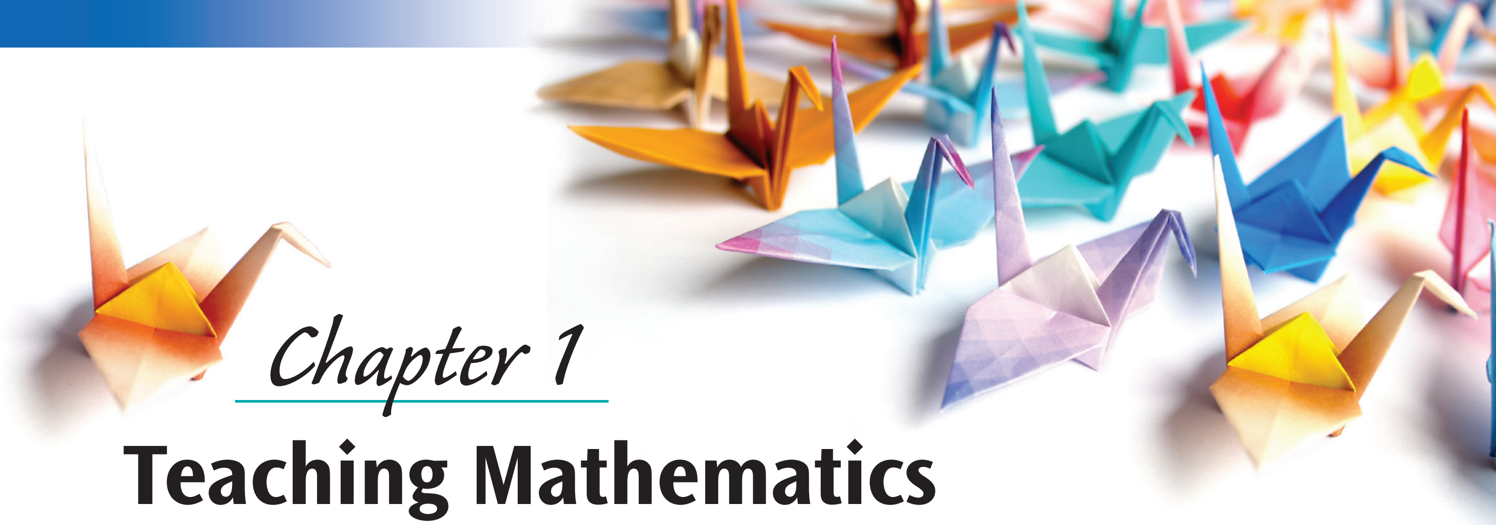

Chapter 1

Teaching Mathematics in the 21st Century

In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures. A lack of mathematical competence keeps those doors closed. . . . All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding.

NCTM (2000, p. 50)

Someday soon you will find yourself in front of a class of students, or perhaps you are already teaching. What general ideas will guide the way you will teach mathematics? This book will help you become comfortable with the mathematics content of the pre-K–8 curriculum. You will also learn about research-based strategies for helping students come to know mathematics and be confident in their ability to do mathematics. These two things—your knowledge of mathematics and how students learn mathematics—are the most important tools you can acquire to be an effective teacher of mathematics. What you teach, however, is largely influenced by state and national standards, as well as local curriculum guides.

For more than two decades, mathematics education has been undergoing steady change. The impetus for this change, in both the content of school mathematics and the way mathematics is taught, can be traced to various sources, including knowledge gained from research. One significant factor in this change has been the professional leadership of the National Council of Teachers of Mathematics (NCTM), the world's largest mathematics education organization, with more than 90,000 members (www.nctm.org). Another factor is the public or political pressure for change in mathematics education due largely to less-than-stellar U.S. student performance in national and international studies. The federal legislation commonly referred to as the No Child Left Behind Act (NCLB) presses for higher levels of achievement, more testing, and increased teacher accountability. Although all agree that we should have high expectations for students, there seems to be little consensus on what the best approach is to improve student learning. According to NCTM, "Learning mathematics is maximized when teachers focus on mathematical thinking and reasoning" (NCTM, 2009, n.d.).

As you prepare to help students learn mathematics, it is important to have some perspective on the forces that effect change in the mathematics classroom. This chapter addresses the leadership that NCTM provides for mathematics education as well as other important influences.

Ultimately, it is you, the teacher, who will shape mathematics for the students you teach. Your beliefs about what it means to know and do mathematics and about how students make sense of mathematics will affect how you approach instruction.

The National Standards-Based Movement

The momentum for reform in mathematics education began in the early 1980s in response to a "back to basics" movement that emphasized "reading, writing, and arithmetic." As a result, problem solving became an important strand in the mathematics curriculum. The work of Jean Piaget and other developmental psychologists helped to focus research on how students can best learn mathematics.

This momentum came to a head in 1989, when NCTM published *Curriculum and Evaluation Standards for School Mathematics* and the standards movement or reform era in mathematics education began. It continues today. No other document has ever had such an enormous effect on school mathematics or on any other area of the curriculum.

In 1991, NCTM published *Professional Standards for Teaching Mathematics*, which articulates a vision of teaching mathematics based on the expectation described in the *Curriculum and Evaluation Standards* that significant mathematics achievement is a vision for all students, not just a few. In 1995, NCTM added to the collection the *Assessment Standards for School Mathematics*, which focuses on the importance of integrating assessment with instruction and indicates the key role that assessment plays in implementing change (see Chapter 5).

