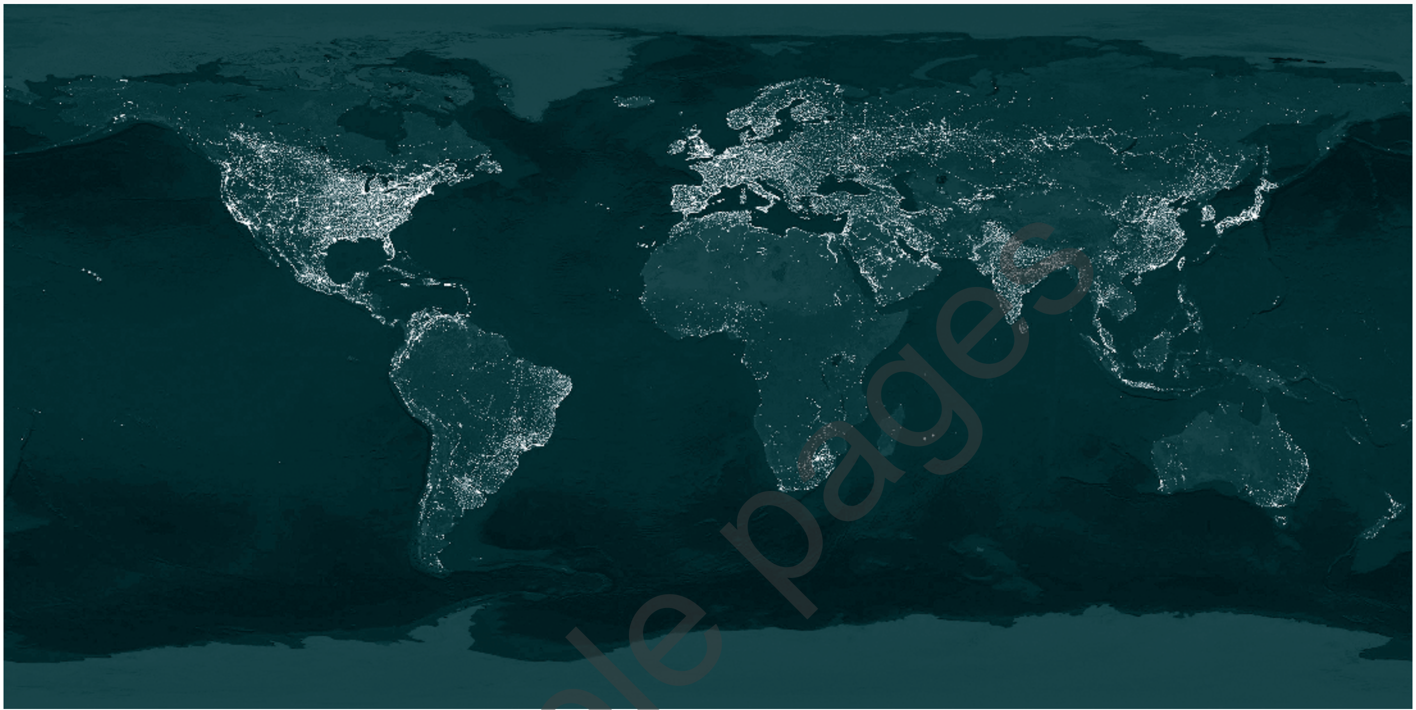
PART FOUR

Electromagnetism



Electricity constitutes a significant portion of humankind's energy, as evidenced by this composite satellite image of Earth at night. Nearly all that electrical energy is produced by generators, devices that exploit an intimate relation between electricity and magnetism.

OVERVIEW

Electromagnetism is one of the fundamental forces, and it governs the behavior of matter from the atomic scale to the macroscopic world. Electromagnetic technology, from computer microchips to cell phones and on to large electric motors and generators, is essential to modern society. Even our bodies rely heavily on electromagnetism: Electric signals pace our heartbeat, electrochemical processes transmit nerve impulses, and the electric structure of cell membranes mediates the flow of materials into and out of the cell.

Four fundamental laws describe electricity and magnetism. Two deal separately with the two phenomena, while the others reveal profound connections that make electricity and magnetism aspects of a single phenomenon that we call electromagnetism. In this part you'll come to understand those fundamental laws, learn how electromagnetism determines the structure and behavior of nearly all matter, and explore the electromagnetic technologies that play so important a role in your life. Finally, you'll see how the laws of electromagnetism lead to electromagnetic waves and thus help us understand the nature of light.

Electric Charge, Force, and Field

Learning Outcomes

After finishing this chapter you should be able to:

LO 20.1 Describe electric charge as a fundamental property of matter.

LO 20.2 Use Coulomb's law to calculate the forces between charges.

LO 20.3 Use the superposition principle to calculate forces involving multiple charges.

LO 20.4 Describe the concept of electric field.

LO 20.5 Determine the fields of electric charge distributions using superposition.

LO 20.6 Describe the electric dipole and the field it produces.

LO 20.7 Determine the fields of continuous charge distributions by integration.

LO 20.8 Determine the motion of charged particles in electric fields.

LO 20.9 Determine forces and torques on electric dipoles in electric fields.

Skills & Knowledge You'll Need

- The concept of force and Newton's second law (Sections 4.2 and 4.3)
- The gravitational field (Section 8.5)
- Integration techniques for physics (Tactics 9.1)
- The concept of torque, expressed as a cross product (Section 11.2)

What holds your body together? What keeps a skyscraper standing? What holds your car on the road as you round a turn? What governs the electronic circuitry in your computer or smartphone, or provides the tension in your climbing rope? What enables a plant to make sugar from sunlight and simple chemicals? What underlies the awesome beauty of lightning? The answer, in all cases, is the **electric force**. With the exception of gravity, all the forces we've encountered in mechanics—including tension forces, normal forces, compression forces, and friction—are based on electric interactions; so are the forces responsible for all of chemistry and biology. The electric force, in turn, involves a fundamental property of matter—namely, electric charge.

What's the fundamental criterion for initiating a lightning strike?

20.1 Electric Charge

LO 20.1 *Describe electric charge as a fundamental property of matter.*

Electric charge is an intrinsic property of the electrons and protons that, along with uncharged neutrons, make up ordinary matter. What is electric charge? At the most fundamental level we don't know. We don't know what mass "really" is either, but we're familiar with it because we've spent our lives pushing objects around. Similarly, our knowledge of electric charge results from observing the behavior of charged objects.

Charge comes in two varieties, which Benjamin Franklin designated *positive* and *negative*. Those names are useful because the total charge on an object—the object's **net charge**—is the algebraic sum of its constituent charges. Like charges repel, and opposites attract, a fact that constitutes a qualitative description of the electric force.

Quantities of Charge

All electrons carry the same charge, and all protons carry the same charge. The proton's charge has *exactly* the same magnitude as the electron's, but with opposite sign. Given that electrons and protons differ substantially in other properties—like mass—this electric relation is remarkable. Exercise 11 shows how dramatically different our world would be if there were even a slight difference between the magnitudes of the electron and proton charges.

The magnitude of the electron or proton charge is the **elementary charge** e. Electric charge is **quantized**; that is, it comes only in discrete amounts. In a famous experiment in 1909, the American physicist R. A. Millikan measured the charge on small oil drops and found it was always a multiple of a basic value we now know as the elementary charge.

Elementary particle theories show that the fundamental charge is actually $\frac{1}{3}e$. Such "fractional charges" reside on quarks, the building blocks of protons, neutrons, and many other particles. Quarks always join to produce particles with integer multiples of the full elementary charge, and it seems impossible to isolate individual quarks.

The SI unit of charge is the **coulomb** (C), named for the French physicist Charles Augustin de Coulomb (1736–1806). From the late 19th century to the early 21st century, the coulomb was defined in terms of electric current and time—a definition that was difficult to implement in practice. The 2019 revision of the SI gave the coulomb a much simpler definition. Now, the elementary charge is defined to be exactly $1.602176634 \times 10^{-19}$ C. The coulomb is therefore the number of elementary charges equal to the inverse of this number. For our purposes, that's about 6.24×10^{18} elementary charges.

Charge Conservation

Electric charge is a conserved quantity, meaning that the net charge in a closed region remains constant. Charged particles may be created or annihilated, but always in pairs of equal and opposite charge. The net charge always remains the same.

GOT IT?

20.1 The proton is a composite particle composed of three quarks, all of which are either *up quarks* (u; charge $+\frac{2}{3}e$) or *down quarks* (d; charge $-\frac{1}{3}e$). (More on quarks in Chapter 39.) Which of these quark combinations is the proton? (a) *udd*; (b) *uuu*; (c) *uud*; (d) *ddd*

20.2 Coulomb's Law

LO 20.2 *Use Coulomb's law to calculate the forces between charges.*

LO 20.3 *Use the superposition principle to calculate forces involving multiple charges.*

Rub a balloon; it gets charged and sticks to your clothing. Charge another balloon, and the two repel (Fig. 20.1). Socks cling to your clothes as they come from the dryer, and bits of Styrofoam cling annoyingly to your hands. Walk across a carpet, and you'll feel a shock when you touch a doorknob. All these are common examples where you're directly aware of electric charge.

Electricity would be unimportant if the only significant electric interactions were these obvious ones. In fact, the electric force dominates all interactions of everyday matter, from the motion of a car to the movement of a muscle. It's just that matter on a large scale

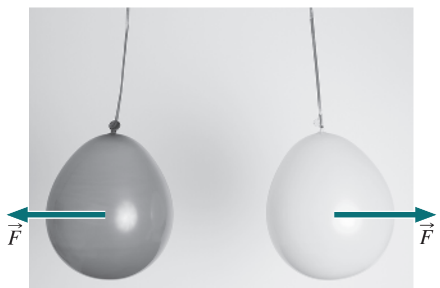


FIGURE 20.1 Two balloons carrying similar electric charges repel each other.

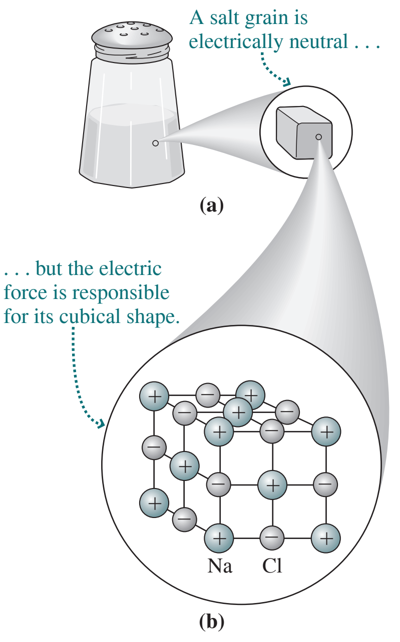


FIGURE 20.2 (a) A single salt grain is electrically neutral, so the electric force isn't obvious. (b) Actually, the electric force determines the structure of salt.

is almost perfectly neutral, meaning it carries zero net charge. Therefore, electric effects aren't obvious. But at the molecular level, the electric nature of matter is immediately evident (Fig. 20.2).

Attraction and repulsion of electric charges imply a force. Joseph Priestley and Charles Augustin de Coulomb investigated this force in the late 1700s. They found that the force between two charges acts along the line joining them, with the magnitude proportional to the product of the charges and inversely proportional to the square of the distance between them. **Coulomb's law** summarizes these results:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r} \quad \text{(Coulomb's law)} \qquad (20.1)$$

$\vec{F}_{12}$ is the force charge q_1 exerts on charge q_2.
k is approximately $9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$.
q_1 and q_2 are two charges.
r is the distance between the two charges.
$\hat{r}$ is a unit vector that points from q_1 toward q_2 regardless of the signs of the charges.

where $\vec{F}_{12}$ is the force charge q_1 exerts on q_2 and r is the distance between the charges. In SI the proportionality constant k has the approximate value $9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$. Force is a vector, and $\hat{r}$ is a unit vector that helps determine its direction. Figure 20.3 shows that $\hat{r}$ lies on a line passing through the two charges and points in the direction *from* q_1 *toward* q_2. Reverse the roles of q_1 and q_2, and you'll see that $\vec{F}_{21}$ has the same magnitude as $\vec{F}_{12}$ but the opposite direction; thus Coulomb's law obeys Newton's third law. Figure 20.3 also shows that the force is in the same direction as the unit vector when the charges have the same sign, but opposite the unit vector when the charges have different signs. Thus Coulomb's law accounts for the fact that like charges repel and opposites attract.

