QUANTITATIVE METHODS FOR BUSINESS

THIRD EDITION

A CUSTOM EDITION FOR THE UNIVERSITY OF SOUTH AUSTRALIA

COMPILED BY DR BELINDA CHIERA AND DR NICK FEWSTER-YOUNG

A PEARSON AUSTRALIA CUSTOM BOOK

3RD EDITION

Quantitative Methods for Business

This custom book is compiled from:

BASIC BUSINESS STATISTICS
5th edition
Berenson, Levine, Szabat, Stephan, O'Brien, Jayne & Watson

MATHEMATICS FOR BUSINESS
10th edition
Salzman & Clendenen

BUSINESS ANALYTICS
3rd edition
Evans

BASIC COLLEGE MATHEMATICS
12th edition
Bittinger, Beecher & Johnson

ACCOUNTING
8th edition
Atrill

HOW TO IMPROVE YOUR MATHS SKILLS
2nd edition
Lakin

UNIVERSITY OF SOUTH AUSTRALIA

table of contents

TOPIC 1 INTRODUCTION 1

Brackets in algebra 2
Chapter 9 Brackets in algebra pp. 72-79
Lakin, S. (2011). *How to improve your maths skills* (2nd ed.). Pearson.

Percentages 11
Chapter 6 Percent notation pp. 359-430
Bittinger, M. L., Beecher, J. A., & Johnson, B. L. (2014). *Basic college mathematics* (Global ed., 12th ed.). Pearson.

TOPIC 2 SIMPLE AND COMPOUND INTEREST 91

Simple interest 93
Chapter 12 Simple interest pp. 451-487
Salzman, S. A., & Clendenen, G. (2014). *Mathematics for business* (10th ed.). Pearson.

Compound interest 131
Chapter 14 Compound interest pp. 523-559
Salzman, S. A., & Clendenen, G. (2014). *Mathematics for business* (10th ed.). Pearson.

TOPIC 3 ANNUITIES AND THEIR APPLICATIONS 169

Annuities and sinking funds 171
Chapter 15 Annuities and sinking funds pp. 561-590
Salzman, S. A., & Clendenen, G. (2014). *Mathematics for business* (10th ed.). Pearson.

TOPIC 4 LINEAR MODELS IN BUSINESS 202

Algebra 203
Chapter 8 An introduction to algebra pp. 65-71
Lakin, S. (2011). *How to improve your maths skills* (2nd ed.). Pearson.

Solving linear equations 211
Chapter 10 Solving linear equations pp. 80-85
Lakin, S. (2011). *How to improve your maths skills* (2nd ed.). Pearson.

Solving simultaneous equations 218
Chapter 12 Solving simultaneous equations pp. 92-102
Lakin, S. (2011). *How to improve your maths skills* (2nd ed.). Pearson.

Break-even analysis and cost-profit analysis 230
Chapter 9 Cost-volume-profit analysis and relevant costing pp. 384-425
Atrill, P., McLaney, E., Harvey, D., & Cong, L. M. (2021). *Accounting* (8th ed.). Pearson.

TOPIC 5 LINEAR PROGRAMMING IN BUSINESS 273

Linear optimization 275
Chapter 13 Linear optimization pp. 493-550
Evans, J. R. (2021). *Business analytics* (Global ed., 3rd ed.). Pearson.

Optimization Analytics 335
Chapter 15 Optimization analytics pp. 593-629
Evans, J. R. (2021). *Business analytics* (Global ed., 3rd ed.). Pearson.

TOPIC 6 DATA TYPES AND VISUALISATION, PROBABILITY AND INDEPENDENCE 373

Presenting data in tables and charts 374
Chapter 1 Defining and collecting data pp. 4-36
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

Chapter 2 Organising and visualising data pp. 37-90
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

Measuring uncertainty and basic probability 465
Chapter 4 Basic probability pp. 147-179
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

TOPIC 7 SUMMARY OF MEASURES 499
Numerical descriptive measures 501
Chapter 3 Numerical descriptive measures pp. 91-143
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