In 2000, NCTM released *Principles and Standards for School Mathematics* as an update of its original standards document. Combined, these two standards documents have prompted a revolutionary reform movement in mathematics education, not just in the United States and Canada but throughout the world.

As these documents influenced state policy and teacher practice, ongoing debate continued about the U.S. curriculum. In particular, many argued that instead of hurrying through many topics every year, the curriculum needed to address content more deeply. Guidance was needed in deciding what mathematics content should be taught at each grade level. In 2006, NCTM released *Curriculum Focal Points*, a little publication with a big message—the mathematics taught at each grade level needs to focus, go into more depth, and explicitly show connections. The standards movement had gained significant momentum and engaged more than just the mathematics education community as business and political leaders became interested in a national vision for K–12 mathematics curriculum.

In 2010, the Council of Chief State School Officers (CCSSO) presented *Common Core State Standards*—grade-level specific standards that incorporated ideas from *Curriculum Focal Points* as well as international curriculum documents. A large majority of U.S. states adopted these as their standards. In less than 25 years, the standards movement transformed the country from having little to no national vision on what mathematics should be taught and when, to a widely shared vision of what students should know and be able to do at each grade level.

In the following sections, we discuss these more recent documents because their message is critical to your work as a teacher of mathematics.

Principles and Standards for School Mathematics

Principles and Standards for School Mathematics (NCTM, 2000) provides guidance and direction for teachers and other leaders in pre-K–12 mathematics education.

The Six Principles

One of the most important features of *Principles and Standards for School Mathematics* is the articulation of six principles fundamental to high-quality mathematics education:

- Equity
- Curriculum
- Teaching
- Learning
- Assessment
- Technology

According to *Principles and Standards*, these principles must be "deeply intertwined with school mathematics programs" (NCTM, 2000, p. 12). The principles make it clear that excellence in mathematics education involves much more than simply listing content objectives.

The Equity Principle

> Excellence in mathematics education requires equity—high expectations and strong support for all students. (NCTM, 2000, p. 12)

The strong message of the Equity Principle is high expectations for all students. All students must have the opportunity and adequate support to learn mathematics "regardless of personal characteristics, backgrounds, or physical challenges" (p. 12). The significance of high expectations for all is interwoven throughout the document.

The Curriculum Principle

> A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades. (NCTM, 2000, p. 14)

Coherence speaks to the importance of building instruction around "big ideas"—both in the curriculum and in daily classroom instruction. Students must be helped to see that mathematics is an integrated whole, not a collection of isolated bits and pieces.

Mathematical ideas can be considered "important" if they help develop other ideas, link one idea to another, or serve to illustrate the discipline of mathematics as a human endeavor.

The Teaching Principle

> Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well. (NCTM, 2000, p. 16)

What students learn about mathematics depends almost entirely on the experiences that teachers provide every day in the classroom. To provide high-quality mathematics education, teachers must (1) understand deeply the mathematics content they are teaching; (2) understand how students learn mathematics, including a keen awareness of the individual mathematical development of their own students and common misconceptions; and (3) select meaningful instructional tasks and generalizable strategies that will enhance learning. "Teachers' actions are what encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions" (p. 18).

The Learning Principle

> Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. (NCTM, 2000, p. 20)

The learning principle is based on two fundamental ideas. First, learning mathematics with understanding is essential. Mathematics today requires not only computational skills but also the ability to think and reason mathematically to solve new problems and learn new ideas that students will face in the future.

Second, students *can* learn mathematics with understanding. Learning is enhanced in classrooms where students are required to evaluate their own ideas and those of others, are encouraged to make mathematical conjectures and test them, and are helped to develop their reasoning and sense-making skills.

The Assessment Principle

> Assessment should support the learning of important mathematics and furnish useful information to both teachers and students . . . Assessment should not merely be done *to* students; rather, it should also be done *for* students, to guide and enhance their learning. (NCTM, 2000, p. 22)

Ongoing assessment highlights for students the most important mathematics concepts. Assessment that includes ongoing observation and student interaction encourages students to articulate and, thus, clarify their ideas. Feedback from daily assessment helps students establish goals and become more independent learners.

Assessment should be a major factor in making instructional decisions. By continuously gathering data about students' understanding of concepts and growth in reasoning, teachers can better make the daily decisions that support student learning. For assessment to be effective, teachers must use a variety of assessment techniques, understand their mathematical goals deeply, and have a research-supported notion of students' thinking or common misunderstandings of the mathematics that is being developed.

The Technology Principle

> Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning. (NCTM, 2000, p. 24)

Calculators, computers, and other emerging technologies are essential tools for doing and learning mathematics. Technology permits students to focus on mathematical ideas, to reason, and to solve problems in ways that are often impossible without these tools. Technology enhances the learning of mathematics by allowing for increased exploration, enhanced representation, and communication of ideas.

The Five Content Standards

Principles and Standards includes four grade bands: pre-K–2, 3–5, 6–8, and 9–12. The emphasis on preschool recognizes the need to highlight the critical years before students enter kindergarten. There is a common set of five content standards throughout the grades:

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

Each content standard includes a set of goals applicable to all grade bands followed by grade-band chapters that provide specific expectations for what students should know. Although the same five content standards apply across all grades, you should not infer that each strand has equal weight or emphasis in every grade band. Number and Operations is the most heavily emphasized strand from pre-K through grade 5 and continues to be important in the middle grades, with a lesser emphasis in grades 9–12. This is in contrast to Algebra, which moves from an emphasis related to number and operations in the early grades and builds to a strong focus in the middle and high school grade bands. Section II of this book (Chapters 8 through 23) is devoted to elaborating on these content standards.