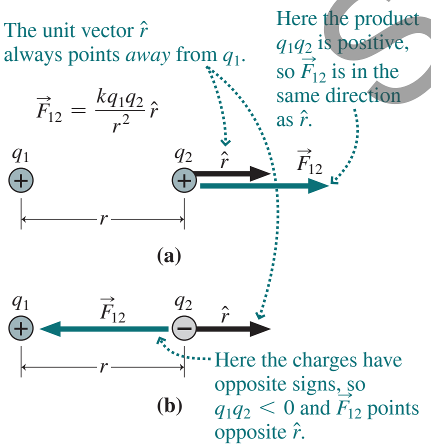


FIGURE 20.3 Quantities in Coulomb's law for calculating the force $\vec{F}_{12}$ that q_1 exerts on q_2.

PROBLEM-SOLVING STRATEGY 20.1 Coulomb's Law

The key to using Coulomb's law is to remember that force is a vector, and to realize that Coulomb's law in the form of Equation 20.1 gives both the magnitude and direction of the electric force. Dealing carefully with vector directions is especially important in situations with more than two charges.

INTERPRET First, make sure you're dealing with the electric force alone. Identify the charge or charges on which you want to calculate the force. Next, identify the charge or charges producing the force. These comprise the **source charge**.

DEVELOP Begin with a drawing that shows the charges, as in Fig. 20.4. If you're given charge coordinates, place the charges on the coordinate system; if not, choose a suitable coordinate system. For each source charge, determine the unit vector(s) in Equation 20.1. If the charges lie along or parallel to a coordinate axis, then the unit vector will be one of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, or $\hat{k}$, perhaps with a minus sign. In Fig. 20.4, the force on q_3 due to q_1 is such a case. When the two charges don't lie on a coordinate axis, like q_1 and q_2 in Fig. 20.4, you can find the unit vector by noting that the displacement vector $\vec{r}_{12}$ points in the desired direction, from the source charge to the charge experiencing the force. Dividing $\vec{r}_{12}$ by its own magnitude then gives the unit vector in the direction of $\vec{r}_{12}$; that is, $\hat{r} = \vec{r}_{12}/r_{12}$.

EVALUATE For each source charge, determine the electric force using Equation 20.1,

$$\vec{F}_{12} = (kq_1q_2/r^2)\hat{r}$$

with $\hat{r}$ the unit vector you've just found.

ASSESS As always, assess your answer to see that it makes sense. Is the direction of the force you found consistent with the signs and placements of the charges giving rise to the force?

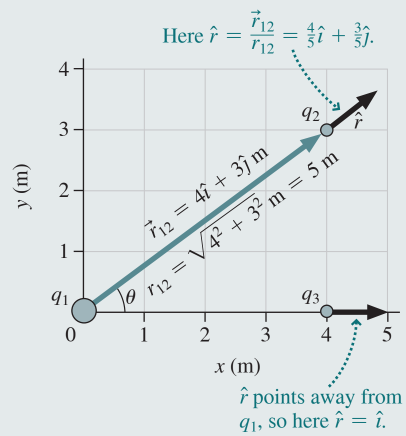


FIGURE 20.4 Finding unit vectors.

GOT IT?

20.2 Charge q_1 is located at $x = 1$ m, $y = 0$. What should you use for the unit vector $\hat{r}$ in Coulomb's law if you're calculating the force that q_1 exerts on a charge q_2 located (1) at the origin and (2) at the point $x = 0$, $y = 1$ m? Explain why you can answer without knowing the sign of either charge.

EXAMPLE 20.1 Finding the Force: Two Charges

A 1.0-μC charge is at $x = 1.0$ cm, and a −1.5-μC charge is at $x = 3.0$ cm. What force does the positive charge exert on the negative one? How would the force change if the distance between the charges tripled?

INTERPRET Following our strategy, we identify the −1.5-μC charge as the one on which we want to find the force and the 1-μC charge as the source charge.

DEVELOP We're given the coordinates $x_1 = 1.0$ cm and $x_2 = 3.0$ cm. Our drawing, Fig. 20.5, shows both charges at their positions on the x-axis. With the source charge q_1 to the left, the unit vector in the direction from q_1 toward q_2 is $\hat{\imath}$.

EVALUATE Now we use Coulomb's law to evaluate the force:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}$$
$$= \frac{(9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0 \times 10^{-6}\ \text{C})(-1.5 \times 10^{-6}\ \text{C})}{(0.020\ \text{m})^2}\hat{\imath}$$
$$= -34\hat{\imath}\ \text{N}$$

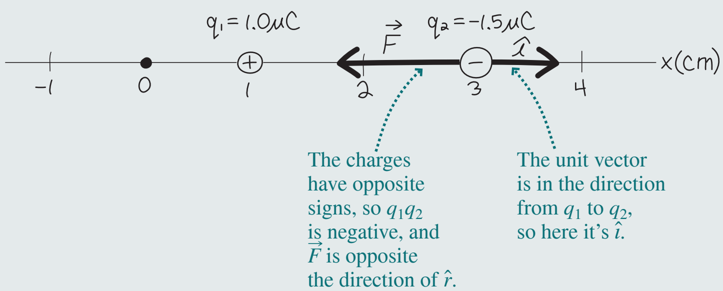


FIGURE 20.5 Sketch for Example 20.1.

This force is for a separation of 2 cm; if that distance tripled, the force would drop by a factor of $1/3^2$, to $-3.8\hat{\imath}$ N.

ASSESS Make sense? Although the unit vector $\hat{\imath}$ points in the $+x$-direction, the charges have opposite signs and that makes the force direction opposite the unit vector, as shown in Fig. 20.5. In simpler terms, we've got two opposite charges, so they attract. That means the force exerted on a charge at $x = 3$ cm by an opposite charge at $x = 1$ cm had better be in the $-x$-direction.

CONCEPTUAL EXAMPLE 20.1 Gravity and the Electric Force

The electric force between elementary particles is far stronger than the gravitational force, yet gravity is much more obvious in everyday life. Why?

EVALUATE Gravity and the electric force obey similar inverse-square laws, and the magnitude of the force is proportional to the product of the masses or charges. There's a big difference, though: There's only one kind of mass, and gravity is always attractive, so large concentrations of mass—like a planet—result in strong gravitational forces. But charge comes in two varieties, and opposites attract, so large accumulations of matter tend to be electrically neutral, in which case large-scale electrical interactions aren't obvious.

ASSESS Ironically, it's the very strength of the electric force that makes it less obvious in everyday life. Opposite charges bind strongly, making bulk matter electrically neutral and its electrical interactions subtle.

MAKING THE CONNECTION Compare the magnitudes of the electric and gravitational forces between an electron and a proton.

EVALUATE Equation 8.1 gives the gravitational force: $F_g = Gm_em_p/r^2$. Equation 20.1 gives the electric force: $|F_E| = ke^2/r^2$, where we wrote e^2 because the electron and proton charges have the same magnitude. We aren't given the distance, but that doesn't matter because both forces have the same inverse-square dependence. The ratio of the force magnitudes is huge: $F_E/F_g = ke^2/Gm_em_p = 2.3 \times 10^{39}$!

Point Charges and the Superposition Principle

Coulomb's law is strictly true only for **point charges**—charged objects of negligible size. Electrons and protons can usually be treated as point charges; so, approximately, can any two charged objects if their separation is large compared with their size. But often we're interested in the electric effects of **charge distributions**—arrangements of charge spread over space. Charge distributions are present in molecules, memory cells in your computer, your heart, and thunderclouds. We need to combine the effects of two or more charges to find the electric effects of such charge distributions.

Figure 20.6 shows two charges q_1 and q_2 that constitute a simple charge distribution. We want to know the net force these exert on a third charge q_3. To find that net force, you might calculate the forces $\vec{F}_{13}$ and $\vec{F}_{23}$ from Equation 20.1, and then vectorially add them. And you'd be right: The force that q_1 exerts on q_3 is unaffected by the presence of q_2, and vice versa, so you can apply Coulomb's law separately to the pairs q_1q_3 and q_2q_3 and then combine the results. That may seem obvious, but nature needn't have been so simple.

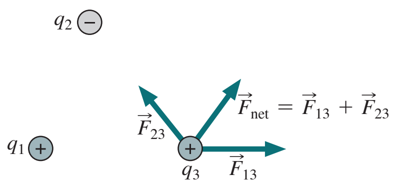


FIGURE 20.6 The superposition principle lets us add vectorially the forces from two or more charges.

The fact that electric forces add vectorially is called the **superposition principle**. Our confidence in this principle is ultimately based on experiments showing that electric and indeed electromagnetic phenomena behave according to the principle. With superposition we can solve relatively complicated problems by breaking them into simpler parts. If the superposition principle didn't hold, the mathematical description of electromagnetism would be far more complicated.

Although the force that one point charge exerts on another decreases with the inverse square of the distance between them, the same is not necessarily true of the force resulting from a charge distribution. The next example provides a case in point.

EXAMPLE 20.2 Finding the Force: Raindrops

Worked Example with Variation Problems

Charged raindrops are ultimately responsible for lightning, producing substantial electric charge within specific regions of a thundercloud. Suppose two drops with equal charge q are on the x-axis at $x = \pm a$. Find the electric force on a third drop with charge Q at an arbitrary point on the y-axis.

INTERPRET Coulomb's law and the superposition principle apply, and we identify Q as the charge for which we want the force. The two charges q are the source charges.

DEVELOP Figure 20.7 is our drawing, showing the charges, the individual force vectors, and their sum. The drawing shows that the distance r in Coulomb's law is the hypotenuse $\sqrt{a^2 + y^2}$. It's clear from symmetry that the net force is in the y-direction, so we need to find only the y-components of the unit vectors. The y-components are clearly the same for each, and the drawing shows that they're given by $\hat{r}_y = y/\sqrt{a^2 + y^2}$.

EVALUATE From Coulomb's law, the y-component of the force from each q is $F_y = (kqQ/r^2)\hat{r}_y$, and the net force on Q becomes

$$\vec{F} = 2\left(\frac{kqQ}{a^2 + y^2}\right)\left(\frac{y}{\sqrt{a^2 + y^2}}\right)\hat{j} = \frac{2kqQy}{(a^2 + y^2)^{3/2}}\hat{j}$$

The factor of 2 comes from the two charges q, which contribute equally to the net force.

ASSESS Make sense? Evaluating $\vec{F}$ at $y = 0$ gives zero force. Here, midway between the two charges, Q experiences equal but opposite forces and the net force is zero. At large distances $y \gg a$, on the other hand, we can neglect a^2 compared with y^2, and the force becomes $\vec{F} = k(2q)Q\hat{j}/y^2$. This is just what we would expect from a single charge $2q$ a distance y from Q—showing that the system of two charges acts like a single charge $2q$ at distances that are large compared with the charge separation. In between our two extremes the behavior of force with distance is more complicated; in fact, its magnitude initially increases as Q moves away from the origin and then begins to decrease.

In drawing Fig. 20.7, we tacitly assumed that q and Q have the same signs. But our analysis holds even if they don't; then the product qQ is negative, and the forces actually point opposite the directions shown in Fig. 20.7.

