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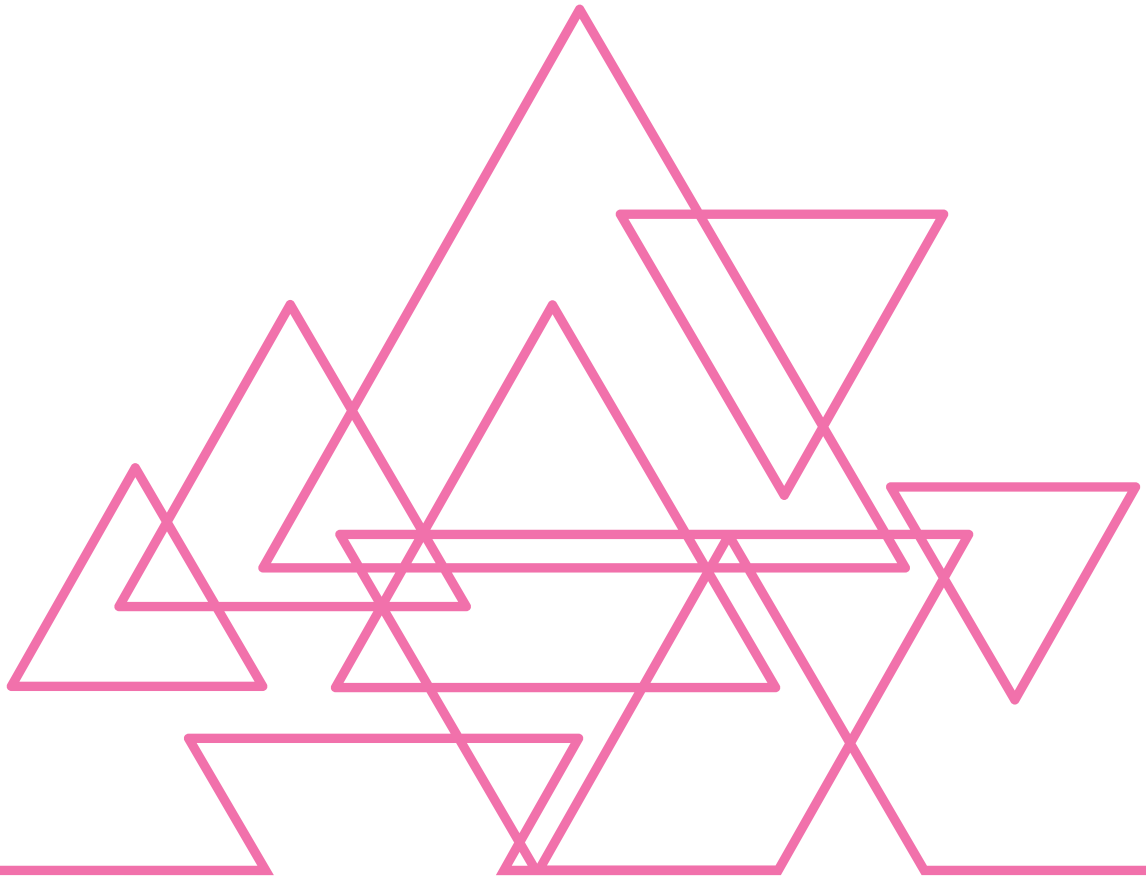

PRIMARY AND MIDDLE YEARS MATHEMATICS

Teaching Developmentally

FIRST AUSTRALIAN EDITION

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SECTION 1 TEACHING MATHEMATICS: FOUNDATIONS AND PERSPECTIVES

The fundamental core of effective instruction of mathematics combines an understanding of how students learn, how to promote that learning by teaching through problem-solving and how to plan for and assess that learning on a daily basis. Introductory chapters in this section provide perspectives on trends in mathematics education and the process of doing mathematics. These chapters develop the core ideas of learning, teaching, planning and assessment. Additional perspectives on mathematics for students with diverse backgrounds and the role of technological tools are also emphasised.

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PREFACE

Welcome to the first Australian edition of *Primary and middle years mathematics: Teaching developmentally*. All students can learn mathematics with understanding and it is their teacher's actions that will enable every student to have this experience. We believe that teachers must create a classroom environment in which students are given opportunities to solve problems and work together, using their ideas and strategies, to solve them. Effective mathematics instruction involves posing tasks that engage students in the mathematics they are expected to learn. Then, by allowing students to interact with and productively struggle with *their own mathematical ideas* and *their own strategies*, they will learn to see the connections among mathematical topics and the real world.