TOPIC 8 LINEAR REGRESSION AND PREDICTIVE ANALYTICS 555
Simple linear regression 557
Chapter 12 Simple linear regression pp. 455-503
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

TOPIC 9 THE NORMAL DISTRIBUTION 607
The normal distribution and other continuous distributions 608
Chapter 6 The normal distribution and other continuous distributions pp. 212-247
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

TOPIC 10 CENTRAL LIMIT THEOREM AND CONFIDENCE INTERVALS FOR THE POPULATION MEAN 645
Sampling distributions 646
Chapter 7 Sampling distributions pp. 248-266
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

Drawing conclusions about populations based only on the sample information 675
Chapter 8 Confidence interval estimation pp. 279-314
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

TOPIC 11 HYPOTHESIS TESTING 712
One-sample 713
Chapter 9 Fundamentals of hypothesis testing: one-sample tests pp. 315-357
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

Two-sample 758
Chapter 10 Hypothesis testing: two-sample tests pp. 358-400
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

Analysis of variance 803
Chapter 11 Analysis of variance pp. 401-450
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

Chi-squared tests 855
Chapter 15 Chi-squared tests pp. 607-641
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

Further tests: non-parametric methods 898
Chapter 19 Further non-parametric tests pp. 740-769
Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

APPENDIX 929

Interest tables 931

Appendix C Interest tables pp. 789-817

Salzman, S. A., & Clendenen, G. (2014). *Mathematics for business* (10th ed.). Pearson.

Review of arithmetic, algebra, logarithms and notation 962

Appendix A Review of arithmetic, algebra and logarithms pp. A1-A3

Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

Further tables 965

Appendix E Tables pp. A15-A41

Berenson, M. L., Levine, D. M., Szabat, K. A., O'Brien, M., Jayne, N., & Watson, J. (2019). *Basic business statistics* (5th ed.). Pearson.

Index 993

TOPIC 1

INTRODUCTION

Brackets in algebra

Chapter 9 Brackets in algebra pp. 72-79

Lakin, S. (2011). *How to improve your maths skills* (2nd ed.). Pearson.

Percentages

Chapter 6 Percent notation pp. 359-430

Bittinger, M. L., Beecher, J. A., & Johnson, B. L. (2014). *Basic college mathematics* (Global ed., 12th ed.). Pearson.

9 Brackets in algebra

Using brackets in algebraic expressions

Here we will continue our study of algebra and show how algebraic expressions use brackets in exactly the same way as 'normal numbers' do – this is what you'd expect, since letters in algebra basically just stand for numbers!

Key topics

- Expanding brackets
- Simplifying expressions
- Multiplying out brackets
- Factorisation

Key terms

algebra and brackets expanding brackets factorising

→ Expanding brackets

Remember from the BODMAS law we introduced in Chapter 1, that things in brackets are always done first, so for example $3 \times (2 + 4)$ is the same as $3 \times 6 = 18$, working out the bit in brackets $(2 + 4)$ first.

Now, note the following: if you multiply both things in the bracket by 3 and add them up, you get $3 \times 2 + 3 \times 4$ which is $6 + 12 = 18$ – the same answer.

So $3 \times (2 + 4)$ is the same as $3 \times 2 + 3 \times 4$.

Does this always work? Check the following are true:

- $4 \times (3 + 2)$ is the same as $4 \times 3 + 4 \times 2$.
- $6 \times (1 + 5)$ is the same as $6 \times 1 + 6 \times 5$.
- $2 \times (3 + 7)$ is the same as $2 \times 3 + 2 \times 7$.

It works when there's a minus inside the brackets too:

- $4 \times (3 - 2)$ is the same as $4 \times 3 - 4 \times 2$.
- $6 \times (1 - 5)$ is the same as $6 \times 1 - 6 \times 5$.
- $2 \times (3 - 7)$ is the same as $2 \times 3 - 2 \times 7$.

In general, this always works. Technically, the word for this is *distributive* but you don't need to know that – but you should know the phrase *expanding the brackets* to describe this process.

Going back to our idea of using symbols to 'stand for something', we can say that for any numbers x, y and z, we have

$$x(y + z) = xy + xz$$

(remember that we don't normally write the multiplication sign $\times$ in algebraic expressions).

