

# **Feedback Control of Dynamic Systems**

---

**Eighth Edition**

**Gene F. Franklin**

Stanford University

**J. David Powell**

Stanford University

**Abbas Emami-Naeini**

SC Solutions, Inc.



**Pearson**

330 Hudson Street, NY, NY 10013

# Contents

## Preface    xiii

## 1 An Overview and Brief History of Feedback Control 1

A Perspective on Feedback Control	1
Chapter Overview	2
1.1 A Simple Feedback System	3
1.2 A First Analysis of Feedback	6
1.3 Feedback System Fundamentals	10
1.4 A Brief History	11
1.5 An Overview of the Book	18
Summary	19
Review Questions	20
Problems	20

## 2 Dynamic Models 24

A Perspective on Dynamic Models	24
Chapter Overview	25
2.1 Dynamics of Mechanical Systems	25
2.1.1 Translational Motion	25
2.1.2 Rotational Motion	32
2.1.3 Combined Rotation and Translation	43
2.1.4 Complex Mechanical Systems (W)**	46
2.1.5 Distributed Parameter Systems	46
2.1.6 Summary: Developing Equations of Motion for Rigid Bodies	48
2.2 Models of Electric Circuits	49
2.3 Models of Electromechanical Systems	54
2.3.1 Loudspeakers	54
2.3.2 Motors	56
2.3.3 Gears	60
△ 2.4 Heat and Fluid-Flow Models	61
2.4.1 Heat Flow	62
2.4.2 Incompressible Fluid Flow	66
2.5 Historical Perspective	73
Summary	76
Review Questions	76
Problems	77

---

\*\*Sections with (W) indicates that additional material is located on the web at [www.FPE8E.com](http://www.FPE8E.com) or [www.pearsonhighered.com/engineering-resources](http://www.pearsonhighered.com/engineering-resources).

<b>3</b>	<b>Dynamic Response 89</b>
A Perspective on System Response 89	
Chapter Overview 90	
3.1	Review of Laplace Transforms 90
3.1.1	Response by Convolution 91
3.1.2	Transfer Functions and Frequency Response 96
3.1.3	The $\mathcal{L}_-$ Laplace Transform 106
3.1.4	Properties of Laplace Transforms 108
3.1.5	Inverse Laplace Transform by Partial-Fraction Expansion 110
3.1.6	The Final Value Theorem 112
3.1.7	Using Laplace Transforms to Solve Differential Equations 114
3.1.8	Poles and Zeros 116
3.1.9	Linear System Analysis Using Matlab 117
3.2	System Modeling Diagrams 123
3.2.1	The Block Diagram 123
3.2.2	Block-Diagram Reduction Using Matlab 127
3.2.3	Mason's Rule and the Signal Flow Graph (W) 128
3.3	Effect of Pole Locations 128
3.4	Time-Domain Specifications 137
3.4.1	Rise Time 137
3.4.2	Overshoot and Peak Time 138
3.4.3	Settling Time 139
3.5	Effects of Zeros and Additional Poles 142
3.6	Stability 152
3.6.1	Bounded Input–Bounded Output Stability 152
3.6.2	Stability of LTI Systems 154
3.6.3	Routh's Stability Criterion 155
△ 3.7	Obtaining Models from Experimental Data: System Identification (W) 162
△ 3.8	Amplitude and Time Scaling (W) 162
3.9	Historical Perspective 162
Summary 163	
Review Questions 165	
Problems 165	

## 4 A First Analysis of Feedback 186

A Perspective on the Analysis of Feedback 186	
Chapter Overview 187	
4.1	The Basic Equations of Control 188
4.1.1	Stability 189
4.1.2	Tracking 190
4.1.3	Regulation 191
4.1.4	Sensitivity 192

4.2	Control of Steady-State Error to Polynomial Inputs: System Type	194
4.2.1	System Type for Tracking	195
4.2.2	System Type for Regulation and Disturbance Rejection	200
4.3	The Three-Term Controller: PID Control	202
4.3.1	Proportional Control (P)	202
4.3.2	Integral Control (I)	204
4.3.3	Derivative Control (D)	207
4.3.4	Proportional Plus Integral Control (PI)	207
4.3.5	PID Control	211
4.3.6	Ziegler–Nichols Tuning of the PID Controller	216
4.4	Feedforward Control by Plant Model Inversion	222
△ 4.5	Introduction to Digital Control (W)	224
△ 4.6	Sensitivity of Time Response to Parameter Change (W)	225
4.7	Historical Perspective	225
	Summary	227
	Review Questions	228
	Problems	229