The Five Process Standards

Following the five content standards, *Principles and Standards* lists five process standards:

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

The process standards refer to the mathematical processes through which students should acquire and use mathematical knowledge. The statement of the five process standards can be found in Table 1.1.

TABLE 1.1

THE FIVE PROCESS STANDARDS FROM *PRINCIPLES AND STANDARDS FOR SCHOOL MATHEMATICS*	
Problem Solving Standard Instructional programs from prekindergarten through grade 12 should enable all students to—	• Build new mathematical knowledge through problem solving • Solve problems that arise in mathematics and in other contexts • Apply and adapt a variety of appropriate strategies to solve problems • Monitor and reflect on the process of mathematical problem solving
Reasoning and Proof Standard Instructional programs from prekindergarten through grade 12 should enable all students to—	• Recognize reasoning and proof as fundamental aspects of mathematics • Make and investigate mathematical conjectures • Develop and evaluate mathematical arguments and proofs • Select and use various types of reasoning and methods of proof
Communication Standard Instructional programs from prekindergarten through grade 12 should enable all students to—	• Organize and consolidate their mathematical thinking through communication • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others • Analyze and evaluate the mathematical thinking and strategies of others • Use the language of mathematics to express mathematical ideas precisely
Connections Standard Instructional programs from prekindergarten through grade 12 should enable all students to—	• Recognize and use connections among mathematical ideas • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole • Recognize and apply mathematics in contexts outside of mathematics
Representation Standard Instructional programs from prekindergarten through grade 12 should enable all students to—	• Create and use representations to organize, record, and communicate mathematical ideas • Select, apply, and translate among mathematical representations to solve problems • Use representations to model and interpret physical, social, and mathematical phenomena

Source: Standards are listed with permission of the National Council of Teachers of Mathematics (NCTM). NCTM does not endorse the content or validity of these alignments. Reprinted with permission from *Principles and Standards for School Mathematics,* copyright © 2000 by the National Council of Teachers of Mathematics, Inc. www.nctm.org.

The process standards should not be regarded as separate content or strands in the mathematics curriculum. Rather, they direct the methods of doing all mathematics and, therefore, should be seen as integral components of all mathematics learning and teaching. To teach in a way that reflects these process standards is one of the best definitions of what it means to teach "according to the *Standards.*"

The Problem Solving standard describes problem solving as the vehicle through which students develop mathematical ideas. Learning and doing mathematics *as you solve problems* is probably the most significant message in the *Standards* documents.

The Reasoning and Proof standard emphasizes the logical thinking that helps us decide if and why our answers make sense. Students need to develop the habit of providing a rationale as an integral part of every answer. It is essential for students to learn the value of justifying ideas through logical argument.

The Communication standard points to the importance of being able to talk about, write about, describe, and explain mathematical ideas. Learning to communicate in mathematics fosters interaction and exploration of ideas in the classroom as students learn through active discussions of their thinking. No better way exists for wrestling with or cementing an idea than attempting to articulate it to others.

The Connections standard has two parts. First, it is important to connect within and among mathematical ideas. For example, fractional parts of a whole are connected to concepts of decimals and percents. Students need opportunities to see how mathematical concepts build on one another in a network of connected ideas.

Second, mathematics should be connected to the real world and to other disciplines. Students should see that mathematics plays a significant role in art, science, language arts, and social studies. This suggests that mathematics should frequently be integrated with other discipline areas and that applications of mathematics should be explored in real world contexts.

The Representation standard emphasizes the use of symbols, charts, graphs, manipulatives, and diagrams as powerful methods of expressing mathematical ideas and relationships. Symbolism in mathematics, along with visual aids such as charts and graphs, should be understood by students as ways of communicating mathematical ideas to others. Moving from one representation to another is an important way to add depth of understanding to a newly formed idea.

Members of NCTM have free online access to the *Principles and Standards* as well as the three previous standards documents. Nonmembers can sign up for 120 days of free access to the *Principles and Standards* at www.nctm.org.

Curriculum Focal Points: A Quest for Coherence

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence (NCTM, 2006) pinpoints mathematical "targets" for each grade level that specify the big ideas for the most significant concepts and skills. At each grade, three essential areas (focal points) are described as the primary focus of that year's instruction. The topics relating to that focus are organized to show the importance of a *coherent* curriculum rather than a curriculum with a list of isolated topics. The expectation is that three focal points along with integrated process skills and connecting experiences form the fundamental content of each grade. Besides focusing instruction, the document provides guidance to professionals about ways to refine and streamline curriculum in light of competing priorities.

Common Core State Standards

As noted earlier, the national dialogue on improving mathematics teaching and learning extends beyond mathematics educators. State policy makers and elected officials have also considered NCTM standards documents, international assessments, and research on the best way to prepare students to be "college and career ready." The state governors (National Governors Association Center for Best Practices) and the Council of Chief State School Officers (CCSSO) collaborated with many other professional groups and entities to develop such benefits as shared expectations for K–12 students across states, a focused set of mathematics content standards and practices, and efficiency of material and assessment development (Porter, McMaken, Hwang, & Yang, 2011). As a result, they created the *Common Core State Standards for Mathematics* (which can be downloaded at www.corestandards.org). Like *Curriculum Focal Points*, this document articulates an overview of *critical areas* for each grade from kindergarten through 8 to provide a coherent curriculum built around big ideas. These larger groups of related standards are called *domains*, and there are eleven that relate to grades K–8 (see Figure 1.1).

