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CHAPTER 1 Introduction

The basic ideas of prestressed concrete are introduced in this chapter. We explain what prestressing is and the advantages and disadvantages of prestressing concrete members. Methods of posttensioning and pretensioning are explained. The chapter includes a short historical note on the development of prestressed concrete, from its beginnings at the end of the 19th Century. The basic ideas of prestressed concrete are
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1.1 Prestressed concrete

1.1.1 Plain concrete and reinforced concrete

When external load is applied to structural members such as beams and slabs, large regions in the member are subjected to tensile stress. Tensile stresses may also be induced in structural members by load-independent effects such as temperature gradients and imposed deformations due to foundation movement. Because of its very low tensile strength, plain concrete cannot be used to construct such members where significant tension is present.

The compressive strength of concrete is reasonably good and if small amounts of reinforcing steel are placed in strategic locations in the concrete to carry the internal tensile forces that develop, an effective load-carrying mechanism is created. The resulting composite material is **reinforced concrete**. The great advantage of concrete as a building material is that it is very cheap, and even when reinforcing steel is added in small quantities, the cost

advantage is still sufficient to ensure that reinforced concrete is presently the most widely used structural material, both in Australia and world-wide.

The flexural behaviour of reinforced concrete is illustrated in Figure 1.1, where the beam is supported on a simple span of length *L* and has two equal point loads *W* acting at the third points of the span which are located at distance $a = L/3$ from the supports.

Figure 1.1 Reinforced concrete beam with external loads

Tensile stresses develop in the lower fibres of the beam as the first increments of load are applied. With increasing load, cracks soon appear throughout the mid-span region, where the moment is largest. At any cracked cross-section X-X between the load points (Figure 1.1(b)), the internal moment *M* is resisted by a tensile force *T* in the steel, located in the lower cracked region of the concrete, and an equal compressive force *C* in the intact compressive concrete above the crack. The steel is effective in carrying the tensile force in a cracked section, but it does not prevent or delay cracking of the tensile concrete. That is not its function. With increasing load the cracked region extends outwards towards each support, and the beam deflection increases. Under full

service load, a well-developed pattern of fine cracks is present in the lower fibres of the beam.

With overload, the existing cracks widen and the cracked region extends even further outwards. At high overload, the steel reinforcement in the mid-span region yields. The cracks then widen even more and the deflection increases rapidly with only a very slight further increase in load. Eventually the ultimate moment M_{u} of the sections is reached in the mid-span region and the beam fails in flexure at its load capacity W_{max} . In the design of reinforced concrete flexural members, the aim is to achieve good service load behaviour, in particular with narrow crack widths and small deflections, and adequate strength to prevent premature failure.

1.1.2 Prestressed concrete

Prestressing is another way of circumventing the poor tensile strength of plain concrete: a system of permanent compressive stresses is introduced into the regions of a concrete member where tensile stresses will subsequently develop when the external service loads act. This pre-compression delays tensile cracking, and may even prevent it altogether at service loads. The downward deflection due to external load is also reduced. Prestressing is, thus, an effective way of improving the service-load behaviour of a reinforced concrete member. Compressive prestress in the concrete cross-sections is usually achieved by the use of highly stressed, high-strength tensile steel or fibre reinforced plastic (FRP) tendons that run through the length of the member. The tendons are permanently anchored to the concrete at each beam end. At each internal cross-section, the tensile force in the tendon produces equilibrating compressive stresses in the concrete. mate moment $M_{\rm H}$ of the sections is reached in the m
beam fails in flexure at its load capacity $W_{\rm H}$. The concrete flexural members, the aim is to achieve good
in particular with narrow crack widths and small del

To illustrate the use of prestressing, we return to the reinforced concrete beam in Figure 1.1 and we consider the effect of stressing two draped external steel prestressing tendons, placed on the side faces as shown in Figure 1.2(a). At the ends, the tendons are anchored to the concrete at the section mid-depth. In-span they are draped around cast-in-place pins located in the lower fibres of the concrete, directly under the load points. In the mid-span region the eccentricity of the tendons, relative to the section centroid, is *e* and the total tensile force in the two tendons is *P*. The prestressed tendons apply forces to the concrete at the pins and at the end anchors. This is shown in Figure 1.2(a).