➔ Expressions involving brackets

Now you know this, you can use this to simplify more complicated expressions. For example, what is $3(x + y) + 2(x - y)$?

Use the rules that we just talked about: $3(x + y)$ is the same as $3x + 3y$. And $2(x - y)$ is the same as $2x - 2y$. So in total, this expression is the same as $3x + 3y + 2x - 2y$. Now if you collect together like terms, as before, then you get $5x + y$.

So $3(x + y) + 2(x - y)$ is the same as $5x + y$, which is much simpler!

Be careful with minus signs. What is $3(x + y) - 2(x - y)$?

Well, $3(x + y)$ is the same as $3x + 3y$, and $2(x - y)$ is the same as $2x - 2y$, just as before. So you have to work out $3x + 3y - (2x - 2y)$ which is the same as $3x + 3y - 2x + 2y$, which now easily simplifies to $x + 5y$.

Don't forget the minus signs when you expand brackets!

smart tip

The hardest part about doing this was the negative numbers. Don't assume in maths that once you've done a topic, you'll never need it again. For example, you will use negative numbers in many other topics (they are just numbers after all) and they often cause problems – so, regularly go back and quickly revise previous chapters to be sure everything stays in your head.

→ Brackets times brackets

We can use what we developed above to expand even more complicated expressions. What is, for example, $(2 + 3) \times (4 + 5)$? If you work this out, it's 5×9 which gives the answer 45.

Note that if you work out $2 \times 4 + 2 \times 5 + 3 \times 4 + 3 \times 5$ you also get the answer 45 (check this yourself!) Why is this going to be true?

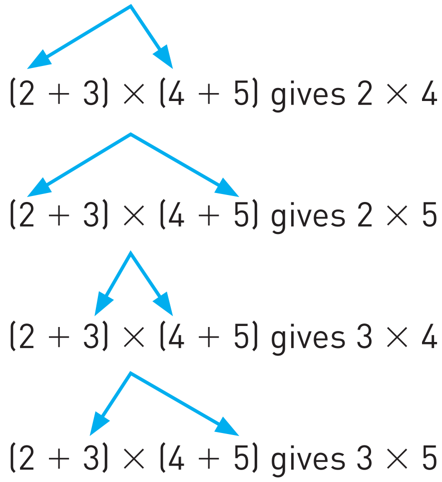
Well, if you used our rules from above about expanding brackets, note that $(2 + 3) \times (4 + 5)$ is the same as $(2 + 3) \times 4 + (2 + 3) \times 5$ (you multiply each bit of the $(4 + 5)$ by what is outside). Because it doesn't matter which order we do multiplication in, this is the same as $4 \times (2 + 3) + 5 \times (2 + 3)$ and we can use our 'expanding brackets' rules again, to get $4 \times 2 + 4 \times 3 + 5 \times 2 + 5 \times 3$, which is the same as $2 \times 4 + 2 \times 5 + 3 \times 4 + 3 \times 5$, just changing the order a little.

What we've done, is basically take every combination of a number from the first bracket and a number from the second bracket:

- The first term is 2×4, which is the first number in $(2 + 3)$ multiplied by the first number in $(4 + 5)$.
- The second term is 2×5, which is the first number in $(2 + 3)$ multiplied by the second number in $(4 + 5)$.
- The third term is 3×4, which is the second number in $(2 + 3)$ multiplied by the first number in $(4 + 5)$.
- The fourth term is 3×5, which is the second number in $(2 + 3)$ multiplied by the second number in $(4 + 5)$.

(Remember that a *term* is one of the things you add up in an expression).

This is illustrated like this:



So in total you have:

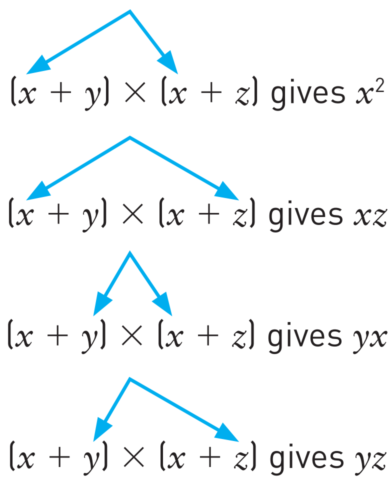
$$2 \times 4 + 2 \times 5 + 3 \times 4 + 3 \times 5 = 45$$

It works in general, for any numbers, and also then for algebraic expressions.