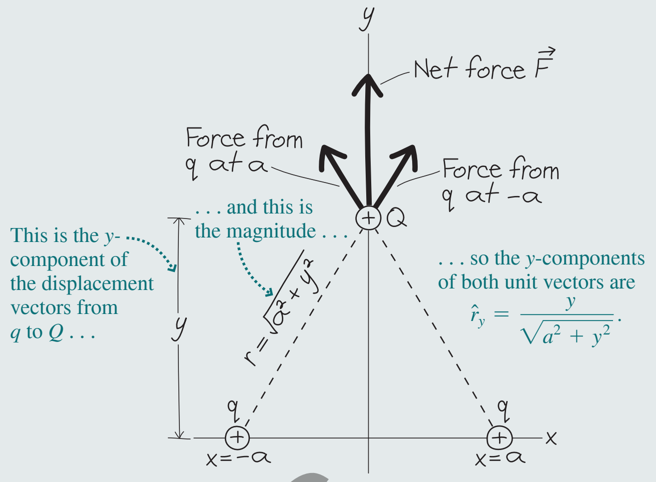


FIGURE 20.7 The force on Q is the vector sum of the forces from the individual charges.

20.3 The Electric Field

LO 20.4 *Describe the concept of electric field.*

In Chapter 8 we defined the gravitational field at a point as the gravitational force per unit mass that an object at that point would experience. In that context, we can think of $\vec{g}$ as the *force per unit mass* that any object would experience due to Earth's gravity. So we can picture the gravitational field as a set of vectors giving the magnitude and direction of the gravitational force per unit mass at each point, as shown in Fig. 20.8*a* below.

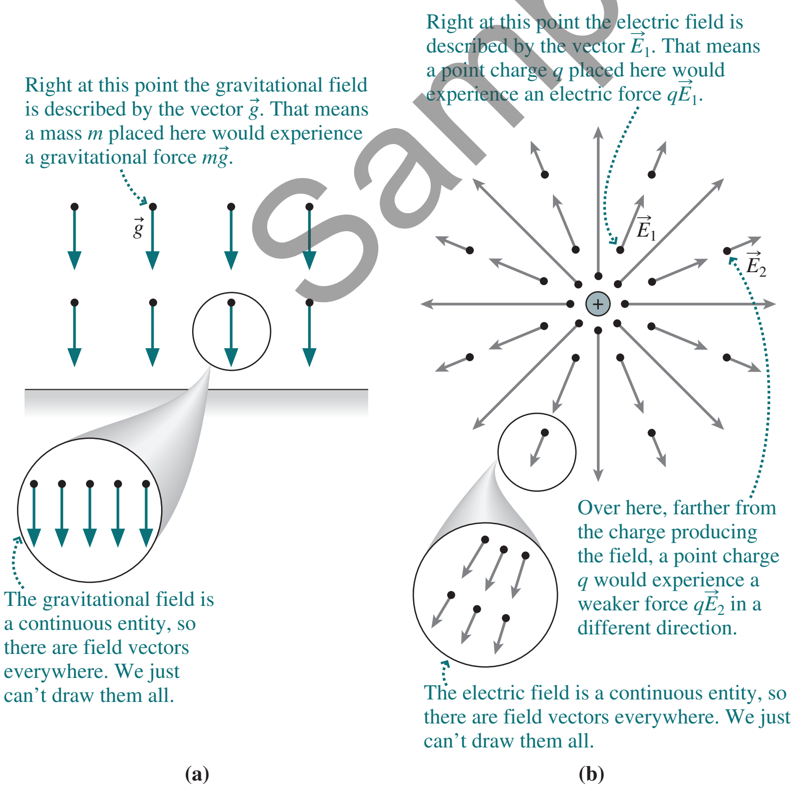
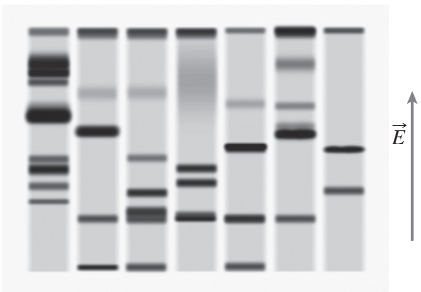


FIGURE 20.8 (a) Gravitational and (b) electric fields, here represented as sets of vectors.

APPLICATION Electrophoresis

Electrophoresis is a widely used application of electric fields for separating molecules by size and molecular weight. It's especially useful in biochemistry and molecular biology for distinguishing larger molecules like proteins and DNA fragments. In the commonly used *gel electrophoresis*, molecules carrying electric charge move through a semisolid but permeable gel under the influence of a uniform electric field; the greater the charge, the greater the electric force. The gel exerts a retarding force that increases with increasing molecular size, with the result that each molecular species moves at a velocity that depends on its size and charge. After a given time, the electric field is switched off. The locations of the molecules then serve as indicators of their size, with the molecules that traveled farthest being the smallest. The photo shows a typical gel electrophoresis result. Here DNA fragments were introduced into the seven channels at the top of the gel and then moved downward; their final locations indicate molecular size. The smaller molecules—those with fewer nucleotide base pairs—end up farther down on the gel. The electric field is shown by the arrow; it needs to point upward because DNA fragments carry a negative charge.



We can do the same thing with the electric force, defining the **electric field** as the force per unit charge:

> The electric field at any point is the force per unit charge that a charge would experience at that point. Mathematically,

$\vec{E}$ is the electric field at any point.

You can determine $\vec{E}$ by measuring the electric force $\vec{F}$ …

$$\vec{E} = \frac{\vec{F}}{q} \quad \text{(electric field)} \tag{20.2a}$$

… on a small test charge q.

The electric field exists at every point in space. When we represent the field by vectors, we can't draw one everywhere, but that doesn't mean there isn't a field at all points. Furthermore, we draw vectors as extended arrows, but each vector represents the field at only one point—namely, the tail end of the vector. Figure 20.8*b* illustrates this for the electric field of a point charge.

The field concept leads to a shift in our thinking about forces. Instead of the action-at-a-distance idea that Earth reaches across empty space to pull on the Moon, the field concept says that Earth creates a gravitational field and the Moon responds to the field at its location. Similarly, a charge creates an electric field throughout the space surrounding it. A second charge then responds to the field at its immediate location. Although the field reveals itself only through its effect on a charge, the field nevertheless exists at all points, whether or not charges are present. Right now you probably find the field concept a bit abstract, but as you advance in your study of electromagnetism you'll come to appreciate that fields are an essential feature of our universe, every bit as real as matter itself.

We can use Equation 20.2a as a prescription for measuring electric fields. Place a point charge at some location, measure the electric force it experiences, and divide by the charge to get the field. In practice, we need to be careful because the field generally arises from some distribution of source charges. If the charge we're using to probe the field—the **test charge**—is large, the field it creates may disturb the source charges, altering their configuration and thus the field they create. For that reason, it's important to use a very small test charge.

If we know the electric field $\vec{E}$ at a point, we can rearrange Equation 20.2a to find the force on any point charge q placed at that point:

$\vec{F}$ is the electric force … … on a charge q …

$$\vec{F} = q\vec{E} \quad \text{(electric force and field)} \tag{20.2b}$$

… at a point where the electric field is $\vec{E}$.

If the charge q is positive, then this force is in the same direction as the field, but if q is negative, then the force is opposite to the field direction.

Equations 20.2 show that the units of electric field are newtons per coulomb. Fields of hundreds to thousands of N/C are commonplace, while fields of 3 MN/C will tear electrons from air molecules. Sometimes we're interested in the magnitude of the field but not its direction. Then we can use Equations 20.2a and 20.2b without the vector signs. We'll often use the term *field strength* to be synonymous with the field's magnitude.

EXAMPLE 20.3 Force and Field: Inside a Lightning Storm

A charged raindrop carrying 10 μC experiences an electric force of 0.30 N in the $+x$-direction. What's the electric field at its location? What would the force be on a -5.0-μC drop at the same point?

INTERPRET In this problem we need to distinguish between electric force and electric field. The electric field exists with or without the charged raindrop present, and the electric force arises when the charged raindrop is in the electric field.

DEVELOP Knowing the electric force and the charge on the raindrop, we can use Equation 20.2a, $\vec{E} = \vec{F}/q$, to get the electric field. Once we know the field, we can use Equation 20.2b,

$\vec{F} = q\vec{E}$, to calculate the force that would act if a different charge were at the same point.

EVALUATE Equation 20.2a gives the electric field:

$$\vec{E} = \frac{\vec{F}}{q} = \frac{0.30\hat{\imath}\ \text{N}}{10\ \mu\text{C}} = 30\hat{\imath}\ \text{kN/C}$$

Acting on a -5.0-μC charge, this field would result in a force

$$\vec{F} = q\vec{E} = (-5.0\ \mu\text{C})(30\hat{\imath}\ \text{kN/C}) = -0.15\hat{\imath}\ \text{N}$$

ASSESS Make sense? The force on the second charge is opposite the direction of the field because now we've got a negative charge in the same field.

TIP **THE FIELD IS INDEPENDENT OF THE TEST CHARGE** Does the electric field in this example point in the $-x$-direction when the charge is negative? No. The field is independent of the particular charge experiencing that field. Here the electric field points in the $+x$-direction *no matter what charge* you put in the field. For a positive charge, the force $q\vec{E}$ points in the *same* direction as the field; for a negative charge, $q < 0$, the force is *opposite* the field.

The Field of a Point Charge

Once we know the field of a charge distribution, we can calculate its effect on other charges. The simplest charge distribution is a single point charge. Coulomb's law gives the force on a test charge q_{test} located a distance r from a point charge q: $\vec{F} = (kqq_{\text{test}}/r^2)\hat{r}$, where $\hat{r}$ is a unit vector pointing *away* from q. The electric field arising from q is the force per unit charge, or

The electric field $\vec{E}$ is the force per unit charge.

For a point charge, $\vec{E}$ depends on the charge q …

$$\vec{E} = \frac{\vec{F}}{q_{\text{test}}} = \frac{kq}{r^2}\hat{r} \quad \text{(field of a point charge)} \qquad (20.3)$$

… and on the distance r from the charge to the point where the field is being evaluated.

The unit vector $\hat{r}$ always points away from q, regardless of q's sign.

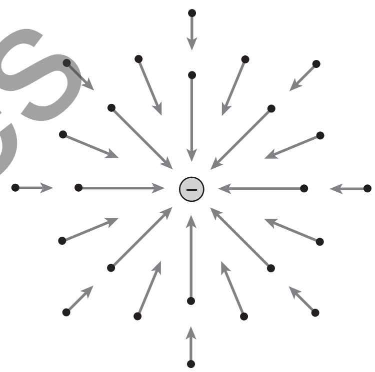


FIGURE 20.9 Field vectors for a negative point charge.

Since it's so closely related to Coulomb's law for the electric force, we also refer to Equation 20.3 as Coulomb's law. The equation contains no reference to the test charge q_{test} because the field of q exists independently of any other charge. Since $\hat{r}$ always points *away* from q, the direction of $\vec{E}$ is radially outward if q is positive and radially inward if q is negative. Figure 20.9 shows some field vectors for a negative point charge, analogous to those of the positive point charge in Fig. 20.8*b*.

GOT IT? **20.3** A positive point charge is located at the origin of an x–y coordinate system, and an electron is placed at a location where the electric field due to the point charge is given by $\vec{E} = E_0(\hat{\imath} + \hat{\jmath})$, where E_0 is positive. Is the direction of the force on the electron (a) toward the origin, (b) away from the origin, (c) parallel to the x-axis, or (d) impossible to determine without knowing the coordinates of the electron's position?