The upward force at each pin, W_p , is slightly inclined from the vertical, and the force *P* at each end is slightly inclined from the horizontal. These forces are self-equilibrating. The overall effect of the prestress is to create an upwards camber in the beam, as in Figure 1.2(b). When the external loads *W* are applied there is a downwards deflection (Figure 1.2(c)), but it is reduced by the prior prestress.

Figure 1.2 Prestressed concrete beam, draped tendons

To investigate the stresses in the concrete due to the prestress, we consider the free body to the left of section X-X in the central region, as shown in Figure 1.3. At X-X the tensile prestressing force *P* in the tendons is horizontal, and at eccentricity *e.* This induces compressive stresses in the concrete, which have a resultant force $C = P$ that also must act at eccentricity *e*. The eccentric force *C* is statically equivalent to a compressive force *C* acting at the centroid of the section, plus a negative moment $M_p = Ce$, as in Figure 1.4(a). The concrete stresses are thus the sum of a uniformly distributed compressive stress C/A_c and the bending stresses due to M_p . At the bottom and top fibres, the stresses are:

$$
\sigma_{\text{pb}} = P \left[\frac{1}{A_c} \sum_{Z} \right]
$$
\n
$$
\sigma_{\text{pa}} = P \left[\frac{1}{A_c} - \frac{e}{Z} \right]
$$
\n(1.1)

Here A_c is the area of the concrete cross-section and Z is its section modulus, which for a rectangular section is $bD^2/6$, where *D* is the section depth and *b* is the section width. The stress distribution in the section is shown in Figure 1.4(b). While the upper fibre stress σ_{pa} is shown as compressive, it will be tensile if the eccentricity *e* is sufficiently large, whereas the bottom fibre stress $\sigma_{\rm nb}$ is always compressive. The effect of any reinforcing steel in an uncracked section is very small and is ignored in Equations 1.1 and 1.2. Figure 1.3 Equilibrating forces at section X-X
 $\sigma_{\text{pb}} = P\left[\frac{1}{A_c} + \frac{e}{Z}\right]$

Here A_c is the area of the concrete cross-section and Z

which for a rectangular section is $bD^2/6$, where D is the

the section width.

The external loads *W* induce a positive moment $M = Wa$ in the central region, with compressive stress in the upper fibres and tensile stress in the lower fibres. However, compressive stress is already present in the lower fibres of the section due to prestress. The resultant stresses at section X-X due to prestress plus external load are as shown in Figure 1.4(c).

Whether or not the resultant stresses remain compressive in the bottom fibres, as shown in Figure 1.4(c), depends on the magnitudes of the prestressing force *P,* the eccentricity *e* and the load-induced moment, *M*. In any case, cracking will be delayed, or possibly even prevented, by the prestress. Also, the initial upwards deflection due to the prestress (Figure 1.2(b)) reduces, and may eliminate completely, the downwards deflection due to the external load *W*.

Figure 1.4 Concrete stresses at section X-X due to prestress and external load

In this example the prestressing tendons have been "draped". The downward eccentricity in the middle region produces a negative moment $M_p = Pe$, which opposes the moment *M* due to the external loads. In the outer regions the eccentricity *e* reduces progressively to zero. The bottom fibre compressive stress due to prestress, σ_{pb} , reduces to the value P/A_c at the end of the beam, while the tensile stress due to external loading reduces to zero.