At this time approximately 44 of the 50 states (and Washington, D.C., and the Virgin Islands) have adopted the *Common Core State Standards*. Notice that these standards are silent on preschool-aged students, so the use of the *Curriculum Focal Points* remains significant in making curricular decisions for this age group.

Mathematical Practice. The *Common Core State Standards* goes beyond specifying mathematics content to include Standards for Mathematical Practice. These are "'processes and proficiencies' with longstanding importance in mathematics education" (CCSSO, 2010, p. 6) that are founded on the five NCTM process standards and the components of mathematical proficiency identified by NRC in their important document *Adding It Up* (National Research Council, 2001). Teachers must develop these mathematical practices in all students (CCSSO, 2010, pp. 7–8) as described briefly in Table 1.2. (A more detailed description of the Standards for Mathematical Practice can be found in Appendix A.)

Learning Progressions. The *Common Core State Standards* were developed with strong consideration given to building coherence through the research on what is known about the development of students' understanding of mathematics

<table>
<tr><th>Kindergarten</th><th>Grade 1</th><th>Grade 2</th><th>Grade 3</th><th>Grade 4</th><th>Grade 5</th><th>Grade 6</th><th>Grade 7</th><th>Grade 8</th></tr>
<tr><td>Counting and Cardinality</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="6">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td></tr>
<tr><td colspan="9">Geometry</td></tr>
<tr><td></td><td></td><td></td><td colspan="3">Number and Operations—Fractions</td><td colspan="2">Ratios and Proportional Relationships</td><td>Functions</td></tr>
</table>

FIGURE 1.1 *Common Core State Standards* domains by grade level.

TABLE 1.2

THE STANDARDS FOR MATHEMATICAL PRACTICE FROM THE *COMMON CORE STATE STANDARDS*	
K–8 Students Should Be Able To:	
Make sense of problems and persevere in solving them	• Explain the meaning of a problem • Describe possible approaches to a solution • Consider similar problems to gain insights • Use concrete objects or illustrations to think about and solve problems • Monitor and evaluate their progress and change strategy if needed • Check their answers using a different method
Reason abstractly and quantitatively	• Explain the relationship between quantities in problem situations • Represent situations using symbols (e.g., writing expressions or equations) • Create representations that fit the problem • Use flexibly the different properties of operations and objects
Construct viable arguments and critique the reasoning of others	• Understand and use assumptions, definitions, and previous results to explain or justify solutions • Make conjectures by building a logical set of statements • Analyze situations and use counterexamples • Justify conclusions in a way that is understandable to teachers and peers • Compare two possible arguments for strengths and weaknesses
Model with mathematics	• Apply mathematics to solve problems in everyday life • Make assumptions and approximations to simplify a problem • Identify important quantities and use tools to map their relationships • Reflect on the reasonableness of their answer based on the context of the problem
Use appropriate tools strategically	• Consider a variety of tools and choose the appropriate tool (e.g., manipulative, ruler, technology) to support their problem solving • Use estimation to detect possible errors • Use technology to help visualize, explore, and compare information
Attend to precision	• Communicate precisely using clear definitions and appropriate mathematics language • State the meanings of symbols • Specify appropriate units of measure and labels of axes • Use a degree of precision appropriate for the problem context
Look for and make use of structure	• Explain mathematical patterns or structures • Shift perspective and see things as single objects or as composed of several objects • Explain why and when properties of operations are true in a context
Look for and express regularity in repeated reasoning	• Notice if calculations are repeated and use information to solve problems • Use and justify the use of general methods or shortcuts • Self-assess to see whether a strategy makes sense as they work, checking for reasonableness prior to getting the answer

Source: Adapted from Council of Chief State School Officers. (2010). *Common Core State Standards.*

over time (Cobb & Jackson, 2011). The resulting selections of topics at particular grades reflects not only rigorous mathematics but also what is known from current research and practice about learning progressions—sometimes referred to as *learning trajectories* (Confrey, Maloney, & Nguyen, 2011; Daro, Mosher, & Corcoran, 2011; Sarama & Clements, 2009) or teaching-learning paths (Cross, Woods, & Schweingruber, 2009). It is these learning progressions that can help teachers know what came before as well as what to expect next as students reach "key waypoints" (Corcoran, Mosher, & Rogat, 2009) on the road to learning mathematics concepts. These progressions identify the interim goals students should reach on the path to desired learning targets (Daro, Mosher, & Corcoran, 2011). Although these paths are not identical for all students, they can inform the order of instructional experiences that will support movement toward understanding and application of mathematics concepts. Go to http://math.arizona.edu/~ime/progressions to find progressions for the domains in the *Common Core State Standards*.

Assessments. New summative assessments are being developed that will be aligned to the *Common Core State Standards*. The assessments will focus on both the grade-level content standards and the standards for mathematical practice. This process would eliminate the need for each state to develop their own assessments for the standards, a problem that has existed since the beginning of the standards era.