20.4 Fields of Charge Distributions

LO 20.5 *Determine the fields of electric charge distributions using superposition.*

LO 20.6 *Describe the electric dipole and the field it produces.*

LO 20.7 *Determine the fields of continuous charge distributions by integration.*

Since the electric force obeys the superposition principle, so does the electric field. That means the field of a charge distribution is the vector sum of the fields of the individual point charges making up the distribution:

The electric field $\vec{E}$ of a distribution of point charges… … is the sum of the fields of the individual point charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots = \sum_i \vec{E}_i = \sum_i \frac{kq_i}{r_i^2}\hat{r}_i \qquad (20.4)$$

Here the $\vec{E}_i$'s are the fields of the point charges q_i located at distances r_i from the point where we're evaluating the field—called, appropriately, the **field point**. The $\hat{r}_i$'s are unit vectors pointing *from* each point charge *toward* the field point. In principle, Equation 20.4 gives the electric field of *any* charge distribution. In practice, the process of summing the individual field vectors is often complicated unless the charge distribution contains relatively few charges arranged in a symmetric way.

Finding electric fields using Equation 20.4 involves the same strategy we introduced for finding the electric force; the only difference is that there's no charge to experience the force. The first step then involves identifying the field point. We still need to find the appropriate unit vectors and form the vector sum in Equation 20.4. Example 20.4 shows how this is done.

Sometimes we're interested in finding not the electric field but a point or points where the field is zero. Conceptual Example 20.2 explores such a case.

EXAMPLE 20.4 Finding the Field: Two Protons

Two protons are 3.6 nm apart. Find the electric field at a point between them, 1.2 nm from one of the protons. Then find the force on an electron at this point.

INTERPRET We follow our electric-force strategy, except that instead of identifying the charge experiencing the force, we identify the field point as being 1.2 nm from one proton. The source charges are the two protons; they produce the field we're interested in.

DEVELOP Let's have the protons define the x-axis, as drawn in Fig. 20.10. Then the unit vector $\hat{r}_1$ from the left-hand proton toward the field point (which we've marked P) is $+\hat{\imath}$, while $\hat{r}_2$ from the right-hand proton toward P is $-\hat{\imath}$.

EVALUATE We now evaluate the field at P using Equation 20.4:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{ke}{r_1^2}\hat{\imath} + \frac{ke}{r_2^2}(-\hat{\imath}) = ke\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)\hat{\imath}$$

We wrote e for q here because the protons' charge is the elementary charge.

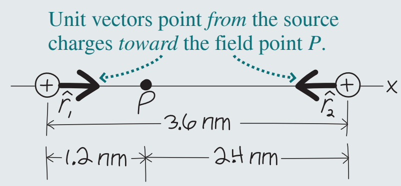


FIGURE 20.10 Finding the electric field at *P*.

Using $e = 1.6 \times 10^{-19}$ C, $r_1 = 1.2$ nm, and $r_2 = 2.4$ nm gives $\vec{E} = 750\hat{\imath}$ MN/C. An electron at P will therefore experience a force $\vec{F} = q\vec{E} = -e\vec{E} = -0.12\hat{\imath}$ nN.

ASSESS Make sense? The field points in the positive x-direction, reflecting the fact that P is closer to the left-hand proton with its stronger field at P. The force on the electron, on the other hand, is in the $-x$-direction; that's because the electron is negative (we used $q = -e$ for its charge), so the force it experiences is opposite the field. That field of almost 1 GN/C sounds huge—but that's not unusual at the microscopic scale, where we're close to individual elementary particles.

CONCEPTUAL EXAMPLE 20.2 Zero Field, Zero Force

A positive charge $+2Q$ is located at the origin, and a negative charge $-Q$ is at $x = a$. In which region of the x-axis is there a point where the force on a test charge—and therefore the electric field—is zero?

INTERPRET We're asked to locate qualitatively a point where the field is zero. Our sketch of the situation, Fig. 20.11, shows that the two charges divide the x-axis into three regions: (1) to the left of $2Q$ ($x < 0$), (2) between the charges ($0 < x < a$), and (3) to the right of $-Q$ ($x > a$). We need to determine which region could include a point where the electric force on a test charge is zero.

EVALUATE Consider what would happen to a positive test charge placed in each of these three regions. Anywhere in region (1), the test charge is closer to the charge with greater magnitude ($2Q$). That charge dominates throughout region (1), where our test charge would experience a repulsive force (to the left). The electric field, then, can't be zero in region (1). Between the two charges, the repulsive force from $2Q$ on a positive test charge points to the right; so does the attractive force from $-Q$. The field, therefore, can't be zero in region (2). That leaves region (3). Could the field be zero here? Put a positive test charge very close to $-Q$, and it experiences an attractive force toward the left. But far away, the distance between $2Q$ and $-Q$ becomes negligible. The fields of both charges drop off as the inverse square of the distance, so at large distances the field of the stronger charge will dominate. Therefore there *is* a point somewhere to the right of $-Q$ where the force on a test charge, and therefore the electric field, will be zero.

ASSESS This answer is consistent with our insight from Example 20.2 that when we get far from a charge distribution, it begins to resemble a point charge with the net charge of the distribution. Here that net charge is $2Q - Q = +Q$, so at large distances we should indeed have a field pointing away from the charge distribution—and that's to the right in region (3). Although we considered a positive test charge, you'll reach the same conclusion with a negative test charge.

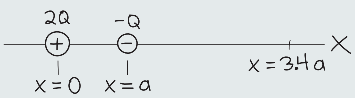


FIGURE 20.11 Where is the electric field zero? We've marked the answer, at $x = 3.4a$.

MAKING THE CONNECTION Find an expression for the position where the electric field in this example is zero.

EVALUATE In Fig. 20.11, we've taken the origin at $2Q$, so at any position x in region (3) we're a distance x from $2Q$ and a distance $x-a$ from $-Q$. Since we're to the right of both charges, the unit vector in Equation 20.3 for the point-charge field—a vector that always points away from the point charge—becomes $+\hat{\imath}$ for both charges. Applying Equation 20.3, $\vec{E} = (kq/r^2)\hat{r}$, for the fields of the two charges and summing gives

$$\vec{E} = \frac{k(2Q)}{x^2}\hat{\imath} + \frac{k(-Q)}{(x-a)^2}\hat{\imath}$$

If we set this expression to zero, we can cancel k, Q, and $\hat{\imath}$; inverting both sides of the remaining equation gives $x^2/2 = (x-a)^2$. Finally, taking the square root and solving for x gives the answer: $x = a\sqrt{2}/(\sqrt{2}-1) \simeq 3.4a$. As a check, note that this point does indeed lie to the right of $x = a$. We've marked this point in Fig. 20.11.

The Electric Dipole

One of the most important charge distributions is the **electric dipole**, consisting of two point charges of equal magnitude but opposite sign. Many molecules are essentially dipoles, so understanding the dipole helps explain molecular behavior (Fig. 20.12). During contraction the heart muscle becomes essentially a dipole, and physicians performing electrocardiography are measuring, among other things, the strength and orientation of that dipole. Antennas used in wireless communications—including radio, TV, wifi, and cellphones—are often based on the dipole configuration.

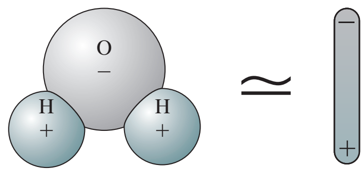


FIGURE 20.12 A water molecule behaves like an electric dipole. Its net charge is zero, but regions of positive and negative charge are separated.

EXAMPLE 20.5 The Electric Dipole: Modeling a Molecule

A molecule may be modeled approximately as a positive charge q at $x = a$ and a negative charge $-q$ at $x = -a$. Evaluate the electric field on the y-axis, and find an approximate expression valid at large distances ($|y| \gg a$).

INTERPRET Here's another example where we'll use our strategy in applying Equation 20.4 to calculate the field of a charge distribution. We identify the field point as being anywhere on the y-axis and the source charges as being $\pm q$.

DEVELOP Figure 20.13 is our drawing. The individual unit vectors point from the two charges toward the field point, but the *negative* charge contributes a field *opposite* its unit vector; we've indicated the individual fields in Fig. 20.13. Here symmetry makes the y-components cancel, giving a net field in the $-x$-direction. So we need only the x-components of the unit vectors, which Fig. 20.13 shows are $\hat{r}_{x-} = a/r$ for the negative charge at $-a$ and $\hat{r}_{x+} = -a/r$ for the positive charge at a.

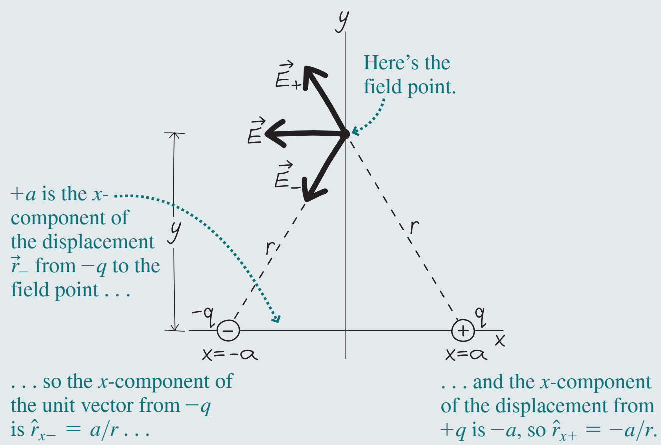


FIGURE 20.13 Finding the field of an electric dipole.

EVALUATE We then evaluate the field using Equation 20.4:

$$\vec{E} = \frac{k(-q)}{r^2}\left(\frac{a}{r}\right)\hat{\imath} + \frac{kq}{r^2}\left(-\frac{a}{r}\right)\hat{\imath} = -\frac{2kqa}{(a^2+y^2)^{3/2}}\hat{\imath}$$

where in the last step we used $r = \sqrt{a^2+y^2}$. For $|y| \gg a$ we can neglect a^2 compared with y^2, giving

$$\vec{E} \simeq -\frac{2kqa}{|y|^3}\hat{\imath} \qquad (|y| \gg a)$$

(*continued*)

ASSESS Make sense? The dipole has no net charge, so at large distances its field can't have the inverse-square drop-off of a point-charge field. Instead the dipole field falls faster, here as $1/|y|^3$. Note that we were careful to put absolute value signs on y^3; that way, our result applies for both positive and negative values of y.

TIP **APPROXIMATIONS** Making approximations requires care. Here we're basically asking for the field when y is so large that a is negligible *compared with* y. So we neglect a^2 compared with y^2 when the two are summed, but we *don't* neglect a when it appears in the numerator, where it isn't being directly compared with y.

Example 20.5 shows that the dipole field at large distances decreases as the inverse *cube* of distance. Physically, that's because the dipole has zero *net* charge. Its field arises entirely from the slight separation of two opposite charges. Because of this separation, the dipole field isn't exactly zero, but it's weaker and more localized than the field of a point charge. Many complicated charge distributions exhibit the essential characteristic of a dipole—that is, they're neutral but consist of separated regions of positive and negative charge—and at large distances, such distributions all have essentially the same field configuration.

At large distances the dipole's physical characteristics q and a enter the equation for the electric field only through the product qa. We could double q and halve a, and the dipole's electric field would remain unchanged. At large distances, therefore, a dipole's electric properties are characterized completely by its **electric dipole moment** p, defined as the product of the charge q and the separation d between the two charges making up the dipole:

$$p = qd \quad \text{(dipole moment)} \tag{20.5}$$

In Example 20.5 the charge separation was $d = 2a$, so there the dipole moment was $p = 2aq$. In terms of the dipole moment, the field in Example 20.5 can then be written

$$\vec{E} = -\frac{kp}{|y|^3}\hat{\imath} \quad \begin{pmatrix}\text{dipole field for } |y| \gg a, \\ \text{on perpendicular bisector}\end{pmatrix} \tag{20.6a}$$

You can show in Problem 54 that the field on the dipole axis is given by

$$\vec{E} = \frac{2kp}{|x|^3}\hat{\imath} \quad \begin{pmatrix}\text{dipole field} \\ \text{for } |x| \gg a\text{, on axis}\end{pmatrix} \tag{20.6b}$$

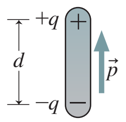


FIGURE 20.14 The dipole moment vector $\vec{p}$ has magnitude $p = qd$ and points from the negative toward the positive charge.