Various other tendon arrangements, different to the one shown in Figure 1.2, can be used to reduce cracking and deflection. For example, Figure 1.5(a) shows a curved tendon located in a parabolically shaped duct that has been cast in the concrete. The tendon is tensioned against the ends of the hardened concrete and then anchored permanently. As we shall see shortly, this form of construction is known as **post-tensioning**. The force in the tendon is *P* and the maximum eccentricity at mid-span, as before, is *e*. The upwards deflection due to the prestress is shown in the Figure.

Figure 1.5 Parabolic and straight tendons

The self-equilibrating force system exerted on the concrete by the curved prestressing tendon is shown in Figure 1.5(a). It consists of slightly inclined end forces P at the anchorage points, and an upward distributed force w_p acting along the beam in a direction perpendicular to the curved cable. If the cable curvature is constant (which is the case if the shape is parabolic) then it can be shown that w_p is a uniformly distributed force.

At mid-span the tendon force *P* is horizontal. It is equilibrated by a horizontal compressive force *C* in the concrete. The distribution of compressive concrete stresses here is therefore the same as that shown already in Figure 1.4(b). Although the eccentricity of the tendon decreases towards the supports, the prestressed tendon induces compressive stresses in the lower fibres at each section so that cracking is delayed or prevented when the load is applied. The total downwards deflection is also reduced.

In Figure 1.5(b) yet another tendon shape is shown. This time the tendon is straight, with the eccentricity *e* constant along the full length of the beam.

This profile is typical when **pre-tensioning** is employed. The straight tendon induces the same stress distribution in every section along the beam, and this is as shown in Figure 1.4(b). There is a uniform compression of P/A_c and bending stresses due to the moment $M_p = P e$. The overall effect is again to induce compressive stresses in the lower fibres and hence to delay or prevent cracking. The initial upwards camber acts to reduce or prevent deflection under external service load. However, the negative prestressing moment is now constant along the beam, and so becomes greater than the positive applied moment in the outer regions near the beam ends. In the design of such beams, care must be taken to ensure that the negative moment due to prestress is not excessive in the end regions.

For the beam in Figure 1.5(b), the self-equilibrating forces due to the straight tendon consists simply of equal and opposite horizontal forces *P* applied with eccentricity *e* at each end of the beam. This is statically equivalent to axial end forces *P*, plus a pair of negative end moments $M_p = -Pe$ at each end.

Figures 1.2 to 1.5 show how various tendon shapes are used to induce an equilibrating system of forces that act on the concrete. These forces act at the ends of the tendon where it is anchored to the concrete, and at any point along the span where the tensioned tendon changes direction. In particular, a concentrated force W_p acts at a kink in the tendon (as in Figure 1.2) and a distributed transverse force w_p acts over any length of member in which the tendon is curved (as in Figure $1.5(a)$). outer regions near the beam ends. In the design of such
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The forces exerted by the prestressed tendon on the beam can be thought of as **equivalent loads**. For example, in Figure 1.2 the equivalent loads are the upwards acting point loads W_p and the inclined end forces *P*, while in Figure 1.5(a) the equivalent loads consist of a uniformly distributed equivalent load w_p and the inclined end forces *P*.

The concept of equivalent loads gives us a simple but very useful view of the effect of prestress on the behaviour of members, both statically determinate and statically indeterminate. It also provides a convenient method for evaluating the stresses that are produced in a beam by the prestress. Furthermore, a simple but extremely useful design technique, called **load balancing**, can be developed from the equivalent load concept. Ideas of equivalent loads and load balancing are discussed in detail in Chapter 4.

THE IDEA OF PRESTRESSING

The idea of prestressing has wider applicability than in the field of prestressed concrete. A simple form of prestressing has been used by coopers for centuries to construct wine barrels by forcing heated metal tension bands over wooden staves. The precompression induced when the bands cool prestresses the staves together and prevents leaking. A variation of this technique is used today in the construction of large circular prestressed concrete liquid-retaining tanks. The procedure involves winding prestressing tendons around precast vertical concrete 'staves'.