Professional Standards for Teaching Mathematics and *Mathematics Teaching Today*

In addition to curriculum-related standards, NCTM has developed related standards documents about teaching. *Professional Standards for Teaching Mathematics* (1991) and its companion document, *Mathematics Teaching Today* (2007a), use detailed classroom stories (vignettes) of real teachers to illustrate the careful, reflective work that is required of effective teachers of mathematics.

Mathematics Teaching Today and its predecessor are excellent resources to help you envision your role as a teacher in creating a classroom that supports teaching through problem solving. As you read the chapters in this book, you will note that the following seven standards are developed in ways that will support your growth as a teacher of mathematics. (See Appendix B for detailed descriptions of these standards.)

1. Knowledge of Mathematics and General Pedagogy
2. Knowledge of Student Mathematical Learning
3. Worthwhile Mathematical Tasks
4. Learning Environment
5. Discourse
6. Reflection on Student Learning
7. Reflection on Teaching Practice

Mathematics Teaching Today lists six major components of the mathematics classroom that are necessary to allow students to develop mathematical understanding:

- Creating an environment that offers all students an equal opportunity to learn
- Focusing on a balance of conceptual understanding and procedural fluency
- Ensuring active student engagement in the NCTM process standards (problem solving, reasoning, communication, connections, and representation)
- Using technology to enhance understanding
- Incorporating multiple assessments aligned with instructional goals and mathematical practices
- Helping students recognize the power of sound reasoning and mathematical integrity (NCTM, 2007a).

PAUSE and REFLECT

Take a moment now to select one or two of the six components that seem especially significant to you and are areas you wish to develop. Why do you think these are the most important to your teaching?

Influences and Pressures on Mathematics Teaching

National and international comparisons of student performance continue to make headlines, provoke public opinion, and pressure legislatures to call for tougher standards backed by testing. The pressures of testing policies exerted on schools and ultimately on teachers may have an impact on instruction.

National and International Studies

Large studies that tell the public how the students are doing in mathematics receive a lot of attention. They influence political decisions as well as provide useful data for mathematics education researchers. Why do these studies matter? Because international and national assessments provide strong evidence that mathematics teaching *must* change if our students are to be competitive in the global market and able to understand the complex issues they must confront as responsible citizens.

National Assessment of Educational Progress. Since the 1960s and at regular intervals, the United States gathers national data on how students are doing in mathematics (and other content areas) through the National Assessment of Educational Progress (NAEP). These data provide an important tool for policy makers and educators to measure the overall improvement of U.S. students over time. Reported in what is called the "Nation's Report Card," NAEP examines both national and state-level trends. NAEP rates fourth-, eighth-, and twelfth-grade students using four achievement levels: below basic, basic, proficient, and advanced (with proficient and advanced representing substantial grade-level achievement). The criterion-referenced test is designed to reflect current curriculum but keeps a few stable items from 1982 for purposes of comparison (Kloosterman, Rutledge, & Kenney, 2009b). In the most recent assessment in 2009, less than half of all U.S. students in grades 4 and 8 performed at the desirable levels of proficient and advanced (39 percent in fourth grade and 34 percent in eighth grade) (National Center for Education Statistics, 2009b). Although No Child Left Behind legislation specifies that all students must be at or above the proficient level by 2014, current NAEP data suggest that goal is likely unattainable. Most troubling, approximately 18 percent of fourth-grade students and 27 percent of eighth-grade students were at the below-basic level. Despite encouraging gains in the NAEP scores over the last 30 years due to important shifts to standards-based instruction (particularly at the elementary level) (Kloosterman, Rutledge, & Kenney, 2009b), U.S. students' performance still reveals some students have disappointing levels of competency. More detailed information can be found at http://nationsreportcard.gov/math_2009.

Trends in International Mathematics and Science Study (TIMSS). In the mid-1990s, 41 nations participated in the Third International Mathematics and Science Study, the largest study of mathematics and science education ever conducted. Data were gathered in grades 4, 8, and 12 from 500,000 students as well as from teachers. The most widely reported results revealed that U.S. students performed above the international average of the TIMSS countries at the fourth grade, below the average at the eighth grade, and significantly below average at the twelfth grade (U.S. Department of Education, 1997a).

TIMSS studies were repeated in 1999 (38 countries), 2003 (46 countries), and again in 2007 (63 countries). (See http://nces.ed.gov/timss for details.) The 2007 TIMSS found that U.S. fourth and eighth graders were above the international average, but were significantly outperformed by eight countries or parts of countries (Hong Kong, Singapore, Chinese Taipei, Japan, Kazakhstan, Russian Federation, England, and Latvia) at the fourth-grade level and by five countries (Chinese Taipei, Korea, Singapore, Hong Kong, and Japan) at the eighth-grade level. Only 15 percent of U.S. fourth graders and 10 percent of U.S. eighth graders performed above the advanced international benchmark. This is in stark contrast with Singapore at 44 percent at the fourth grade and 32 percent at the eighth grade. The impressive performance by Singapore has led some educators to talk about "Singapore mathematics" as a methodology to be emulated.

A report on the original TIMSS curriculum analysis labeled the U.S. mathematics curriculum "a mile wide and an inch deep" (Schmidt, McKnight, & Raizen, 1996, p. 62), meaning it was found to be unfocused, pursuing many more topics than other countries while engaging in a great deal of repetition. They found the U.S. curriculum attempted to do everything and, as a consequence, rarely provided depth of study, making reteaching all too common.