Because the dipole isn't spherically symmetric, its field depends not only on distance but also on orientation; for instance, Equations 20.6 show that the field along the dipole axis at a given distance is twice as strong as along the bisector. So it's important to know the orientation of a dipole in space, and therefore we generalize our definition of the dipole moment to make it a vector of magnitude $p = qd$ in the direction from the negative toward the positive charge (Fig. 20.14).

GOT IT? **20.4** Far from a charge distribution, you measure an electric field strength of 800 N/C. What will the field strength be if you double your distance from the charge distribution, if the distribution consists of (1) a point charge or (2) a dipole?

Continuous Charge Distributions

Although any charge distribution ultimately consists of pointlike electrons and protons, it would be impossible to sum all the field vectors from the 10^{23} or so particles in a typical piece of matter. Instead, it's convenient to make the approximation that charge is spread continuously over the distribution. If the charge distribution extends throughout a volume, we describe it in terms of the **volume charge density** ρ, with units of C/m^3. For charge distributions spread over surfaces or lines, the corresponding quantities are the **surface charge density** σ (C/m^2) and the **line charge density** λ (C/m).

To calculate the field of a continuous charge distribution, we divide the charged region into very many small charge elements dq, each small enough that it's essentially a point charge. Each dq then produces an electric field $d\vec{E}$ given by Equation 20.3: $d\vec{E} = (k\,dq/r^2)\hat{r}$. We then form the vector sum of all the $d\vec{E}$'s (Fig. 20.15). In the limit of infinitely many infinitesimally small dq's and their corresponding $d\vec{E}$'s, that sum becomes an integral and we have

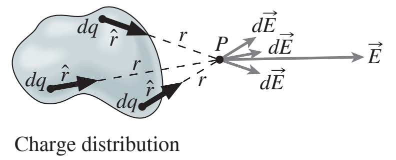


FIGURE 20.15 The electric field at P is the vector sum of the fields $d\vec{E}$ arising from the individual charge elements dq, each calculated using the appropriate distance r and unit vector $\hat{r}$.

$$\vec{E} = \int d\vec{E} = \int \frac{k\,dq}{r^2}\hat{r} \quad \left(\begin{array}{c}\text{field of a continuous}\\ \text{charge distribution}\end{array}\right) \qquad (20.7)$$

The electric field $\vec{E}$ of a continuous distribution of charges …

… is determined by integrating the fields $d\vec{E}$ of infinitesimal charge elements dq.

dq is an infinitesimal charge element.

$\hat{r}$ is a unit vector that points away from dq, regardless of its sign.

r is the distance from dq to the point where the field is being evaluated.

The limits of this integral include the entire charge distribution.

Calculating the field of a continuous charge distribution involves the same strategy we've already used: We identify the field point and the source charges—although now the source is a continuous charge distribution. Summing the individual field contributions now presents us with an integral, and that means writing the unit vectors $\hat{r}$ and distances r in terms of coordinates over which we can integrate. Setting up the integral involves the same strategy we outlined in Chapter 9 to find the center of mass of a continuous distribution of matter, and used again in Chapter 10 to find rotational inertias.

EXAMPLE 20.6 Evaluating the Field: A Charged Ring

A ring of radius a carries a charge Q distributed evenly over the ring. Find an expression for the electric field at any point on the axis of the ring.

INTERPRET We identify the field point as lying anywhere on the ring's axis, and the source charge as the entire ring.

DEVELOP Let's take the x-axis to coincide with the ring axis, with the center of the ring at $x = 0$ (Fig. 20.16). The figure shows that the y-components of the field contributions from pairs of charge elements on opposite sides of the ring cancel; therefore, the net field points in the $+x$-direction (for $x > 0$) and we need only the x-components of the unit vectors. Those are the same for all unit vectors—namely, $\hat{r}_x = x/r$.

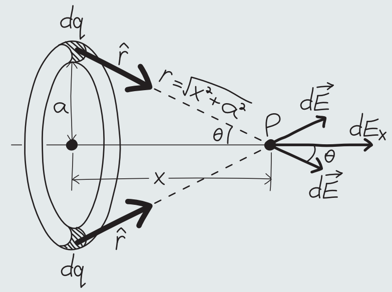


FIGURE 20.16 The electric field of a charged ring points along the ring axis, since field components perpendicular to the axis cancel in pairs.

EVALUATE We're now ready to set up the integral in Equation 20.7. Here each charge element contributes the same amount $dE_x = (k\,dq/r^2)\hat{r}_x = (k\,dq/r^2)(x/r)$ to the field. Figure 20.16 shows that $r = \sqrt{x^2 + a^2} = (x^2 + a^2)^{1/2}$, so the integral becomes

$$E = \int_{\text{ring}} dE_x = \int_{\text{ring}} \frac{kx\,dq}{(x^2 + a^2)^{3/2}} = \frac{kx}{(x^2 + a^2)^{3/2}} \int_{\text{ring}} dq$$

The last step follows because we have a fixed field point P, so its coordinate x is a constant for the integration. But the remaining integral is just the sum of all the charge elements on the ring—namely, the total charge Q. So our result becomes

$$E = \frac{kQx}{(x^2 + a^2)^{3/2}} \quad \text{(on-axis field, charged ring)}$$

This is the magnitude; the direction is along the x-axis, away from the ring if Q is positive and toward it if Q is negative.

ASSESS Make sense? At $x = 0$ the field is zero. A charge placed at the ring center is pulled (or pushed) equally in all directions—no net force, so no electric field. But for $x \gg a$, we get $E = kQ/x^2$—just what we expect for a point charge Q. As always, a finite-sized charge distribution looks like a point charge at large distances. Problem 73 shows how you can use the result of this example to find the electric field on the axis of a charged disc, and Problem 75 shows that, once again, the field at large distances becomes that of a point charge.

EXAMPLE 20.7 **Line Charge: A Power Line's Field**
Worked Example with Variation Problems

A long, straight electric power line coincides with the x-axis and carries a uniform line charge density λ (unit: C/m). Find the electric field on the y-axis using the approximation that the wire is infinitely long.

INTERPRET We identify the field point as being a distance y from the wire, and the source charge as the whole wire.

DEVELOP Figure 20.17 is our drawing, showing a coordinate system with the field point P along the y-axis. We divide the wire into small charge elements dq and note that field contributions from two such elements dq on opposite sides of the y-axis contribute fields $d\vec{E}$ whose x-components cancel. Then we need only the y-component of each unit vector, and Figure 20.17 shows that's $\hat{r}_y = y/r$, where $r = \sqrt{x^2 + y^2}$.

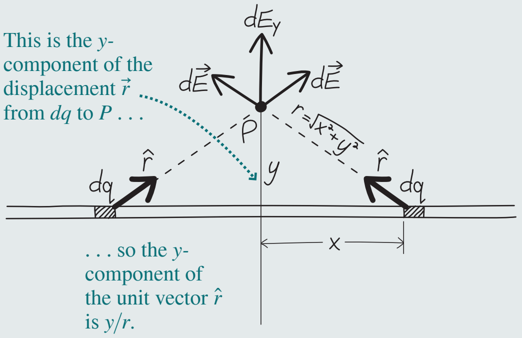


FIGURE 20.17 The field of a charged line is the vector sum of the fields $d\vec{E}$ from all the individual charge elements dq along the line.

EVALUATE We're now ready to set up the integral in Equation 20.7. As described in Chapter 9's integral strategy, we need to relate dq to a geometric variable so we can do the integral. Here our wire has charge density λ C/m, so if a charge element has length dx, then its charge is $dq = \lambda\, dx$. Putting all this together gives the y-component of the field from an arbitrary dq anywhere on the wire:

$$dE_y = \frac{k\,dq}{r^2}\hat{r}_y = \frac{k\lambda\,dx}{r^2}\frac{y}{r} = \frac{k\lambda y}{(x^2 + y^2)^{3/2}}dx$$

where we used $r = \sqrt{x^2 + y^2}$. Since the x-components cancel, we can sum—that is, integrate—the y-components to get the net field:

$$E = E_y = \int_{-\infty}^{+\infty} \frac{k\lambda y\,dx}{(x^2 + y^2)^{3/2}} = k\lambda y \int_{-\infty}^{+\infty} \frac{dx}{(x^2 + y^2)^{3/2}}$$

$$= k\lambda y \left[\frac{x}{y^2\sqrt{x^2 + y^2}}\right]_{-\infty}^{+\infty} = k\lambda y\left[\frac{1}{y^2} - \left(-\frac{1}{y^2}\right)\right] = \frac{2k\lambda}{y}$$

Here we used the integral table in Appendix A and applied the limits $x = \pm\infty$. Our result is the field's magnitude; the direction is away from the line for positive λ and toward the line for negative λ.

ASSESS Make sense? For an infinite line there's nothing to favor one direction along the line over another, so the only way the field can point is radially, away from or toward the line (Fig. 20.18). And because the line is infinite, it never resembles a point no matter how far away we are. As a result the field falls more slowly than the field of a point charge—in this case, as $1/y$. If we let r designate the radial distance from the line rather than the diagonal in Fig. 20.17, then the field decreases as $1/r$. An infinite line is impossible, but our result holds approximately for finite lines of charge as long as we're much closer to the line than its length, and not near an end. Far from a *finite* line, on the other hand, its field will resemble that of a point charge. You can explore the finite charged line in Problem 72.

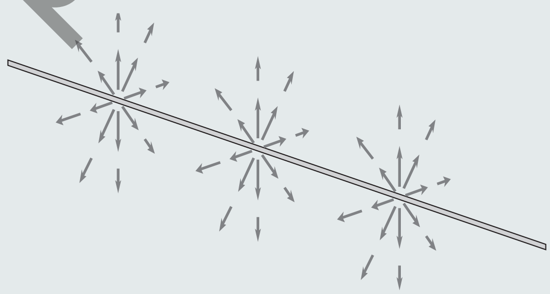


FIGURE 20.18 Field vectors for an infinite line of positive charge point radially outward, with magnitude decreasing inversely with distance.

20.5 Matter in Electric Fields

LO 20.8 *Determine the motion of charged particles in electric fields.*

LO 20.9 *Determine forces and torques on electric dipoles in electric fields.*

Electric fields give rise to forces on charged particles. Because matter consists of such particles, much of the behavior of matter is fundamentally determined by electric fields.

Point Charges in Electric Fields

The motion of a single charge in an electric field is governed by the definition of the electric field, $\vec{F} = q\vec{E}$, and Newton's law, $\vec{F} = m\vec{a}$. Combining these equations gives the acceleration of a particle with charge q and mass m in an electric field $\vec{E}$:

$$\vec{a} = \frac{q}{m}\vec{E} \qquad (20.8)$$

This equation shows that it's the charge-to-mass ratio, q/m, that determines a particle's response to an electric field. Electrons, nearly 2000 times less massive than protons but carrying the same charge, are readily accelerated by electric fields. Many practical devices, from X-ray machines to fluorescent lights, use electrons accelerated in electric fields.

When the electric field is uniform, problems involving the motion of charged particles reduce to the constant-acceleration problems of Chapter 2. An ink-jet printer is one application; a pair of oppositely charged plates creates a uniform field that "steers" charged ink droplets to the right place on the page (Fig. 20.19).