One of the first suggestions to introduce prestress into structural concrete was made by P H Jackson in 1886 in San Francisco. A patent taken out in Berlin in 1888 by Doehring anticipated the idea of the production method which uses a pretensioning bed. Various proposals and tests followed, but this early developmental work was unsuccessful because mild steel reinforcing bars were used as the prestressing medium.

In the 1920s, R H Dill in the USA recognised that high strength wire could be used to produce a satisfactory prestressed member. However, the first successful practical designs in prestressed concrete were carried out in Europe by Eugene Freyssinet in the 1930s, when the time-dependent creep and shrinkage behaviour of concrete came to be better understood. In the United States, prestressed concrete was first used in the construction of circular water tanks. In the late 1930s the Preload Corporation developed the technique of winding wires around cast-in-place circular concrete walls. prevents leaking. A variation of this technique is
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precast vertical concrete 'staves'.
One of the first suggestions to int

Shortly after World War 2, Freyssinet designed a number of successful and highly acclaimed bridges in France, which led to wide acceptance of prestressed concrete. An upsurge in interest in prestressed concrete at that time can be attributed in part to the scarcity and high cost of steel and other structural materials in those post-war years. In the United States, prestressed concrete was first used in bridge construction in the late 1940s. Interesting historical information on the development of prestressed concrete, on personalities involved in its early development, and on the range of structures previously constructed in prestressed concrete, is to be found in the T Y Lin Symposium on Prestressed Concrete, reported in the Prestressed Concrete Journal (Lin, 1976).

1.2 Prestressing as a design option

Prestressed concrete, like reinforced concrete, takes advantage of the compressive strength of concrete, while circumventing its weakness in tension. Prestressed concrete is usually made from concrete of medium to high strength, with a small quantity of very high strength prestressing steel tendon. Ordinary non-prestressed reinforcing steel is also included in the member, both as subsidiary longitudinal reinforcement, that becomes effective after cracking, and as transverse stirrups to improve shear strength.

As a design option, prestressing has associated with it a number of advantages, and some disadvantages, that the designer always needs to keep in mind.

1.2.1 Advantages of prestressing

Improved service load behaviour

We have seen how prestressing can improve the service load behaviour of a concrete member. Even quite moderate levels of prestressing can reduce crack widths to very small values under working load. Deflections can be controlled to remain within a desired range or entirely eliminated, for a chosen service load, by introducing the appropriate level of prestress. It should be noted that prestress does not in itself have any significant effect on flexural strength although the presence of prestressing steel in the tensile region of an under-reinforced beam contributes to its moment capacity. ing the discussion of the discussion of the discussion and the discussion and the discussion has associated with it a number of advantages, s, that the designer always needs to keep in mind.

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Efficient use of high strength steel and concrete.

Prestressed concrete makes possible the efficient and economic use of both very high strength steel and high strength concrete in an efficient structural medium. High strength steel is not efficient if used as ordinary reinforcement. This is because unacceptably large deflections and excessively wide cracks may occur in the member under service load, long before the steel is stressed even into its working load range.

More slender concrete structures with larger spans.

With the effective use of high strength materials together with the control of deflections and crack widths by prestressing, it becomes possible on the one

hand to produce concrete structures that are more slender than reinforced concrete structures, or, on the other hand, to use significantly larger spans than would otherwise be possible. Prestressing is a particularly useful design option for large-span structures where self-weight is a high proportion of the total load and also in structures subjected to large imposed dead load. It is particularly advantageous when used in medium-span and long-span bridges.

Improved recovery after overload

Prestress ensures that there is crack closure and good deflection recovery following an unexpected short-term overload, even when the overload is quite severe. This is not the case with reinforced concrete construction.