For example, what is $(x + y)(w + z)$? Taking our combinations just like above, this is the same as $xw + xz + yw + yz$.

Again be careful with minus signs: what is $(x - y)(w - z)$? Again taking combinations but being careful we get all the signs right, this is $xw - xz - yw + yz$.

If you have the same letter in both brackets you can end up with something like $x \times x$. Remember that this is written as x^2. So something like $(x + y)(x + z)$ is the same as $x^2 + xz + yx + yz$ – see below:



So in total you have:

$$x^2 + xz + yx + yz$$

It works just the same way if there are numbers in front of the symbols. For example, what is $(x + 2y)(3x + z)$?

The first term is $x \times 3x$ which we would normally write as $3x^2$ ($x \times 3x$ is the same as $x \times 3 \times x$, which is the same as $3 \times x \times x$ since the order of multiplication doesn't matter, which we can write as $3x^2$ ignoring the multiplication signs).

Similarly the second term is xz, the third term is $6xy$ (it is conventional to write symbols in alphabetical order but if you write $6yx$ that is fine) and the fourth term is $2yz$.

Hence the final answer is $3x^2 + xz + 6xy + 2yz$.

> Note that we always write algebraic terms with the number first then the letters (often in alphabetical order) – so for example we would write $6xy$ rather than $y6x$ – it looks much nicer and makes it easier to follow!

Also, remember to collect terms together. If you do $(x + y)(x + y)$ then you get $x^2 + xy + yx + y^2$. Now, xy is just the same as yx, so these two terms can be collected together and you get $x^2 + 2xy + y^2$.

An interesting one to note is $(x + y)(x - y)$. If you expand this, you get $x^2 - xy + yx - y^2$. Since xy is the same as yx, the two middle terms just cancel to leave you with just $x^2 - y^2$. For example, $(5 + 3)(5 - 3)$ is the same as $5^2 - 3^2 = 25 - 9 = 16$ (check this is right!)

→ Factorisation

Here you've seen how you can expand brackets, so for example we worked out that $x(y + z)$ is the same as $xy + xz$

The reverse process, so given $xy + xz$ and saying that this is the same as $x(y + z)$, is called *factorisation*.

All you have to do is look for something that is in every term of our expression. For example, to do it with numbers first, consider the expression

$$3 \times 4 + 3 \times 5$$

There is a '3' in each term, so we 'bring this out to the front'. What this means is to do the 'multiplying out the brackets' in reverse.

The 3 is in every term, so we write it first, then inside the brackets we fill in all the rest of the terms, but getting rid of the multiplying by 3 in each case. That means we write the 3 first, then inside the brackets write $4 + 5$ (what is left from $3 \times 4 + 3 \times 5$ when we get rid of the multiplying by 3 in each term).

Hence this expression is $3 \times (4 + 5)$. Check that this is the same.

It works exactly the same for symbols too. What is $yz + yx$? Following exactly our rules, we note that there is a y in both terms. 'Bring it out to the front' and you are left with $y(z + x)$ as you would expect.

Don't forget it doesn't matter what order you do multiplying in, and you can 'bring out' any term.

For example, what is $xyz + 3y$? The only thing in both terms is y. 'Bring it out' and you are left with $y(xz + 3)$ (getting rid of the y from the xyz term leaves xz and similarly, getting rid of it from the $3y$ term leaves just 3).

What about something like $6x^2 + 8xy$? You have to be careful here.

The numbers aren't the same, but notice that 2 divides into them both. Also, remember that x^2 is the same as $x \times x$. So, you *could* write this expression as $2 \times 3 \times x \times x + 2 \times 4 \times x \times y$.

