus-Time Graphs

FIGURE 1.17 is a motion diagram, made at 1 frame per minute, of a student walking to school. You can see that she leaves home at a time we choose to call $t = 0$ min and makes steady progress for a while. Beginning at $t = 3$ min there is a period where the distance traveled during each time interval becomes less—perhaps she slowed down to speak with a friend. Then she picks up the pace, and the distances within each interval are longer.

FIGURE 1.17 The motion diagram of a student walking to school and a coordinate axis for making measurements.

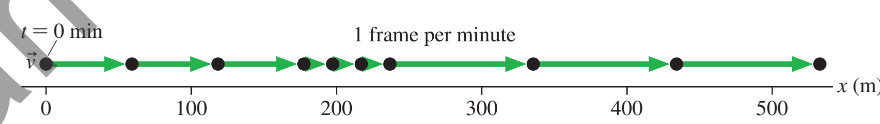


TABLE 1.1 Measured positions of a student walking to school

Time t (min)	Position x (m)	Time t (min)	Position x (m)
0	0	5	220
1	60	6	240
2	120	7	340
3	180	8	440
4	200	9	540

Figure 1.17 includes a coordinate axis, and you can see that every dot in a motion diagram occurs at a specific position. **TABLE 1.1** shows the student's positions at different times as measured along this axis. For example, she is at position $x = 120$ m at $t = 2$ min.

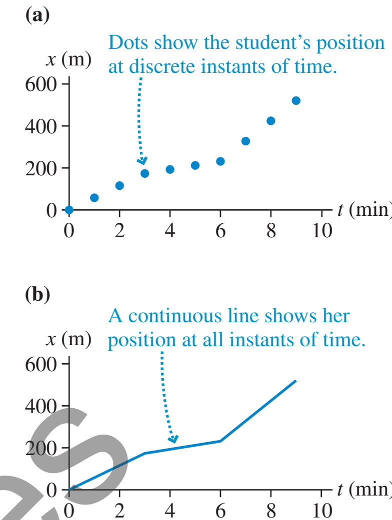
The motion diagram is one way to represent the student's motion. Another is to make a graph of the measurements in Table 1.1. **FIGURE 1.18a** is a graph of x versus t for the student. The motion diagram tells us only where the student is at a few discrete points of time, so this graph of the data shows only points, no lines.

NOTE A graph of "a versus b" means that a is graphed on the vertical axis and b on the horizontal axis. Saying "graph a versus b" is really a shorthand way of saying "graph a as a function of b."

However, common sense tells us the following. First, the student was *somewhere specific* at all times. That is, there was never a time when she failed to have a well-defined position, nor could she occupy two positions at one time. Second, the student moved *continuously* through all intervening points of space. She could not go from $x = 100$ m to $x = 200$ m without passing through every point in between. It is thus quite reasonable to believe that her motion can be shown as a continuous line passing through the measured points, as shown in **FIGURE 1.18b**. A continuous line or curve showing an object's position as a function of time is called a **position-versus-time graph** or, sometimes, just a *position graph*.

NOTE A graph is *not* a "picture" of the motion. The student is walking along a straight line, but the graph itself is not a straight line. Further, we've graphed her position on the vertical axis even though her motion is horizontal. Graphs are *abstract representations* of motion. We will place significant emphasis on the process of interpreting graphs, and many of the exercises and problems will give you a chance to practice these skills.

FIGURE 1.18 Position graphs of the student's motion.