When the field isn't uniform, it's generally more difficult to calculate particle trajectories. An important exception is a particle moving perpendicular to a field that points radially. Under appropriate conditions, the result is uniform circular motion (see Section 5.3), as shown in the next example.

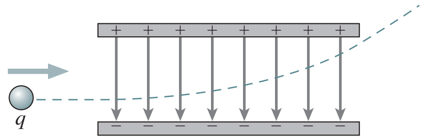


FIGURE 20.19 A pair of parallel charged plates creates a uniform electric field that deflects a charged particle. Can you tell the sign of the charge q?

EXAMPLE 20.8 Particle Motion: An Electrostatic Analyzer

Two oppositely charged curved metal plates establish an electric field given by $E = E_0(b/r)$, where E_0 and b are constants with the units of electric field and length, respectively. The field points toward the center of curvature, and r is the distance from the center. Find an expression for the speed v with which a proton entering vertically from below in Fig. 20.20 will leave the device moving horizontally.

INTERPRET This problem is about charged-particle motion in an electric field that points radially. We're asked for the condition that will have a proton exiting the field region moving horizontally. Figure 20.20 shows that this requires its trajectory to be a circular arc.

DEVELOP Equation 20.8, $\vec{a} = (q/m)\vec{E}$, determines the acceleration of a charged particle in an electric field. Here we want uniform circular motion, so our plan is to write this equation with the given field and the acceleration v^2/r that we know applies in circular motion. Then we'll solve for v.

EVALUATE Under these conditions, Equation 20.8 becomes

$$a = \frac{v^2}{r} = \frac{eE}{m} = \frac{e}{m}E_0\frac{b}{r}$$

We then solve to get $v = \sqrt{eE_0b/m}$.

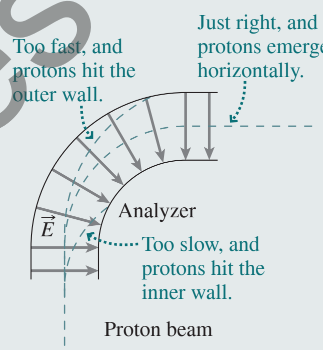


FIGURE 20.20 An electrostatic analyzer.

ASSESS Make sense? Strengthen the field by increasing E_0 or b, and the electric force becomes greater. For a given speed, that would result in more bending of the trajectory; to maintain the desired trajectory, we must therefore increase the speed. Note that the radius r canceled from our equations, showing that it doesn't matter where the protons enter the device. That's because the $1/r$ decrease in field strength matches the $1/r$ dependence of the acceleration. This device is called an electrostatic analyzer because it can sort charged particles by speed and charge-to-mass ratio. Spacecraft use such analyzers to characterize charged particles in interplanetary space.

GOT IT?

20.5 An electron, a proton, a deuteron (a neutron combined with a proton), a helium-3 nucleus (2 protons, 1 neutron), a helium-4 nucleus (2 protons, 2 neutrons), a carbon-13 nucleus (6 protons, 7 neutrons), and an oxygen-16 nucleus (8 protons, 8 neutrons) all find themselves in the same electric field. Rank in order their accelerations from lowest to highest under the assumption (only approximately correct) that the neutron and proton have the same mass and that the mass of a composite particle is the sum of the masses of its constituent neutrons and protons. Note any that have the same acceleration.

Dipoles in Electric Fields

Earlier in this chapter we calculated the field of an electric dipole, which consists of two opposite charges of equal magnitude. Here we study a dipole's response to electric fields. Since the dipole provides a model for many molecules, our results help explain molecular behavior.

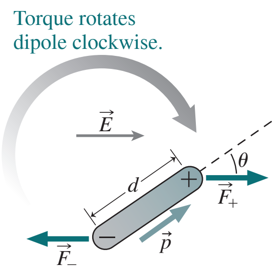


FIGURE 20.21 A dipole in a uniform electric field experiences a torque but no net force.

Figure 20.21 shows a dipole with charges $\pm q$ separated a distance d, located in a uniform electric field. The dipole moment vector $\vec{p}$ has magnitude qd and points from the negative to the positive charge (recall Fig. 20.14). Since the field is uniform, it's the same at both ends of the dipole. Since the dipole charges are equal in magnitude but opposite in sign, they experience equal but opposite forces $\pm q\vec{E}$—and therefore there's no net force on the dipole.

However, Fig. 20.21 shows that the dipole does experience a torque that tends to align it with the field. In Chapter 11 we described torque as the cross product of the position vector with the force: $\vec{\tau} = \vec{r} \times \vec{F}$, where the magnitude of the torque vector is $rF\sin\theta$ and its direction is given by the right-hand rule. Figure 20.21 thus shows that the torque about the center of the dipole due to the force on the positive charge has magnitude $\tau_+ = rF\sin\theta = (\frac{1}{2}d)(qE)\sin\theta$. The torque associated with the negative charge has the same magnitude, and both torques are in the same direction since both tend to rotate the dipole clockwise. Thus the net torque has magnitude $\tau = qdE\sin\theta$. Applying the right-hand rule shows that this torque is into the page. But qd is the magnitude of the dipole moment $\vec{p}$, and Fig. 20.21 shows that θ is the angle between the dipole moment vector and the electric field $\vec{E}$; therefore, we can write the torque vectorially as

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad \text{(torque on a dipole)} \qquad (20.9)$$

Because of this torque, the electric field does work on a dipole as it rotates. The electric force is conservative, so that work results in a change in potential energy. In Chapter 7 we defined potential-energy change as the negative of the work done by a conservative force: $\Delta U = -W$. Here we're dealing with rotational motion, and Equation 10.19 shows that the work done in a rotation from angular position θ_1 to θ_2 is given by $W = \int_{\theta_1}^{\theta_2}\tau\, d\theta$. Figure 20.21 shows that we're taking $\theta = 0$ when the dipole is aligned with the field. The figure also shows that the direction of increasing θ is counterclockwise or, in terms of rotational vectors, out of the page. The torque, in contrast, is clockwise or, vectorially, into the page. Thus the sign of the torque is opposite the angular change, so we need to write $\tau = -pE\sin\theta$ in the integral for the work. Let's now consider a dipole that's initially perpendicular to the field, so $\theta_1 = \pi/2$. Then the work done by the electric force as the dipole rotates to an arbitrary angle θ becomes

$$W = \int_{\pi/2}^{\theta} \tau\, d\theta = \int_{\pi/2}^{\theta} (-pE\sin\theta)\, d\theta = -pE[-\cos\theta]_{\pi/2}^{\theta} = pE\cos\theta$$

The potential-energy change is the negative of this work, and we note that $pE\cos\theta$ can be expressed as the dot product $\vec{p}\cdot\vec{E}$, so we can write the potential energy as

$$U = -\vec{p}\cdot\vec{E} \qquad (20.10)$$

where $U = 0$ corresponds to the dipole at right angles to the field.

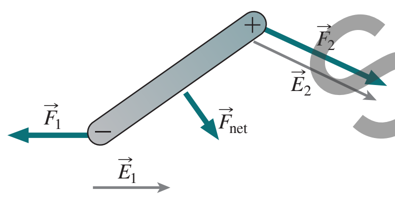


FIGURE 20.22 When the electric field differs in magnitude or direction at the two ends of the dipole, the dipole experiences a nonzero net force as well as a torque.

When the electric field isn't uniform, the charges at opposite ends of the dipole experience forces that differ in magnitude and/or aren't exactly opposite in direction. Then the dipole experiences a net force as well as a torque (Fig. 20.22). An important instance of this effect is the force on a dipole in the field of another dipole (Fig. 20.23). Because the dipole field falls off rapidly with distance and because the dipole responding to the field has closely spaced charges of equal magnitude but opposite sign, the dipole–dipole force is quite weak and falls extremely rapidly with distance. This weak force, which Fig. 20.23 shows to be attractive, is partly responsible for the van der Waals interaction between gas molecules that we mentioned in Chapter 17.

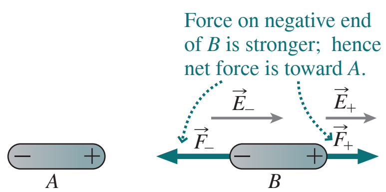


FIGURE 20.23 Dipole *B* aligns with the field of dipole *A* and then experiences a net force toward *A*.

Conductors, Insulators, and Dielectrics

Bulk matter contains vast numbers of point charges—namely, electrons and protons. In some matter—notably metals, ionic solutions, and ionized gases—individual charges are free to move throughout the material. In these **conductors**, the application of an electric field results in the ordered motion of electric charge that we call **electric current**. We'll consider conductors and current in later chapters.

Materials in which charge is not free to move are **insulators**, since they can't carry electric current. Insulators still contain charges—it's just that their charges are bound into neutral molecules. Some molecules, like water, have intrinsic dipole moments and therefore rotate in response to an applied electric field. Even if they don't have dipole moments, molecules may respond to an electric field by stretching and acquiring **induced dipole moments** (Fig. 20.24). In either case, the application of an electric field results in the alignment of molecular dipoles with the field (Fig. 20.25). The fields of the dipoles, pointing from their positive to their negative charges, then reduce the applied electric field within the material. We'll explore the consequences of this effect further in Chapter 23. Materials in which molecules either have intrinsic dipole moments or acquire induced moments are called **dielectrics**.

If the electric field applied to a dielectric becomes too great, individual charges are ripped free, and the material then acts like a conductor. Such **dielectric breakdown** can cause severe damage in materials and in electric equipment (Fig. 20.26). On a larger scale, lightning results from dielectric breakdown in air.

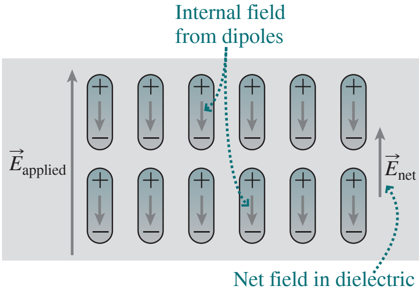


FIGURE 20.25 Alignment of molecular dipoles in a dielectric reduces the electric field within the dielectric.

FIGURE 20.26 Dielectric breakdown in a solid piece of Plexiglas produced this striking fractal pattern that marks permanent changes in the material.

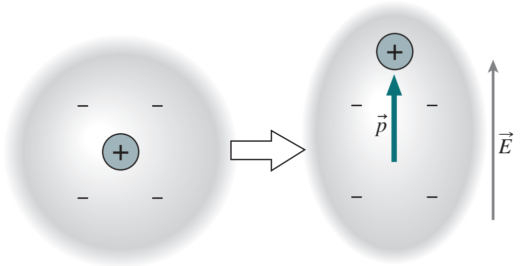
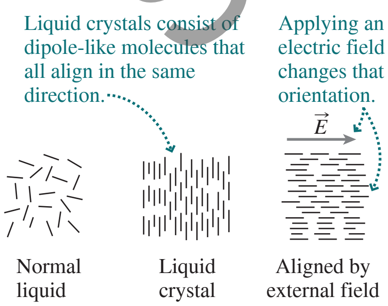


FIGURE 20.24 A molecule stretches in response to an electric field, acquiring a dipole moment.

APPLICATION Microwave Cooking and Liquid Crystals

The torque on dipoles in electric fields forms the basis of two widespread contemporary technologies: the microwave oven and the liquid-crystal display (LCD).

A microwave oven works by generating an electric field whose direction changes several billion times per second. Water molecules, whose dipole moment is much greater than most others, attempt to align with the field. But the field is changing, so the molecules swing rapidly back and forth. As they jostle against each other, the energy they gain from the field is dissipated as heat that cooks the food.