Now you can see that there is a $2 \times x$ in both terms, so you can 'bring this out' and you get $(2 \times x)$ $(3 \times x + 4 \times y)$ which you would then write as $2x(3x + 4y)$

After a bit of practice you will be able to do this in one go, but it does take some practice.

smart tip

When doing algebra, it's often a good idea to see if your answers seem right by checking with some actual values. For example, suppose you had to expand $x(y + z)$. You should get $xy + xz$, which you can check by choosing some random values of x, y and z, say $x = 2$, $y = 3$, $z = 4$. Then $x(y + z) = 2 \times (3 + 4) = 2 \times 7 = 14$, and also $xy + xz = 2 \times 3 + 2 \times 4 = 6 + 8 = 14$, the same thing, so you can be confident you got it right! If you didn't get the same thing (say you wrongly wrote $xy + z$ as the answer, then you get $2 \times 3 + 4 = 6 + 4 = 10$) then you know you've got it wrong. Checking your answers like this can really help give you reassurance that you are doing the right thing.

Summary

Using normal numbers, we need to use brackets sometimes. Since algebra is just using symbols to represent numbers, then we need brackets here too, and we'd expect them to follow exactly the same rules as they would with normal numbers. We aren't creating anything magical and new, just doing with the symbols the same as we do with normal numbers!

Exercises

1 Verify the following are true.

Example: $2 \times (3 + 5) = 2 \times 3 + 2 \times 5$

Solution: $2 \times (3 + 5) = 2 \times 8 = 16$. Also, $2 \times 3 + 2 \times 5 = 6 + 10 = 16$, so the two things are equal.

(a) $3 \times (4 + 2) = 3 \times 4 + 3 \times 2$
(b) $2 \times (1 + 1) = 2 \times 1 + 2 \times 1$
(c) $3 \times (4 - 2) = 3 \times 4 - 3 \times 2$
(d) $2 \times (3 - 4) = 2 \times 3 - 2 \times 4$
(e) $-2 \times (3 - 2) = (-2) \times 3 + (-2) \times (-2)$
(f) $2 \times (1 - 1) = 2 \times 1 - 2 \times 1$

2 Expand the brackets in the following algebraic expressions.

Example: $x(y + 3z)$

Solution: Multiply each term by x, so you get the answer of $xy + 3xz$.

(a) $x(y + z)$
(b) $a(b + c)$
(c) $r(s - t)$

(d) $2(x + y)$ (e) $4(x - y)$ (f) $2(2x + 3y)$

(g) $2x(y + z)$ (h) $3x(x - y)$ (i) $xy(x + y)$

(j) $x(3 + y)$ (k) $z(x - 2)$ (l) $2y(x - z)$

3 Expand the brackets in the following algebraic expressions.

Example: $4(x + 2y) + 2(3x - y)$

Solution: This is the same as $4x + 8y + 6x - 2y$ which simplifies to $10x + 6y$.

(a) $2(x + y) + 3(x + y)$ (b) $3(x - 2y) + 2(x + 3y)$

(c) $2(x + y) - 3(x - y)$ (d) $5(2x - 3y) - 2(3x - 6y)$

(e) $x(x + y) + y(2x + y)$ (f) $x(x + y) - y(x - y)$

4 Multiply out the following brackets:

Example: $(x + 2y)(3x + z)$

Solution: Working out the four terms in order as done in the examples above (first term multiplied by first term, etc), you get $3x^2 + xz + 6xy + 2yz$.

Note: remember to collect together like terms where possible.

(a) $(t + u)(r + s)$ (b) $(x + y)(z - w)$ (c) $(x + 2y)(2z + 3w)$

(d) $(x + 2y)(2x + y)$ (e) $(x - 2y)(2x + y)$ (f) $(2x + z)(2x - z)$

5 Factorise the following expressions, checking your final answer.

Example: $2xy + 4xz$

Solution: The two terms have a 2 and an x in common, so 'bringing this out to the front', we get $2x(y + 2z)$. Checking by multiplying out, this is indeed the same as $2xy + 4xz$ so we were right.

(a) $2x + 2y$ (b) $3xy + 2xz$ (c) $3xz + 6xy$

(d) $xyz + xyw$ (e) $10xy + 5y$ (f) $x^2y + xy^2$