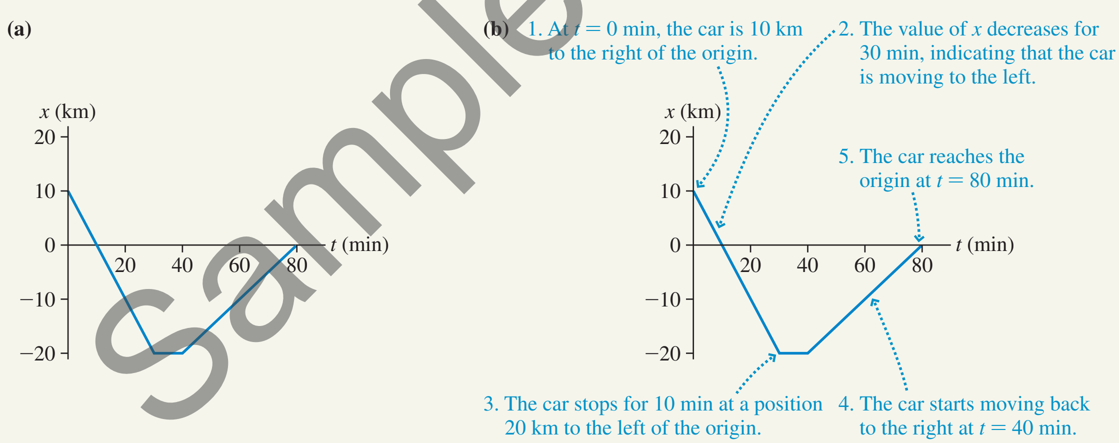
EXAMPLE 1.7 ■ Interpreting a position graph

The graph in **FIGURE 1.19a** represents the motion of a car along a straight road. Describe the motion of the car.

MODEL We'll model the car as a particle with a precise position at each instant.

VISUALIZE As **FIGURE 1.19b** shows, the graph represents a car that travels to the left for 30 minutes, stops for 10 minutes, then travels back to the right for 40 minutes.

FIGURE 1.19 Position-versus-time graph of a car.



1.7 Solving Problems in Physics

Physics is not mathematics. Math problems are clearly stated, such as "What is $2 + 2$?" Physics is about the world around us, and to describe that world we must use language. Now, language is wonderful—we couldn't communicate without it—but language can sometimes be imprecise or ambiguous.

The challenge when reading a physics problem is to translate the words into symbols that can be manipulated, calculated, and graphed. **The translation from words to symbols is the heart of problem solving in physics.** This is the point where ambiguous words and phrases must be clarified, where the imprecise must be made precise, and where you arrive at an understanding of exactly what the question is asking.

Using Symbols

Symbols are a language that allows us to talk with precision about the relationships in a problem. As with any language, we all need to agree to use words or symbols in the same way if we want to communicate with each other. Many of the ways we use symbols in science and engineering are somewhat arbitrary, often reflecting historical roots. Nonetheless, practicing scientists and engineers have come to agree on how to use the language of symbols. Learning this language is part of learning physics.

We will use subscripts on symbols, such as x_3, to designate a particular point in the problem. Scientists usually label the starting point of the problem with the subscript "0," not the subscript "1" that you might expect. When using subscripts, make sure that all symbols referring to the same point in the problem have the *same numerical subscript*. To have the same point in a problem characterized by position x_1 but velocity v_{2x} is guaranteed to lead to confusion!

Drawing Pictures

You may have been told that the first step in solving a physics problem is to "draw a picture," but perhaps you didn't know why, or what to draw. The purpose of drawing a picture is to aid you in the words-to-symbols translation. Complex problems have far more information than you can keep in your head at one time. Think of a picture as a "memory extension," helping you organize and keep track of vital information.

Although any picture is better than none, there really is a *method* for drawing pictures that will help you be a better problem solver. It is called the **pictorial representation** of the problem. We'll add other pictorial representations as we go along, but the following procedure is appropriate for motion problems.

TACTICS BOX 1.4

Drawing a pictorial representation

1. **Draw a motion diagram.** The motion diagram develops your intuition for the motion.
2. **Establish a coordinate system.** Select your axes and origin to match the motion. For one-dimensional motion, you want either the x-axis or the y-axis parallel to the motion. The coordinate system determines whether the signs of v and a are positive or negative.
3. **Sketch the situation.** Not just any sketch. Show the object at the *beginning* of the motion, at the *end,* and at any point where the character of the motion changes. Show the object, not just a dot, but very simple drawings are adequate.
4. **Define symbols.** Use the sketch to define symbols representing quantities such as position, velocity, acceleration, and time. *Every* variable used later in the mathematical solution should be defined on the sketch. Some will have known values, others are initially unknown, but all should be given symbolic names.
5. **List known information.** Make a table of the quantities whose values you can determine from the problem statement or that can be found quickly with simple geometry or unit conversions. Some quantities are implied by the problem, rather than explicitly given. Others are determined by your choice of coordinate system.
6. **Identify the desired unknowns.** What quantity or quantities will allow you to answer the question? These should have been defined as symbols in step 4. Don't list every unknown, only the one or two needed to answer the question.