One of the most interesting components of the study was the videotaping of eighth-grade classrooms in the United States, Australia, and five of the highest-achieving countries. The results indicate that teaching is a cultural activity and, despite similarities, the differences in the ways countries taught mathematics were often striking. In all countries, problems or tasks were frequently used to begin the lesson. However, as a lesson progressed, the way these problems were handled in the United States was in stark contrast to high-achieving countries.

Analysis revealed that although the world is for all purposes unrecognizable from what it was 100 years ago, the U.S. approach to teaching mathematics during the same time frame was essentially unchanged (Stigler & Hiebert, 2009). They found the U.S. teacher begins with a review of previous materials or homework and then demonstrates a problem. Students practice similar problems at their desks, the teacher checks the seatwork, and then assigns problems for either the remainder of the class session or homework. (Sound familiar?) In more than 99.5 percent of the U.S. lessons, the teacher reverts to showing students how to solve the problems. In not one of the 81 U.S. lessons was any high-level mathematics content observed; in contrast, 30 to 40 percent of lessons in Germany and Japan contained high-level mathematics content. Although all teachers knew the research team was coming to videotape, 89 percent of the U.S. lessons consisted exclusively of low-level content. Other countries incorporated a variety of methods, but they frequently used a problem-solving approach with an emphasis on conceptual understanding and students engaged in problem solving (Hiebert et al., 2003). Teaching in the high-achieving countries more closely resembles the recommendations of the NCTM standards than does the teaching in the United States.

Curriculum

As described in the beginning of this chapter, curriculum documents (standards) have a significant influence on what is taught, and even how it is taught. In addition, the textbook is a very influential factor in determining the what, when, and how of actual teaching. What is becoming increasingly complicated is how teachers and school systems attempt to align existing textbooks or other curriculum materials with the *Common Core State Standards*, *Curriculum Focal Points*, or other key documents.

Textbooks greatly influence teaching practice. A teacher using one textbook may be more likely to cover many topics, spend one day on each topic, use a teacher-directed instructional approach, and focus on procedures. Using a different textbook (that is more standards-based), a teacher may devote more time to a concept, teach it more deeply, and use a student-centered approach. Writing, speaking, working in groups, and problem solving are more likely to be commonplace components in current curriculum offerings. The selection of curriculum materials is an important endeavor.

In Section II of this book you will find features describing activities from two standards-based (problem-solving oriented) curriculum programs: *Investigations in Number, Data, and Space* (Grades K–5) and *Connected Mathematics Project* (Grades 6–8). These features are included to offer you some insight into how a textbook can support your implementation of the standards (both the content and the processes/practices).

A Changing World Economy

In his book *The World Is Flat* (2007), Thomas Friedman discusses the need for people to have skills that are lasting and will survive the ever-changing landscape of available jobs. These are what he calls "the untouchables"—the individuals who outlast all the ups and downs of the economy. He suggests people who fit into several broad categories that he defines will not be challenged by a shifting job market. One of his safety-ensuring categories is "math lovers." Friedman points out that in a world that is digitized and surrounded by algorithms, math lovers will always have career opportunities and options.

Now it becomes the job of the teacher to develop this passion in students. As Lynn Arthur Steen, a well-known mathematician and educator, states, "As information becomes ever more quantitative and as society relies increasingly on computers and the data they produce, an innumerate citizen today is as vulnerable as the illiterate peasant of Gutenberg's time" (1997, p. xv).

The changing world influences what should be taught in pre-K–8 mathematics classrooms. As we prepare pre-K–8 students for jobs that possibly do not currently exist, we do know that there are few jobs for people where they just do simple computation. We can predict that there will be work that requires interpreting complex data, designing algorithms to make predictions, and using the ability to approach new problems in a variety of ways.

An Invitation to Learn and Grow

The mathematics education described in the NCTM *Principles and Standards* and new *Common Core State Standards* may not be the same as the mathematics and the mathematics teaching you experienced in grades K through 8. Along the way, you may have had excellent teachers of mathematics who reflect the current reform spirit. Examples of good standards-based curriculum have been around since the early 1990s. But for the most part, after more than two decades the goals of the reform movement have yet to be realized in many of the school districts in North America.

As a practicing or prospective teacher facing the challenge of teaching standards-based mathematics from a problem-solving approach, this book may require you to confront some of your personal beliefs—about what it means to *do mathematics*, how one goes about *learning mathematics*, how to *teach mathematics through reasoning and sense making*, and what it means to *assess mathematics* so that it leads to targeted instructional change.

As part of this personal assessment, you should understand that mathematics is unfortunately seen as the subject that people love to hate. At parties or even at parent–teacher conferences, other adults will respond to the fact that you are a teacher of mathematics with comments such as "I could never do math," or "I can't even balance my checking account." Instead of just dismissing these disclosures, consider what action you can take. Would people confide that they don't read and hadn't read a book in years? That is not likely. Families' and teachers' attitudes toward mathematics may enhance or detract from students' ability to do math. It is important for you and for students' families to know that mathematics ability is not inherited—anyone can learn mathematics. Moreover, learning mathematics is an essential life skill. You need to find ways of countering these statements, especially if they are stated in the presence of students, pointing out the importance of the topic and the fact that all people have the capacity to learn mathematics. Only in that way can the long-standing sequence that passes this apprehension from family member to child (or in rare cases teacher to student) be broken. There is much joy to be had in solving mathematical problems, and you need to model this and nurture that passion in your students.

