MATH ESSENTIAL 1: Units, Significant Figures, and Solving End of Chapter Problems

ME1.1 UNITS

Quantities of interest in physical chemistry such as pressure, volume, or temperature are characterized by their magnitude and their units. In this textbook, we use the SI (from the French *Le Système international d'unités*) system of units. All physical quantities can be defined in terms of the seven base units listed in Table ME1.1. For more details, see http://physics.nist.gov/cuu/Units/units.html. The definition of temperature is based on the coexistence of the solid, gaseous, and liquid phases of water at a pressure of 1 bar.

TABLE ME1.1 Base SI Units

Base Unit	Unit	Definition of Unit
Unit of length	meter (m)	The meter is the length of the path traveled by light in vacuum during a time interval of 1/299,792,458 of a second.
Unit of mass	kilogram (kg)	The kilogram is the unit of mass; it is defined by taking the value of Planck's constant h to be exactly 6.62607015 $\times$ 10^{-34} kg · m^2/s.
Unit of time	second (s)	The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.
Unit of electric current	ampere (A)	The ampere is the constant current that, if maintained in two straight parallel conductors of infinite length, is of negligible circular cross section, and if placed 1 meter apart in a vacuum would produce between these conductors a force equal to 2 $\times$ 10^{-7} kg m s^{-2} per meter of length. In this definition, 2 is an exact number.
Unit of thermodynamic temperature	kelvin (K)	The kelvin, unit of thermodynamic temperature, is defined by taking the value of the Boltzmann constant k to be exactly 1.380649 $\times$ 10^{-23} J/K.
Unit of amount of substance	mole (mol)	The mole is the SI unit of substance. One mole contains exactly 6.02214076 $\times$ 10^{23} elementary entities. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.
Unit of luminous intensity	candela (cd)	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540. $\times$ 10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

Quantities of interest other than the seven base quantities can be expressed in terms of the units meter, kilogram, second, ampere, kelvin, mole, and candela. The most important of these derived units, some of which have special names as indicated, are listed in Table ME1.2. A more inclusive list of derived units can be found at http://physics.nist.gov/cuu/Units/units.html.

TABLE ME1.2 Derived Units

Unit	Definition	Relation to Base Units	Special Name	Abbreviation
Area	Size of a surface	m^2		m^2
Volume	Amount of three-dimensional space an object occupies	m^3		m^3
Velocity	Measure of the rate of motion	$m\ s^{-1}$		$m\ s^{-1}$
Acceleration	Rate of change of velocity	$m\ s^{-2}$		$m\ s^{-2}$
Linear momentum	Product of mass and linear velocity of an object	$kg\ m\ s^{-1}$		$kg\ m\ s^{-1}$
Angular momentum	Product of the moment of inertia of a body about an axis and its *angular* velocity with respect to the same axis	$kg\ m^2\ s^{-1}$		$kg\ m^2\ s^{-1}$
Force	Any interaction that, when unopposed, will change the motion of an object	$kg\ m\ s^{-2}$	newton	N
Pressure	Force acting per unit area	$kg\ m^{-1}\ s^{-2}$ $N\ m^{-2}$	pascal	Pa
Work	Product of force on an object and movement along the direction of the force	$kg\ m^2\ s^{-2}$	joule	J
Kinetic energy	Energy an object possesses because of its motion	$kg\ m^2\ s^{-2}$	joule	J
Potential energy	Energy an object possesses because of its position or condition	$kg\ m^2\ s^{-2}$	joule	J
Power	Rate at which energy is produced or consumed	$kg\ m^2\ s^{-3}$	watt	W
Mass density	Mass per unit volume	$kg\ m^{-3}$		$kg\ m^{-3}$
Radian	Angle at the center of a circle whose arc is equal in length to the radius	$m/m = 1$		$m/m = 1$
Steradian	Angle at the center of a sphere subtended by a part of the surface equal in area to the square of the radius	$m^2/m^2 = 1$		$m^2/m^2 = 1$
Frequency	Number of repeat units of a wave per unit time	s^{-1}	hertz	Hz
Electrical charge	Physical property of matter that causes it to experience an electrostatic force	A s	coulomb	C
Electrical potential	Work done in moving a unit positive charge from infinity to that point	$kg\ m^2\ s^{-3}/A$ W/A	volt	V
Electrical resistance	Ratio of the voltage to the electric current that flows through a conductive material	$kg\ m^2\ s^{-3}/A^2\ W/A^2$	ohm	Ω