Computer displays, TVs, cameras, cell phones, watches, and many other devices display visual images using liquid crystals. These unique materials combine the fluidity of a liquid with the order of a solid. The liquid crystal consists of long molecules whose chemical structure results in a dipole-like charge separation. In response to each others' electric fields, the molecules tend to align. As the figure shows, an external electric field can rotate the the dipoles that comprise the liquid crystal, thus, altering the material's optical properties. With optical components we'll study in Chapter 29, different sections of a liquid-crystal display can then be made to appear visible or invisible. Liquid-crystal displays consume very little power, but they generate no light of their own and therefore most have a built-in light source. This photo of an iPhone shows its high-resolution display; also shown is a microphoto of the liquid crystals.

Chapter 20 Summary

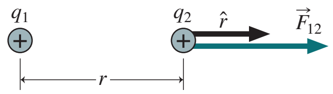
Big Idea

This chapter introduces several big ideas. First is **electric charge**, a fundamental property of matter that comes in positive and negative forms. Like charges repel and opposites attract; this is the **electric force**. It's convenient to define the **electric field** as the force per unit charge that a charge would experience if placed in the vicinity of other charges. Both force and field obey the **superposition principle**, meaning that the effects of several charges add vectorially.

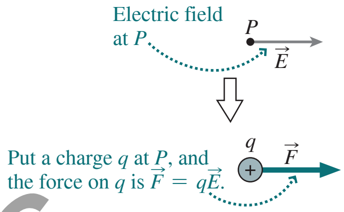
Key Concepts and Equations

Coulomb's law describes the electric force between point charges:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}$$

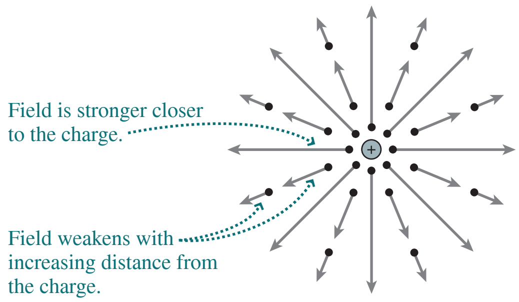


The electric field is the force per unit charge, $\vec{E} = \vec{F}/q$, and therefore the force a given charge q experiences in a field is $\vec{F} = q\vec{E}$.

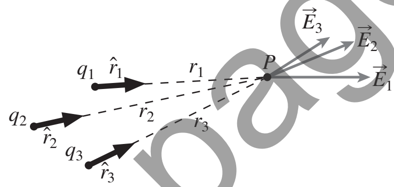


The field of a point charge follows from Coulomb's law:

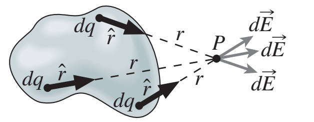
$$\vec{E} = \frac{kq}{r^2}\hat{r}$$



Fields of charge distributions are found by summing fields of individual point charges, or by integrating in the case of continuously distributed charge:



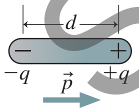
$$\vec{E}(P) = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \sum_i \frac{kq}{r_i^2}\hat{r}_i$$



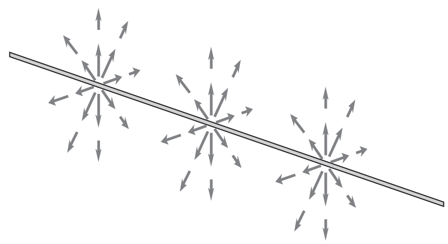
$$\vec{E}(P) = \int d\vec{E} = \int \frac{k\,dq}{r^2}\hat{r}$$

Applications

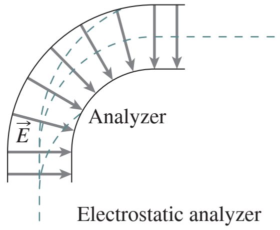
A **dipole** consists of equal but opposite charges $\pm q$ a distance d apart. For distances large compared with d, the dipole field drops as $1/r^3$, and the dipole is completely characterized by its **dipole moment** $p = qd$.



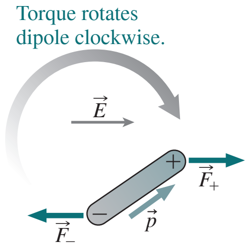
The field of an infinite line drops as $1/r$: $E = 2k\lambda/r$, with λ the charge per unit length. This is a good approximation to the field near an elongated structure like a wire.



Point charges respond to electric fields with acceleration proportional to the charge-to-mass ratio q/m.

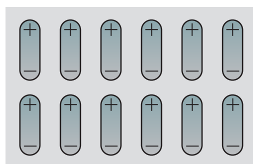


A dipole in an electric field experiences a torque that tends to align it with the field: $\vec{\tau} = \vec{p} \times \vec{E}$.



If the field is nonuniform, there's also a net force on the dipole.

Dielectrics are insulating materials whose molecules act like electric dipoles.



BIO *Biology and/or medicine-related problems* **DATA** *Data problems* **ENV** *Environmental problems* **CH** *Challenge problems* **COMP** *Computer problems*

Learning Outcomes *After finishing this chapter you should be able to:*

LO 20.1 Describe electric charge as a fundamental property of matter.
For Thought and Discussion Questions 20.1, 20.2, 20.8; Exercises 20.11, 20.12, 20.13, 20.14, 20.15

LO 20.2 Use Coulomb's law to calculate the forces between charges.
Exercises 20.16, 20.17, 20.18, 20.19, 20.20; Problems 20.44, 20.57, 20.58

LO 20.3 Use the superposition principle to calculate forces involving multiple charges.
For Thought and Discussion Question 20.3; Problems 20.46, 20.47, 20.48, 20.49, 20.52

LO 20.4 Describe the concept of electric field.
For Thought and Discussion Questions 20.4, 20.5; Exercises 20.21, 20.22, 20.23, 20.24, 20.25, 20.26; Problem 20.50

LO 20.5 Determine the fields of electric charge distributions using superposition.
For Thought and Discussion Question 20.7; Exercises 20.27, 20.28; Problems 20.51, 20.56, 20.59

LO 20.6 Describe the electric dipole and the field it produces.
For Thought and Discussion Questions 20.6, 20.9; Problems 20.53, 20.54, 20.55, 20.67, 20.69, 20.70, 20.72

LO 20.7 Determine the fields of continuous charge distributions by integration.
Exercises 20.29, 20.30, 20.31; Problems 20.61, 20.64, 20.68, 20.71, 20.73, 20.74, 20.75, 20.76, 20.77, 20.78

LO 20.8 Determine the motion of charged particles in electric fields.
Exercises 20.32, 20.33, 20.34, 20.35; Problems 20.60, 20.63

LO 20.9 Determine forces and torques on electric dipoles in electric fields.
For Thought and Discussion Questions 20.9, 20.10; Problems 20.62, 20.65, 20.66

For Thought and Discussion

1. Conceptual Example 20.1 shows that the gravitational force between an electron and a proton is about 10^{-40} times weaker than the electric force between them. Since matter consists largely of electrons and protons, why is the gravitational force important at all?
2. A free neutron is unstable and soon decays to other particles, one of them a proton. Must there be others? If so, what electric properties must it or they have?
3. Where in Fig. 20.5 could you put a third charge so it would experience no net force? Would it be in stable or unstable equilibrium?
4. Equation 20.3 gives the electric field of a point charge. Does the direction of (a) $\hat{r}$ or (b) $\vec{E}$ depend on whether the charge is positive or negative?
5. Is the electric force on a charged particle always in the direction of the field? Explain.
6. Why does a dipole, which has no net charge, produce an electric field?
7. The ring in Example 20.6 carries total charge Q, and the point P is the same distance $r = \sqrt{x^2 + a^2}$ from all parts of the ring. So why isn't the electric field of the ring just kQ/r^2?
8. A spherical balloon is initially uncharged. If you spread positive charge uniformly over the balloon's surface, would it expand or contract? What would happen if you spread negative charge instead?
9. Why should there be a force between two dipoles, which each have zero net charge?
10. Dipoles A and B are both located in the field of a point charge Q, as shown in Fig. 20.27. Does either experience a net torque? A net force? If each dipole is released from rest, qualitatively describe its subsequent motion.

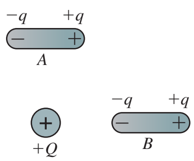


FIGURE 20.27 For Thought and Discussion 10

Exercises and Problems

Exercises

Section 20.1 Electric Charge

11. Suppose the electron and proton charges differed by one part in one billion. Estimate the net charge on your body, assuming it contains equal numbers of electrons and protons.
12. A typical lightning flash delivers about 25 C of negative charge from cloud to ground. How many electrons are involved?
13. Protons and neutrons are made from combinations of the two most common quarks, the u quark (charge $+\frac{2}{3}e$) and the d quark (charge $-\frac{1}{3}e$). How could three of these quarks combine to make (a) a proton and (b) a neutron?
14. Earth carries a net charge of about -5×10^5 C. How many more electrons are there than protons on Earth?
15. **BIO** As they fly, honeybees may acquire electric charges of about 180 pC. Electric forces between charged honeybees and spider webs can make the bees more vulnerable to capture by spiders. How many electrons would a honeybee have to lose to acquire a charge of +180 pC?

Section 20.2 Coulomb's Law

16. The electron and proton in a hydrogen atom are 52.9 pm apart. Find the magnitude of the electric force between them.
17. An electron at Earth's surface experiences a gravitational force $m_e g$. How far away can a proton be and still produce the same force on the electron? (Your answer should show why gravity is unimportant on the molecular scale!)
18. You break a piece of Styrofoam packing material, and it releases lots of little spheres whose electric charge makes them stick annoyingly to you. If two of the spheres carry equal charges and repel with a force of 20 mN when they're 17 mm apart, what's the magnitude of the charge on each?
19. A charge q is at the point $x = 5$ m, $y = 0$ m. Write expressions for the unit vectors you would use in Coulomb's law if you were finding the force that q exerts on other charges located at (a) $x = 5$ m, $y = 2.5$ m; (b) the origin; and (c) $x = 7$ m, $y = 3.5$ m. You're not given the sign of q. Why doesn't this matter?

20. A proton is at the origin and an electron is at the point $x = 0.41$ nm, $y = 0.36$ nm. Find the electric force on the proton.

Section 20.3 The Electric Field

21. An electron experiences an electric force of 0.61 nN. What's the field strength at its location?
22. Find the magnitude of the electric force on a 6.0-μC charge in a 50-N/C electric field.
23. A 75-nC charge experiences a 144-mN force in a certain electric field. Find (a) the field strength and (b) the force that a 35-μC charge would experience in the same field.
24. BIO The electric field inside a cell membrane is 8.0 MN/C. What's the force on a singly charged ion in this field?
25. A -3.0-μC charge experiences a $9.0\hat{\imath}$-N electric force in a certain electric field. What force would a proton experience in the same field?
26. The electron in a hydrogen atom is 52.9 pm from the proton. At this distance, what's the strength of the electric field due to the proton?

Section 20.4 Fields of Charge Distributions

27. In Fig. 20.28, point P is midway between the two charges. Find the electric field in the plane of the page (a) 5.0 cm to the left of P, (b) 5.0 cm directly above P, and (c) at P.

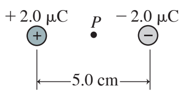


FIGURE 20.28 Exercise 27

28. The water molecule's dipole moment is 6.17×10^{-30} C·m. What would be the separation distance if the molecule consisted of charges $\pm e$? (The effective charge is actually less because H and O atoms share the electrons.)
29. The electric field 22 cm from a long wire carrying a uniform line charge density is 1.9 kN/C. What's the field strength 38 cm from the wire?
30. Find the line charge density on a long wire if the electric field 39 cm from the wire has magnitude 210 kN/C and points toward the wire.
31. Find the magnitude of the electric field due to a charged ring of radius a and total charge Q on the ring axis at distance a from the ring's center.