Students and adults alike need to think of themselves as mathematicians, in the same way as many think of themselves as readers. As all people interact with our increasingly mathematical and technological world, they need to construct, modify, or integrate new information in many forms. Solving novel problems and approaching circumstances with a mathematical perspective should come as naturally as reading new materials to comprehend facts, insights, or news. Consider how important this is to interpreting and successfully surviving in our economy. Thinking and talking about mathematics instead of focusing on the "one right answer" is a strategy that will serve us well in becoming a society where all citizens are confident that they can do math.

Becoming a Teacher of Mathematics

This book and this course of study are critical to your professional teaching career. The mathematics education course you are taking now or the professional development you are experiencing will be the foundation for much of the mathematics instruction you do in your classroom for the next decade. The authors of this book take that seriously, as we know you do. Therefore, this section lists and describes the characteristics, habits of thought, skills, and dispositions you will need to succeed as a teacher of mathematics.

Knowledge of Mathematics. You will need to have a profound, flexible, and adaptive knowledge of mathematics content (Ma, 1999). This statement is not intended to scare you if you feel that mathematics is not your strong suit, but it is meant to help you prepare for a serious semester of learning about mathematics and how to teach it. The "school effects" for mathematics are great, meaning that unlike other subject areas, where students have frequent interactions with their family or others outside of school on topics such as reading books, exploring nature, or discussing current events, in the area of mathematics what we do in school is often "it" for many students. This adds to the earnestness of your responsibility, because a student's learning for the year in mathematics will likely come only from you.

If you are not sure of a fractional concept or other aspect of mathematics content knowledge, now is the time to make changes in your depth and flexibility of understanding to best prepare for your role as an instructional leader. This book and your professor will help you in that process.

Persistence. You need the ability to stave off frustration and demonstrate persistence. This is the very skill that your students must have to conduct mathematical investigations. As you move through this book and work the problems yourself, you will learn methods and strategies that will help you anticipate the barriers to student learning and identify strategies to get them past these stumbling blocks. It is likely that what works for you as a learner will work for your students. As you experience the material in this book, if you ponder, struggle, talk about your thinking, and reflect on how it all fits or doesn't fit your prior knowledge, then you enhance your repertoire as a teacher. Remember you need to demonstrate these characteristics so your students can model them. Creating opportunities for your students to struggle is part of learning (Stigler & Hiebert, 2009).

Positive Attitude. Arm yourself with a positive attitude toward the subject of mathematics. Research shows that teachers with positive attitudes teach math in more successful ways that result in their students liking math more (Karp, 1991). If in your heart you say, "I never liked math," that will be evident in your instruction. The good news is that research shows that changing attitudes toward mathematics is relatively easy (Tobias, 1995) and that attitude changes are long-lasting (Dweck, 2006). Through expanding your knowledge of the subject and trying new ways to approach problems, you can learn to enjoy mathematical activities. Not only can you acquire a positive attitude toward mathematics, it is essential that you do.

Readiness for Change. Demonstrate a readiness for change, even for change so radical that it may cause disequilibrium. You may find that what is familiar will become unfamiliar and, conversely, what is unfamiliar will become familiar. For example, you may have always referred to "reducing fractions" as the process of changing $\frac{2}{4}$ to $\frac{1}{2}$, but this phrase is not appropriate because it is misleading—the fractions are not getting smaller. Such terminology can lead to mistaken connections. ("Did the reduced fraction go on a diet?") A careful look will point out that *reducing* is not the term to use; rather, you are writing an equivalent fraction that is simplified. Even though you have used the term *reducing* for years, you need to become familiar with more precise language such as "simplifying fractions."

On the other hand, what is unfamiliar will become more comfortable. It may feel uncomfortable for you to be asking students, "Did anyone solve it differently?" especially if you are worried that you won't understand their approach. Yet this is essential to effective teaching. As you bravely use this strategy, it will become comfortable (and you will learn new things!).

Another potentially difficult change is toward an emphasis on concepts as well as procedures. What happens in a procedure-focused classroom when a student doesn't understand division of fractions? A teacher with only procedural knowledge is often left to repeat the procedure louder and slower. "Just change the division sign to multiplication, flip over the second fraction, and multiply." We know this approach doesn't work well, so let's consider an example using $3\frac{1}{2} \div \frac{1}{2} =$ ______. You might relate this division problem to a whole number division problem such as $25 \div 5 =$ ______. A corresponding story problem might be, "How many orders of 5 pizzas are there in a group of 25 pizzas?" Then ask students to put words around the fraction division problem, such as "You plan to serve each guest $\frac{1}{2}$ a pizza. If you have $3\frac{1}{2}$ pizzas, how many guests can you serve?" Yes, there are seven halves in $3\frac{1}{2}$ and therefore 7 guests can be served. Are you surprised that you can do this problem in your head?

To respond to students' challenges, uncertainties, and frustrations you may need to unlearn and relearn mathematical concepts, developing comprehensive understanding and substantial representations along the way. Supporting your mathematics content knowledge on solid, well-supported terrain is your best hope of making a lasting difference—so be ready for change. What you already understand will provide you with many "Aha" moments as you read this book and connect new information to your current mathematics knowledge.