If SI units are used throughout the calculation of a quantity, the result will have SI units. For example, consider a unit analysis of the electrostatic force between two charges:

$$F = \frac{q_1 q_2}{8\pi\varepsilon_0 r^2} = \frac{C^2}{8\pi \times kg^{-1}s^4A^2m^{-3} \times m^2} = \frac{A^2 s^2}{8\pi \times kg^{-1}s^4A^2m^{-3} \times m^2}$$

$$= \frac{1}{8\pi}\,kg\ m\ s^{-2} = \frac{1}{8\pi}\,N$$

Therefore, in carrying out a calculation, it is only necessary to make sure that all quantities are expressed in SI units rather than carrying out a detailed unit analysis of the entire calculation.

ME1.2 UNCERTAINTY AND SIGNIFICANT FIGURES

In carrying out a calculation, it is important to take into account the uncertainty of the individual quantities that go into the calculation. The uncertainty is indicated by the number of significant figures. For example, the mass 1.356 g has four significant figures. The mass 0.003 g has one significant figure, and the mass 0.01200 g has four significant figures. By convention, the uncertainty of a number is ± 1 in the rightmost digit. A zero at the end of a number that is not to the right of a decimal point is not significant. For example, 150 has two significant figures, but 150. has three significant figures. Some numbers are exact and have no uncertainty. For example, 1.00×10^6 has three significant figures because the 10 and 6 are exact numbers. By definition, the mass of one atom of ^{12}C is exactly 12 atomic mass units.

If a calculation involves quantities with a different number of significant figures, the following rules regarding the number of significant figures in the result apply:

- In addition and subtraction, the result has the number of digits to the right of the decimal point corresponding to the number that has the smallest number of digits to the right of the decimal point. For example $101 + 24.56 = 126$ and $0.523 + 0.10 = 0.62$.
- In multiplication or division, the result has the number of significant figures corresponding to the number with the smallest number of significant figures. For example, $3.0 \times 16.00 = 48$ and $0.05 \times 100. = 5$.

It is good practice to carry forward a sufficiently large number of significant figures in different parts of the calculation and to round off to the appropriate number of significant figures at the end.

ME1.3 SOLVING END-OF-CHAPTER PROBLEMS

Because calculations in physical chemistry often involve multiple inputs, it is useful to carry out calculations in a manner that they can be reviewed and easily corrected. For example, the input and output for the calculation of the pressure exerted by gaseous benzene with a molar volume of 2.00 L at a temperature of 595 K using the Redlich–Kwong equation of state $P = \frac{RT}{V_m - b} - \frac{a}{\sqrt{T}} \frac{1}{V_m(V_m + b)}$ in Mathematica is shown below. The statement in the first line clears the previous values of all listed quantities, and the semicolon after each input value suppresses its appearance in the output.

```
In[36]:=   Clear[r, t, vm, a, b, prk]
           r = 8.314 × 10^−2;
           t = 595;
           vm = 2.00;
           a = 452;
           b = .08271;
                   rt         a        1
           prk = ------  -  ----  ----------
                 vm − b      √t    vm(vm + b)
out[42]=   21.3526
```

Invoking the rules for significant figures, the final answer is $P = 21.4$ bar.

The same problem can be solved using Microsoft Excel as shown in the following table.

	A	B	C	D	E	F
1	R	T	V_m	a	b	=((A2*B2)/(C2-E2))-(D2/SQRT(B2))*(1/(C2*(C2+E2)))
2	0.08314	595	2	452	0.08271	21.35257941