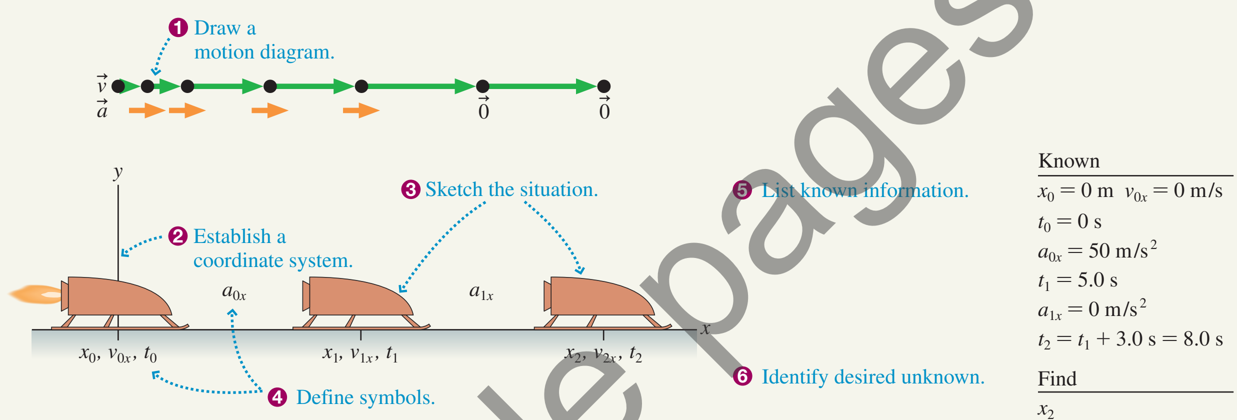
It's not an overstatement to say that a well-done pictorial representation of the problem will take you halfway to the solution. The following example illustrates how to construct a pictorial representation for a problem that is typical of problems you will see in the next few chapters.

EXAMPLE 1.8 ■ Drawing a pictorial representation

Draw a pictorial representation for the following problem: A rocket sled accelerates horizontally at 50 m/s^2 for 5.0 s, then coasts for 3.0 s. What is the total distance traveled?

VISUALIZE **FIGURE 1.20** is the pictorial representation. The motion diagram shows an acceleration phase followed by a coasting phase. Because the motion is horizontal, the appropriate coordinate system is an x-axis. We've chosen to place the origin at the starting point. The motion has a beginning, an end, and a point where the motion changes from accelerating to coasting, and these are the three sled positions sketched in the figure. The quantities x, v_x, and t are needed at each of three *points,* so these have been defined on the sketch and distinguished by subscripts. Accelerations are associated with *intervals* between the points, so only two accelerations are defined. Values for three quantities are given in the problem statement, although we need to use the motion diagram, where we find that $\vec{a}$ points to the right, to know that $a_{0x} = +50$ m/s^2 rather than -50 m/s^2. The values $x_0 = 0$ m and $t_0 = 0$ s are choices we made when setting up the coordinate system. The value $v_{0x} = 0$ m/s is part of our *interpretation* of the problem. Finally, we identify x_2 as the quantity that will answer the question. We now understand quite a bit about the problem and would be ready to start a quantitative analysis.

FIGURE 1.20 A pictorial representation.



We didn't *solve* the problem; that is not the purpose of the pictorial representation. The pictorial representation is a systematic way to go about interpreting a problem and getting ready for a mathematical solution. Although this is a simple problem, and you probably know how to solve it if you've taken physics before, you will soon be faced with much more challenging problems. Learning good problem-solving skills at the beginning, while the problems are easy, will make them second nature later when you really need them.