## 5

### The Root-Locus Design Method 248

	A Perspective on the Root-Locus Design Method	248
	Chapter Overview	249
5.1	Root Locus of a Basic Feedback System	249
5.2	Guidelines for Determining a Root Locus	254
5.2.1	Rules for Determining a Positive ( $180^\circ$ ) Root Locus	256
5.2.2	Summary of the Rules for Determining a Root Locus	262
5.2.3	Selecting the Parameter Value	263
5.3	Selected Illustrative Root Loci	266
5.4	Design Using Dynamic Compensation	279
5.4.1	Design Using Lead Compensation	280
5.4.2	Design Using Lag Compensation	285
5.4.3	Design Using Notch Compensation	288
△ 5.4.4	Analog and Digital Implementations (W)	290
5.5	Design Examples Using the Root Locus	290
5.6	Extensions of the Root-Locus Method	301
5.6.1	Rules for Plotting a Negative ( $0^\circ$ ) Root Locus	301
△ 5.6.2	Successive Loop Closure	304
△ 5.6.3	Time Delay (W)	309
5.7	Historical Perspective	309

Summary	311
Review Questions	313
Problems	313

## 6 The Frequency-Response Design Method 331

A Perspective on the Frequency-Response Design Method	331
Chapter Overview	332
6.1 Frequency Response	332
6.1.1 Bode Plot Techniques	340
6.1.2 Steady-State Errors	352
6.2 Neutral Stability	354
6.3 The Nyquist Stability Criterion	357
6.3.1 The Argument Principle	357
6.3.2 Application of The Argument Principle to Control Design	358
6.4 Stability Margins	371
6.5 Bode's Gain-Phase Relationship	380
6.6 Closed-Loop Frequency Response	385
6.7 Compensation	386
6.7.1 PD Compensation	387
6.7.2 Lead Compensation ( $W$ )	388
6.7.3 PI Compensation	398
6.7.4 Lag Compensation	398
6.7.5 PID Compensation	404
6.7.6 Design Considerations	411
△ 6.7.7 Specifications in Terms of the Sensitivity Function	413
△ 6.7.8 Limitations on Design in Terms of the Sensitivity Function	418
△ 6.8 Time Delay	421
6.8.1 Time Delay via the Nyquist Diagram ( $W$ )	423
△ 6.9 Alternative Presentation of Data	423
6.9.1 Nichols Chart	423
6.9.2 The Inverse Nyquist Diagram ( $W$ )	428
6.10 Historical Perspective	428
Summary	429
Review Questions	431
Problems	432

## 7 State-Space Design 457

A Perspective on State-Space Design	457
Chapter Overview	458
7.1 Advantages of State-Space	458
7.2 System Description in State-Space	460
7.3 Block Diagrams and State-Space	466

7.4	Analysis of the State Equations	469
7.4.1	Block Diagrams and Canonical Forms	469
7.4.2	Dynamic Response from the State Equations	481
7.5	Control-Law Design for Full-State Feedback	486
7.5.1	Finding the Control Law	487
7.5.2	Introducing the Reference Input with Full-State Feedback	496
7.6	Selection of Pole Locations for Good Design	500
7.6.1	Dominant Second-Order Poles	500
7.6.2	Symmetric Root Locus (SRL)	502
7.6.3	Comments on the Methods	511
7.7	Estimator Design	512
7.7.1	Full-Order Estimators	512
7.7.2	Reduced-Order Estimators	518
7.7.3	Estimator Pole Selection	522
7.8	Compensator Design: Combined Control Law and Estimator (W)	525
7.9	Introduction of the Reference Input with the Estimator (W)	537
7.9.1	General Structure for the Reference Input	539
7.9.2	Selecting the Gain	548
7.10	Integral Control and Robust Tracking	549
7.10.1	Integral Control	549
△	7.10.2 Robust Tracking Control: The Error-Space Approach	551
△	7.10.3 Model-Following Design	563
△	7.10.4 The Extended Estimator	567
△	7.11 Loop Transfer Recovery	570
△	7.12 Direct Design with Rational Transfer Functions	576
△	7.13 Design for Systems with Pure Time Delay	580
7.14	Solution of State Equations (W)	583
7.15	Historical Perspective	585
	Summary	586
	Review Questions	589
	Problems	590

## 8 Digital Control 614

	A Perspective on Digital Control	614
	Chapter Overview	614
8.1	Digitization	615
8.2	Dynamic Analysis of Discrete Systems	618
8.2.1	$z$ -Transform	618
8.2.2	$z$ -Transform Inversion	619