Section 20.5 Matter in Electric Fields

32. In his famous 1909 experiment that demonstrated quantization of electric charge, R. A. Millikan suspended small oil drops in an electric field. With field strength 20 MN/C, what mass drop can be suspended when the drop carries 10 elementary charges?
33. How strong an electric field is needed to accelerate electrons in an X-ray tube from rest to one-tenth the speed of light in a distance of 4.7 cm?
34. A proton moving to the right at 3.3×10^5 m/s enters a region where a 60-kN/C electric field points to the left. (a) How far will the proton get before it momentarily stops? (b) Describe its subsequent motion.
35. An electrostatic analyzer like that of Example 20.8 has $b = 7.5$ cm. What value of E_0 will enable the device to select protons moving at 84 m/s?

Example Variations

The following problems are based on two examples from the text. Each set of four problems is designed to help you make connections that enhance your understanding of physics and to build your confidence in solving problems that differ from ones you've seen before. The first problem in each set is essentially the example problem but with different numbers. The second problem presents the same scenario as the example but asks a different question. The third and fourth problems repeat this pattern but with entirely different scenarios.

36. **Example 20.2:** Charges on raindrops vary widely in both magnitude and sign. Consider a case where the two drops on the x-axis in Example 20.2 are 2.18 mm apart and have charge $q = 645$ nC, while the third drop is 12.3 mm up the y-axis and has charge $Q = -1.87$ μC. Find the electric force on the upper drop.
37. **Example 20.2:** Suppose that all three raindrops in Example 20.2 have equal charges and that their positions form an equilateral triangle with side 3.36 mm. If the electric force on the upper charge is $96.2\,\hat{\jmath}$ N, (a) what's the magnitude of the charge? (b) Can you determine the sign of the charge from the information given?
38. **Example 20.2:** (a) Repeat Example 20.2 to find the force on Q, now taking the right-hand charge on the x-axis to be $-q$. (b) For $y \gg a$, how does the force you found in (a) depend on the distance y?
39. **Example 20.2:** (a) Use calculus to show that the maximum force in the situation of Example 20.2 occurs when $y = a/\sqrt{2}$, and (b) find the magnitude of that maximum force.
40. **Example 20.7:** A 1.00-km length of power line carries a total charge of 264 mC distributed uniformly over its length. Find the magnitude of the electric field 54.3 cm from the axis of the power line, and not near either end (staying away from the ends means you can approximate the field as that of an infinitely long wire).
41. **Example 20.7:** A uniformly charged wire is 2.18 m long and 0.15 mm in diameter. You measure the electric field 1.20 cm from the wire's axis, not near either end, and you find it to be 455 kN/C, pointing toward the wire. Find the total charge on the wire.
42. **Example 20.7:** A thin rod of length L lies on the x-axis with its center at the origin, as shown in Fig. 20.29. The rod carries charge Q distributed uniformly over its length. (a) Modify the calculation of Example 20.7 to find an expression for the electric field at point A in Fig. 20.29, located on the positive y-axis an arbitrary distance y from the origin (but with y large enough to put point A outside the thin rod). (b) Show that your result reduces to the field of a point charge Q when $y \gg L$.

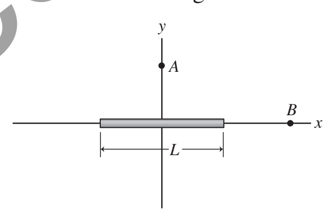


FIGURE 20.29 Problems 42 and 43

43. **Example 20.7:** A thin rod of length L lies on the x-axis with its center at the origin, as shown in Fig. 20.29. The rod carries charge Q distributed uniformly over its length. (a) Find an expression for the electric field at point B in Fig. 20.29, located on the positive x-axis a distance x from the origin, where $x > L/2$, so that point B is beyond the right end of the rod. (b) Show that your result reduces to the field of a point charge Q when $x \gg L$.

Problems

44. Two charges, of which one has a magnitude three times as large as the other's, are located 14.5 cm apart and experience an attractive force of 156 N. (a) What's the magnitude of the larger charge? (b) Can you determine the sign of the larger charge?
45. A proton is on the x-axis at $x = 1.3$ nm. An electron is on the y-axis at $y = 0.86$ nm. Find the net force the two exert on a helium nucleus (charge $+2e$) at the origin.
46. A charge $3q$ is at the origin, and a charge $-2q$ is on the positive x-axis at $x = a$. Where would you place a third charge so it would experience no net electric force?
47. A negative charge $-q$ lies midway between two positive charges $+Q$. What must Q be such that the electric force on all three charges is zero?
48. In Fig. 20.30, take $q_1 = 68$ μC, $q_2 = -34$ μC, and $q_3 = 15$ μC. Find the electric force on q_3.
49. In Fig. 20.30, take $q_1 = 21$ μC and $q_2 = 18$ μC. If the force on q_1 points in the $-x$-direction, find (a) q_3 and (b) the magnitude of the force on q_1.

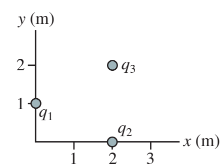


FIGURE 20.30 Problems 48 and 49

50. BIO DATA DNA fragments introduced into an electrophoresis apparatus (see Application, page 390) generally carry negative charges equivalent to two extra electrons per base pair of nucleotides in the fragment. The table below shows the forces on several DNA fragments in an electrophoresis apparatus, as a function of the number of base pairs. Plot these data, establish a best-fit line, and use the resulting slope to determine the strength of the electric field in the electrophoresis apparatus.

Base pairs	400	800	1200	2000	3000	5000
Force (pN)	0.235	0.472	0.724	1.15	1.65	2.87

51. A proton is at the origin and an ion at $x = 8.0$ nm. If the electric field is zero at $x = -4.0$ nm, what's the ion's charge?
52. Four equal charges Q are at the corners of a square of side a. Find an expression for the magnitude of the force on each charge.
53. A dipole lies on the y-axis and consists of an electron at $y = 0.60$ nm and a proton at $y = -0.60$ nm. Find the electric field (a) midway between the two charges; (b) at the point $x = 2.0$ nm, $y = 0$ nm; and (c) at the point $x = -20$ nm, $y = 0$ nm.
54. Show that the field on the x-axis for the dipole of Example 20.5 is given by Equation 20.6b, for $|x| \gg a$.
55. You're 1.44 m from a charge distribution that is well under 1 cm in size. You measure an electric field strength of 296 N/C due to this distribution. You then move to a distance of 2.16 m from the distribution, where you measure a field strength of 87.7 N/C. What's the net charge of the distribution? *Hint*: Don't try to calculate the charge. Determine instead how the field decreases with distance, and from that infer the charge.
56. Three identical charges q form an equilateral triangle of side a, with two charges on the x-axis and one on the positive y-axis. (a) Find an expression for the electric field at points on the y-axis above the uppermost charge. (b) Show that your result reduces to the field of a point charge $3q$ for $y \gg a$.
57. Two identical small metal spheres initially carry charges q_1 and q_2. When they're 1.0 m apart, they experience a 2.5-N attractive force. Then they're brought together so charge moves from one to the other until they have the same net charge. They're again placed 1.0 m apart, and now they repel with a 2.5-N force. What were the original charges q_1 and q_2?
58. COMP Two 32.0-μC charges are attached to opposite ends of a spring with spring constant $k = 135$ N/m and equilibrium length 49.3 cm. By how much does the spring stretch? *Hint*: You'll need to use a computer or advanced calculator to solve the cubic equation that arises in this problem.
59. A positive charge Q is located at the origin, and another charge q is at $x = a$, where $a > 0$. Given that the electric field is zero at $x = 2a$, find an expression for q in terms of Q.
60. An electron is moving in a circular path around a long, uniformly charged wire carrying 1.4 nC/m. What's the electron's speed?
61. Find the line charge density on a long wire if a 6.5-μg particle carrying 2.2 nC describes a circular orbit about the wire with speed 270 m/s.
62. A dipole with dipole moment 1.6 nC · m is oriented at 30° to a 5.0-MN/C electric field. Find (a) the magnitude of the torque on the dipole and (b) the work required to rotate the dipole until it's antiparallel to the field.
63. You have a job examining patent applications. You're presented with the device in Fig. 20.31, which its inventor claims will separate isotopes of a particular element. Atoms are first stripped completely of their electrons, then accelerated from rest through an electric field chosen to give the desired isotope exactly the right speed to pass through the electrostatic analyzer (see Example 20.8). Will the device work?

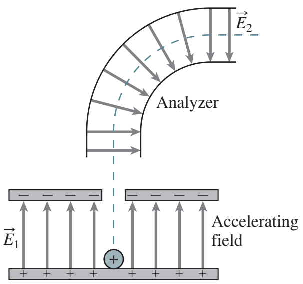


FIGURE 20.31 Problem 63

64. BIO A 5.0-μm strand of DNA carries charge $+e$ per nm of length. Treating it as a charged line, what's the electric field strength 21 nm from the DNA, not near either end?
65. Heating in a microwave oven occurs as water molecules rotate back and forth to align their dipole moments with a rapidly changing electric field. Given water's dipole moment of 6.17×10^{30} C·m, what's the energy change when a water molecule, with its dipole moment initially opposite a 2.95-kN/C electric field, swings to align with the field?
66. CH A dipole with charges $\pm q$ and separation $2a$ is located a distance x from a point charge $+Q$, oriented as shown in Fig. 20.32. Find expressions for the magnitude of (a) the net torque and (b) the net force on the dipole, both in the limit $x \gg a$. (c) What's the direction of the net force?

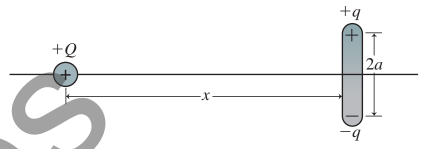


FIGURE 20.32 Problem 66

67. You're taking physical chemistry, and your professor is discussing molecular dipole moments. Water, he says, has a dipole moment of "1.85 debyes," while carbon monoxide's dipole moment is only "0.12 debye." Your physics professor wants these moments expressed in SI. She tells you that the atomic separation in these two covalent compounds is about the same, and asks what that indicates about the way shared charge is distributed. What do you tell her?
68. The electric field on the axis of a uniformly charged ring has magnitude 340 kN/C at a point 10 cm from the ring center. The magnitude 25 cm from the center is 110 kN/C; in both cases the field points away from the ring. Find (a) the ring's radius and (b) its charge.
69. CH An *electric quadrupole* consists of two oppositely directed dipoles in close proximity. (a) Calculate the field of the quadrupole shown in Fig. 20.33 for points to the right of $x = a$ and (b) show that for $x \gg a$ the quadrupole field falls off as $1/x^4$.

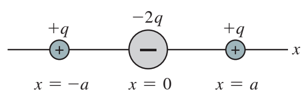


FIGURE 20.33 Problem 69

70. CH Four charges lie at the corners of a square of side a, with the center of the square at the origin. The two charges with $y = a/2$ have magnitude Q and are positive. The two charges with $y = -a/2$ also have magnitude Q but are negative. (a) Find an expression for the magnitude of the electric field for points on the y-axis with $y > a/2$. (b) Show that, for $y \gg a$, your result exhibits the $1/y^3$ falloff you would expect for an electric dipole. (c) Compare the result of (b) with Equation 20.6b and write an expression for the magnitude of the dipole moment of this four-charge distribution. *Hint*: Be careful with your approximation in (b)! If you get 0 for your answer, then you've gone too far. You can neglect a^2 when compared with y^2, but you can't neglect a when compared with y or you'll be throwing out the charge separation that makes this distribution resemble a dipole at large distances.
71. A straight wire 12 m long carries 28 μC distributed uniformly over its length. (a) What's the line charge density on the wire? Find the electric field strength (b) 20 cm from the wire axis, not near either end, and (c) 450 m from the wire. Make suitable approximations in both cases.