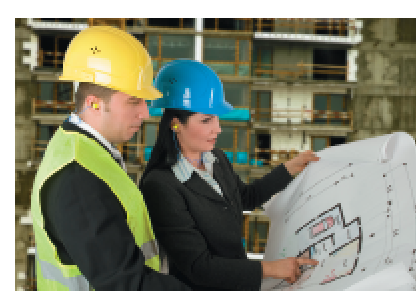
Representations

A picture is one way to *represent* your knowledge of a situation. You could also represent your knowledge using words, graphs, or equations. Each **representation of knowledge** gives us a different perspective on the problem. The more tools you have for thinking about a complex problem, the more likely you are to solve it.

There are four representations of knowledge that we will use over and over:

1. The *verbal* representation. A problem statement, in words, is a verbal representation of knowledge. So is an explanation that you write.
2. The *pictorial* representation. The pictorial representation, which we've just presented, is the most literal depiction of the situation.
3. The *graphical* representation. We will make extensive use of graphs.
4. The *mathematical* representation. Equations that can be used to find the numerical values of specific quantities are the mathematical representation.

NOTE The mathematical representation is only one of many. Much of physics is more about thinking and reasoning than it is about solving equations.



A new building requires careful planning. The architect's visualization and drawings have to be complete before the detailed procedures of construction get under way. The same is true for solving problems in physics.

A Problem-Solving Strategy

One of the goals of this textbook is to help you learn a *strategy* for solving physics problems. The purpose of a strategy is to guide you in the right direction with minimal wasted effort. The four-part problem-solving strategy—**Model, Visualize, Solve, Review**—is based on using different representations of knowledge. You will see this problem-solving strategy used consistently in the worked examples throughout this textbook, and you should endeavor to apply it to your own problem solving.

GENERAL PROBLEM-SOLVING STRATEGY

MODEL It's impossible to treat every detail of a situation. Simplify the situation with a model that captures the essential features. For example, the object in a mechanics problem is often represented as a particle.

VISUALIZE This is where expert problem solvers put most of their effort.

- Draw a *pictorial representation.* This helps you visualize important aspects of the physics and assess the information you are given. It starts the process of translating the problem into symbols.
- Use a *graphical representation* if it is appropriate for the problem.
- Go back and forth between these representations; they need not be done in any particular order.

SOLVE Only after modeling and visualizing are complete is it time to develop a *mathematical representation* with specific equations that must be solved. All symbols used here should have been defined in the pictorial representation.

REVIEW Is your result believable? Does it have proper units? Does it make sense?

Throughout this textbook we will emphasize the first two steps. They are the *physics* of the problem, as opposed to the mathematics of solving the resulting equations. This is not to say that those mathematical operations are always easy—in many cases they are not. But our primary goal is to understand the physics.

Textbook illustrations are obviously more sophisticated than what you would draw on your own paper. To show you a figure very much like what *you* should draw, the final example of this section is in a "pencil sketch" style. We will include one or more pencil-sketch examples in nearly every chapter to illustrate exactly what a good problem solver would draw.

EXAMPLE 1.9 ■ Launching a weather rocket

Use the first two steps of the problem-solving strategy to analyze the following problem: A small rocket, such as those used for meteorological measurements of the atmosphere, is launched vertically with an acceleration of 30 m/s^2. It runs out of fuel after 30 s. What is its maximum altitude?

MODEL We need to do some interpretation. Common sense tells us that the rocket does not stop the instant it runs out of fuel. Instead, it continues upward, while slowing, until it reaches its maximum altitude. This second half of the motion, after running out of fuel, is like the ball that was tossed upward in the first half of Example 1.6. Because the problem does not ask about the rocket's descent, we conclude that the problem ends at the point of maximum altitude. We'll model the rocket as a particle.

VISUALIZE **FIGURE 1.21** shows the pictorial representation in pencil-sketch style. The rocket is speeding up during the first half of the motion, so $\vec{a}_0$ points upward, in the positive y-direction. Thus the initial acceleration is $a_{0y} = 30\ m/s^2$. During the second half, as the rocket slows, $\vec{a}_1$ points downward. Thus a_{1y} is a negative number.

FIGURE 1.21 Pictorial representation for the rocket.