8.2.3	Relationship Between $s$ and $z$	621
8.2.4	Final Value Theorem	623
8.3	Design Using Discrete Equivalents	625
8.3.1	Tustin's Method	625
8.3.2	Zero-Order Hold (ZOH) Method	629
8.3.3	Matched Pole-Zero (MPZ) Method	631
8.3.4	Modified Matched Pole-Zero (MMPZ) Method	635
8.3.5	Comparison of Digital Approximation Methods	636
8.3.6	Applicability Limits of the Discrete Equivalent Design Method	637
8.4	Hardware Characteristics	637
8.4.1	Analog-to-Digital (A/D) Converters	638
8.4.2	Digital-to-Analog Converters	638
8.4.3	Anti-Alias Prefilters	639
8.4.4	The Computer	640
8.5	Sample-Rate Selection	641
8.5.1	Tracking Effectiveness	642
8.5.2	Disturbance Rejection	643
8.5.3	Effect of Anti-Alias Prefilter	643
8.5.4	Asynchronous Sampling	644
△ 8.6	Discrete Design	644
8.6.1	Analysis Tools	645
8.6.2	Feedback Properties	646
8.6.3	Discrete Design Example	648
8.6.4	Discrete Analysis of Designs	650
8.7	Discrete State-Space Design Methods (W)	652
8.8	Historical Perspective	652
	Summary	653
	Review Questions	655
	Problems	655

## 9

## Nonlinear Systems 661

	A Perspective on Nonlinear Systems	661
	Chapter Overview	662
9.1	Introduction and Motivation: Why Study Nonlinear Systems?	663
9.2	Analysis by Linearization	665
9.2.1	Linearization by Small-Signal Analysis	665
9.2.2	Linearization by Nonlinear Feedback	670
9.2.3	Linearization by Inverse Nonlinearity	671
9.3	Equivalent Gain Analysis Using the Root Locus	672
9.3.1	Integrator Antiwindup	679

9.4	Equivalent Gain Analysis Using Frequency Response: Describing Functions	684
9.4.1	Stability Analysis Using Describing Functions	690
△ 9.5	Analysis and Design Based on Stability	694
9.5.1	The Phase Plane	695
9.5.2	Lyapunov Stability Analysis	701
9.5.3	The Circle Criterion	709
9.6	Historical Perspective	715
	Summary	716
	Review Questions	717
	Problems	717

# 10

## Control System Design: Principles and Case Studies 729

	A Perspective on Design Principles	729
	Chapter Overview	729
10.1	An Outline of Control Systems Design	731
10.2	Design of a Satellite's Attitude Control	737
10.3	Lateral and Longitudinal Control of a Boeing 747	755
10.3.1	Yaw Damper	760
10.3.2	Altitude-Hold Autopilot	767
10.4	Control of the Fuel–Air Ratio in an Automotive Engine	773
10.5	Control of a Quadrotor Drone	781
10.6	Control of RTP Systems in Semiconductor Wafer Manufacturing	797
10.7	Chemotaxis, or How <i>E. Coli</i> Swims Away from Trouble	811
10.8	Historical Perspective	821
	Summary	823
	Review Questions	825
	Problems	825

## Appendix A Laplace Transforms 843

A.1	The $\mathcal{L}_-$ Laplace Transform	843
A.1.1	Properties of Laplace Transforms	844
A.1.2	Inverse Laplace Transform by Partial-Fraction Expansion	852
A.1.3	The Initial Value Theorem	855
A.1.4	Final Value Theorem	856

**Appendix B Solutions to the Review Questions 858**

**Appendix C Matlab Commands 875**

**Bibliography 881**

**Index 890**

**List of Appendices on the web at [www.FPE8e.com](http://www.FPE8e.com)  
and [www.pearsonhighered.com/engineering-resources](http://www.pearsonhighered.com/engineering-resources)**

**Appendix WA: A Review of Complex Variables**

**Appendix WB: Summary of Matrix Theory**

**Appendix WC: Controllability and Observability**

**Appendix WD: Ackermann's Formula for Pole Placement**

**Appendix W2.1.4: Complex Mechanical Systems**

**Appendix W3.2.3: Mason's Rule and Signal-Flow Graph**

**Appendix W.3.6.3.1: Routh Special Cases**

**Appendix W3.7: System Identification**

**Appendix W3.8: Amplitude and Time Scaling**

**Appendix W4.1.4.1: The Filtered Case**

**Appendix W4.2.2.1: Truxal's Formula for the Error  
Constants**

**Appendix W4.5: Introduction to Digital Control**

**Appendix W4.6: Sensitivity of Time Response to Parameter  
Change**

**Appendix W5.4.4: Analog and Digital Implementations**

**Appendix W5.6.3: Root Locus with Time Delay**

**Appendix W6.7.2: Digital Implementation of  
Example 6.15**

**Appendix W6.8.1: Time Delay via the Nyquist Diagram**

**Appendix W6.9.2: The Inverse Nyquist Diagram**

**Appendix W7.8: Digital Implementation of Example 7.31**

**Appendix W7.9: Digital Implementation of Example 7.33**

**Appendix W7.14: Solution of State Equations**

**Appendix W8.7: Discrete State-Space Design Methods**