Improved strength in shear and torsion

The presence of compressive prestress in the concrete delays the formation of inclined cracks as well as vertical cracks. Prestress can therefore be used to reduce or eliminate shear and torsion problems, for example by increasing the inclined cracking load to a level that exceeds the flexural strength of the member. Even if inclined cracking does occur, the shear and torsional capacity of the member is improved by the presence of prestress. The capacity of a member in shear and torsion can be increased substantially by introducing special vertical prestress. Improved recovery after overload

Prestress ensures that there is crack closure and good

lowing an unexpected short-term overload, even whe

severe. This is not the case with reinforced concrete

Improved strength in shea

Improved fatigue resistance

Fatigue failure in a concrete flexural member is far more likely to occur in the tensile steel than in the compressive concrete. The fatigue resistance therefore depends largely on the fatigue properties of the tensile steel, and on the amplitude of the stress cycles that occur in the tensile steel due to the load cycles. Prestress increases greatly the minimum stress level and so reduces the amplitude of the tensile stress cycle. The fatigue resistance of a member can be greatly improved by introducing (or increasing the level of) longitudinal prestress.

Other uses

Prestressing has a variety of useful applications apart from the construction of slender concrete structures and members. For example, it is used in the repair and refurbishment of existing structures of concrete, steel and even timber.

1.2.2 Some cautionary aspects of prestressing

Prestress improves the structural performance of a concrete member in various ways, but if it is used inappropriately, or without proper understanding, serious problems can occur. If, for example, high prestress is applied at a large eccentricity to a flexural member, the camber (upward deflection) may become unacceptably large and it will increase with time because of concrete creep. Excessive camber can be a form of unserviceability. Cracking may also occur in the upper surface of the member during prestressing, before the full service load acts. In extreme situations, if an excessively high prestress is used, catastrophic failure can occur by crushing of the concrete during the prestressing operation.

Crushing of the concrete in the anchorage regions in post-tensioned members is a potential problem that can arise during construction. Very high, localised forces are induced in the concrete directly behind the mechanical anchorages. Alternatively, longitudinal cracks can occur in the anchorage region, in both post-tensioned and pre-tensioned members, and extend into the span and effectively destroy the member.

The production of prestressed concrete requires special equipment and refined construction operations. The services of a skilled labour force are also required. High-quality materials need to be used, with careful attention paid to quality assurance. Thus there are additional costs involved when prestressing is chosen as a design option. The disadvantages must be considered carefully and weighed against the structural advantages and cost savings. The decision to use prestressing needs to be based on a proper analysis of overall cost effectiveness and structural efficiency. method to be used, with careful attention and the street of the same method of the concrete during the prestressing operations, if an excessively high prestress is used, catastrophic sching of the concrete during to prestr

1.3 Use of high-strength tendons and cables

The advantages of prestressing were well appreciated by the early pioneers of reinforced concrete construction. Ingenious attempts to introduce prestressing into concrete were made in the late 19th Century, when reinforced concrete was still being developed as a building material. In early experiments, prestress was applied to mild steel reinforcement but this was not successful because the prestressing force was completely lost due to the large inelastic deformations that occur in concrete over time as a result of creep and shrinkage. It was not appreciated then that concrete undergoes such large, long-term inelastic strains.

Progressive loss of prestress due to creep and shrinkage

To illustrate the effect of creep and shrinkage we consider the concrete member shown in Figure 1.6, of length *L* and axially prestressed by means of a post-tensioned tendon in a duct at the centroid of the section. The initial condition of zero stress prior to prestressing is represented in Figure 1.6(a). The tendon is tensioned by jacking against the right end of the concrete to produce a tensile force *P* in the tendon and an equal and opposite compressive axial force *C* in the concrete. The tendon ends are then anchored to the concrete. Just after the prestressing operation the deformations consist of (a) an initial tensile tendon strain of $\varepsilon_{\text{po}} = P/A_{\text{p}}E_{\text{p}}$, where A_{p} and E_{p} are the area of the tendon and its elastic modulus, respectively, and (b) an initial compressive elastic strain in the concrete of $\varepsilon_{\text{co}} = P/A_{\text{c}}E_{\text{c}}$, where A_{c} and E_{c} are the area and elastic modulus of the concrete. The corresponding deformations are shown in Figure 1.6(b).