Reflective Disposition. Make time to be self-conscious and reflective. As Steve Leinwand wrote, "If you don't feel inadequate, you're probably not doing the job" (2007, p. 583). No matter whether you are a preservice teacher or an experienced teacher, there is more to learn about the content and methodology of teaching mathematics. The ability to examine oneself for areas that need improvement or to reflect on successes and challenges is critical for growth and development. The best teachers are always trying to improve their practice through the latest article, the newest book, the most recent conference, or by signing up for the next series of professional development opportunities. These teachers don't say, "Oh, that's what I am already doing"; instead, they identify and celebrate one small tidbit that adds to their repertoire. The best teachers never finish learning all that they need to know, they never exhaust the number of new mental connections that they make, and, as a result, they never see teaching as stale or stagnant. An ancient Chinese proverb states, "The best time to plant a tree is twenty years ago; the second best time is today." So, as John Van de Walle said with every new edition, "Enjoy the journey!"

RESOURCES *for Chapter 1*

RECOMMENDED READINGS

Articles

Hoffman, L., & Brahier, D. (2008). Improving the planning and teaching of mathematics by reflecting on research. *Mathematics Teaching in the Middle School*, *13* (7), 412–417.
This article addresses how a teacher's philosophy and beliefs influence his or her mathematics instruction. Using TIMSS and NAEP studies as a foundation, the authors talk about posing higher-level problems, asking thought-provoking questions, facing students' frustration, and using mistakes to enhance understanding of concepts. They pose a set of reflective questions that are good for self-assessment or discussions with peers.

Books

Lambdin, D., & Lester, F. K., Jr. (2010). *Teaching and learning mathematics: Translating research for elementary school teachers*. Reston, VA: NCTM.
Using the most current research on the teaching and learning of mathematics, this book translates research into meaningful chapters for classroom teachers. Built around major questions on a variety of topics, the authors highlight the importance of research in helping teachers be reflective and to assist in the day-to-day judgments teachers make as they support all learners.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.
The hallmark of this book is the formulation of five strands of "mathematical proficiency": conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Educators and policy makers will cite this book for many years to come.

ONLINE RESOURCES

Dare to Compare (NCES Kids' Zone)
http://nces.ed.gov/nceskids/eyk/index.asp
See how your students perform compared to peers from around the world on items used on past administrations of the grades 4 and 8 NAEP and grades 4, 8, and 12 TIMSS.

Illuminations
http://illuminations.nctm.org
A companion website to NCTM provides lessons, interactive applets, dynamic paper, and links to websites for learning and teaching mathematics.

Illustrative Mathematics Project
http://illustrativemathematics.org
This site provides tools and support for the *Common Core State Standards*. It includes multiple ways to look at the standards across grades and domains as well as provides task and problems that will illustrate individual standards.

National Council of Teachers of Mathematics
www.nctm.org
Here you can discover everything about NCTM and its resources to support your work. Also find an overview of several standards-based documents, position statements, research-based clips and briefs, free access to interactive digital lessons, professional development resources, membership and conference information, online publications store, links to related sites, and much more.

Progressions Documents for the Common Core Math Standards
http://math.arizona.edu/~ime/progressions
This site provides the learning progressions based on mathematical structure and students' cognitive development at given grades across the domains in the *Common Core State Standards*.

PROFESSIONAL LEARNING OPPORTUNITIES *for Chapter 1*

DISCUSSION QUESTION

1. In NCTM's *Mathematics Teaching Today* are six components of the mathematics classroom (p. 7) that are suggested as necessary to allow students to develop mathematical understanding. Examine these shifts and discuss which aspects of each shift seem the most significant to you including *why* these are the most significant to you and/or your students.

PROFESSIONAL DEVELOPMENT ACTIVITIES

1. Ask teachers to examine a textbook at any grade level of their choice. If possible, use a teacher's edition. Ask them to page through any chapter and look for signs of the five NCTM process standards or the eight CCSSO Standards for Mathematical Practice. Ask them to what extent students who are being taught from that textbook are likely to be doing and learning mathematics in ways described by those processes

or practices. What would you have to do to supplement the general approach of that text to meet these goals?

2. Examine the Standards for Teaching Mathematics (Appendix B). As teachers read the list, have them code each in the following manner using these three symbols. (You can use a gallery walk approach for this too.)
   - ! the practices that excite you
   - ? the practices that you wonder about or question
   - ✓ the practices that are reflective of your current practice (individually, grade level team, school)

   Discuss in small groups. Ask teachers to set one to two goals for their mathematics instructional practice informed by this coding and discussion.

3. Read the article "Improving the Planning and Teaching of Mathematics by Reflecting on Research" listed in the Recommended Readings section. Begin by discussing the major findings from the TIMSS study. Then use the set of reflective questions in the article to prompt analysis of teachers' philosophies and actions in their classrooms.

PD TOOLKIT™

Among the many tools on PDToolkit, the following tools provide support in teaching mathematics in light of the NCTM standards:

- Video *Using Manipulatives as Models* as a basis to discuss the NCTM standards in action
- *NCTM Process Standards* classroom observation tool
- Teacher reflection tools with the indicated foci:
  - Student Understanding
  - Reflecting on Teaching and Learning
  - Reactions to Learning with Technology
  - Professional Growth

COACH/TEACHER LEADER CONSIDERATIONS

- Informed by the goal setting from Professional Development Activity 2, the coach should make a plan with the teacher about how to best support her/him in the process of reaching her/his goal(s) and should regularly check in regarding progress toward the goal(s), celebrating progress along the way.