Figure 1.6 Strains during and after post-tensioning, axially placed tendon

Following stressing, the concrete continues to shorten due to creep and shrinkage. The tensioned tendon shortens by an equal amount, so that the prestressing force decreases with time. The situation after a long period of time is shown in Figure 1.6(c), where the final inelastic concrete strain is ε_{ct} . The decrease in tensile strain in the tendon is equal to ε_{ct} and the loss of prestress is $\Delta P = A_p E_p(\epsilon_{\text{ct}})$ in both the tendon and the concrete.

It is instructive to consider typical numerical values. If mild steel bars are used, with a yield stress of 250 MPa and an initial level of prestress of say 200 MPa, the initial elastic steel strain is about 1000 microstrain, i.e. 0.001. The long-term shrinkage strain of concrete in Australia is in the order 0.0006 to 0.0010. Assuming a shrinkage strain of, say, 0.0008, we see that the prestressing force drops sharply over time to just one-fifth of its original value. When we add long-term creep, the result is an almost total loss of prestress. Prestressing losses are always significant, but with mild steel as the prestressing medium, the prestress can be completely lost within weeks.

Use of very high strength prestressing steel to manage losses

It was not until very high strength prestressing steels were introduced in the 1930s that loss of prestress due to creep and shrinkage could be limited, and practical success was achieved in the manufacture of prestressed concrete. At present, prestressing tendons with an ultimate strength of about 1750 MPa are in common use. An initial prestress to 75 per cent of the ultimate strength, say to 1300 MPa, gives an initial strain of about 0.007. With creep and shrinkage losses of, say, 0.001, the loss of prestress is still significant, in the order of 15 per cent, but the residual tension in the steel is ample to maintain good permanent prestress in the concrete. ss of 250 MPa and an initial level of prestress of say
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Leonhardt (1980) used the word 'springiness' to describe the large extension needed in the tendon during the prestressing operation, relative to the small contraction of the concrete. The difference in strains needed in the two materials is almost an order of magnitude. It is important to have this springiness present if the prestressing process is to be successful.

Types of high-strength prestressing steel

Prestressing steel in use today comes in the form of small diameter, **high strength steel wire** and large diameter **high strength alloy bar**. To increase

the capacity of the wire product, individual wires are often fabricated into **multi-wire strand**, typically with seven wires per strand. To further increase the tensile capacity of the prestressing unit, many strands can be used together to form a single post-tensioning **cable**. Details of the properties of wires, strands and cables, as well as information on creep and shrinkage of concrete, will be presented in Chapter 2. The terms **tendon** and **cable** are both used for the prestressing steel in a prestressed member. Depending on context, these terms may indicate either an individual component, for example a cable in a single duct, or the entire assemblage.

Development work has been going on for some years to produce prestressing tendons from materials other than steel, and we can expect to see their increased use in the near future. In this text, however, attention is focused on the use of steel tendons. France Context, these terms may more
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Development work has been going on for some years
tendons from materials other than steel, and we can
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1.4 Methods of prestressing

In Figures 1.2 to 1.5 we have seen how a range of different tendon shapes can be used to prestress a concrete beam and so improve its working load behaviour. We now look at some of the methods that are used to introduce prestress into concrete members and structures:

Post-tensioning

In general, the term **post-tensioning** refers to construction processes in which the cable is tensioned *after* the concrete has been placed and hardened. The cables are typically contained in ducts that are cast in the concrete member. They are prestressed by jacking their ends against the concrete. A simple example of a post-tensioned beam with a single curved cable was shown in Figure 1.5(a).

In large members, such as bridge girders, it may be necessary to use many individual cables to achieve a sufficiently high prestressing force. Each cable may follow a separate path through the member, in its own duct. The individual paths are chosen so as to avoid congestion, and may even follow a three dimensional path, curved both in plan and elevation. The paths of the individual cables are chosen so that, when acting together, the resultant prestressing