Creating a classroom in which students design solution pathways, engage in productive struggle and connect one mathematical idea to another is complex. Questions arise, such as, 'How do I get students to wrestle with problems if they just want me to show them how to do it? What kinds of tasks lend themselves to this type of engagement? Where can I learn the mathematics content I need in order to be able to teach in this way?' With these and other questions firmly in mind, we have several objectives in this first Australian edition:

1. Illustrate what it means to teach mathematics using a problem-based approach.
2. Serve as a go-to reference for the mathematics content suggested for Foundation to Year 9 as recommended in the Australian Curriculum: Mathematics (ACARA, 2016), and for the research-based strategies that illustrate how students best learn this content.
3. Present a practical resource of robust, problem-based activities and tasks that can engage students in the use of significant mathematical concepts and skills.
4. Report on technology that makes teaching mathematics in a problem-based approach more visible, including access to ready-to-use activity pages and quality websites.

▶ FEATURES OF THE AUSTRALIAN EDITION

The chapters in this text are separated into two distinct sections. The first section consists of seven chapters and covers important ideas that cross the boundaries of specific areas of content. The second section, consisting of sixteen chapters, offers teaching suggestions and activities for the major mathematics topics in the Foundation to Year 9 curriculum.

Chapters in Section 1 offer perspectives on the challenging task of helping students learn mathematics. Having a feel for the discipline of mathematics – that is, to know what it means to 'do mathematics' – is critical to learning how to teach mathematics well. In addition, understanding constructivist and sociocultural perspectives on learning mathematics and how they are applied to teaching through problem-solving provides a foundation and rationale for how to teach and assess students in Foundation to Year 9.

You will be teaching students for whom English is an additional language or dialect, who are gifted or have additional needs. In this text, you will learn how to apply instructional strategies in ways that support and challenge *all* learners. Formative assessment strategies, strategies for diverse learners and effective use of technological tools are addressed in specific chapters in Section 1 (Chapters 5, 6 and 7, respectively), and throughout Section 2.

Each chapter of Section 2 focuses on one of the major content areas in the Foundation–Year 9 mathematics curriculum. It begins with identifying the big ideas for that content, and also provides guidance on how students best learn that content through problem-based activities to engage them in understanding mathematics. Reflecting on the activities as you read can help you think about the mathematics from the perspective of the student. As often as possible, take out a pencil and paper and try the problems so that you actively engage in *your learning* about *students learning* mathematics. In so doing, we are hopeful that this book will increase your own understanding of mathematics, the students you teach and how to teach them well.

FOCUS ON AUSTRALIAN CURRICULUM: MATHEMATICS CONTENT DESCRIPTIONS AND PROFICIENCY STRANDS

The Australian Curriculum: Mathematics is described in Chapter 1 along with other standards documents. In Section 2, references to the Australian Curriculum are embedded in the text and every activity lists the Australian Curriculum content descriptions that can be developed in that activity. Additionally, within the chapters you will find Australian Curriculum proficiency strand margin notes that identify content that illustrates what these strands look like in classroom teaching.

Appendix B provides the Australian Curriculum: Mathematics content descriptions for Foundation–Year 9, the chapter where the content description is addressed and activities from this textbook that can be used to support students' learning of the content. We believe you will find this an invaluable resource for planning instruction.

▶ EDUCATOR RESOURCES

A suite of resources is provided to assist with delivery of the content, as well as to support learning and teaching.

Test bank. The test bank provides a wealth of testing material. Each chapter offers a wide variety of question types, arranged by section. Questions can be integrated into Blackboard, Canvas or Moodle learning management systems.

Instructor manual. The instructor manual provides detailed concepts, discussion topics and activities to enrich lessons.

Digital image PowerPoint slides. All the diagrams and tables from the course content are available for lecturer use.

▶ ACKNOWLEDGEMENTS

Many talented people have contributed to the success of this book. We have received thoughtful feedback from many mathematics teacher educators on both the US ninth edition and on the manuscript for this first Australian edition. Each reviewer challenged us to think through important issues, and many specific suggestions have found their way into the book. Thank you to:

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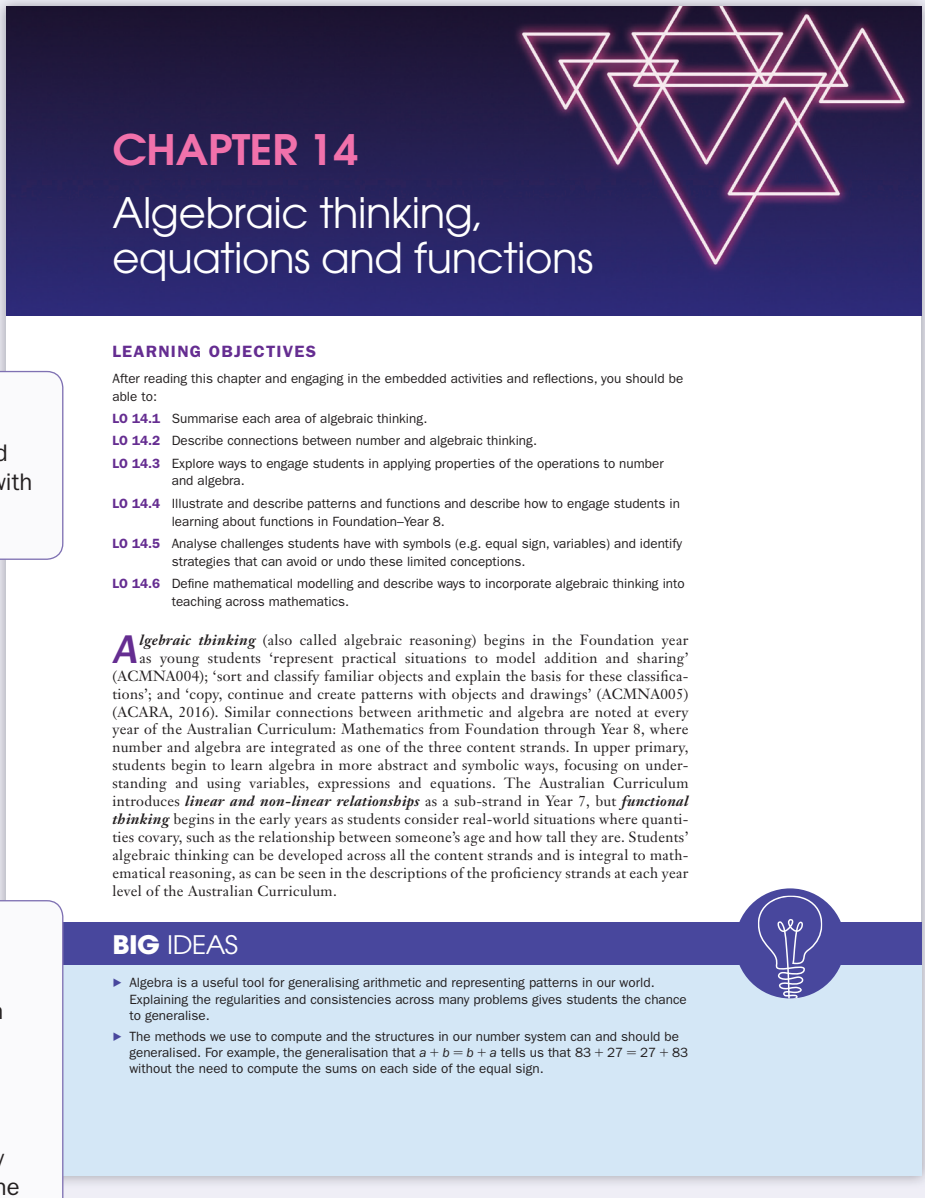
We are privileged to work with fantastic colleagues at Pearson, particularly Stephen Heasley, Nicole Le Grand and Melanie Dankel, who offered us valuable support and advice. Stephen, our portfolio manager, provided the vision for this first Australian edition and helped us define the direction of this edition. Our development editor, Nicole, consistently offered us advice and encouragement about our chapters. Melanie, lead editor and copy editor, helped us to further refine our chapters.

From Amy Brass: I am grateful for my husband, Jory Brass, who is my biggest supporter and challenges me in the best ways possible to think more deeply about education. I am also grateful for my parents and my in-laws, who kindly asked for updates about the textbook and who also offered their support.

HOW TO USE THIS TEXT

By flipping through the text, you will notice many section headings, a large number of figures and various special features. All are designed to make the text more useful as a long-term resource. Here are a few things to look for.



CHAPTER 14

Algebraic thinking, equations and functions

LEARNING OBJECTIVES

After reading this chapter and engaging in the embedded activities and reflections, you should be able to:

LO 14.1 Summarise each area of algebraic thinking.

LO 14.2 Describe connections between number and algebraic thinking.

LO 14.3 Explore ways to engage students in applying properties of the operations to number and algebra.

LO 14.4 Illustrate and describe patterns and functions and describe how to engage students in learning about functions in Foundation–Year 8.

LO 14.5 Analyse challenges students have with symbols (e.g. equal sign, variables) and identify strategies that can avoid or undo these limited conceptions.

LO 14.6 Define mathematical modelling and describe ways to incorporate algebraic thinking into teaching across mathematics.

Algebraic thinking (also called algebraic reasoning) begins in the Foundation year as young students 'represent practical situations to model addition and sharing' (ACMNA004); 'sort and classify familiar objects and explain the basis for these classifications'; and 'copy, continue and create patterns with objects and drawings' (ACMNA005) (ACARA, 2016). Similar connections between arithmetic and algebra are noted at every year of the Australian Curriculum: Mathematics from Foundation through Year 8, where number and algebra are integrated as one of the three content strands. In upper primary, students begin to learn algebra in more abstract and symbolic ways, focusing on understanding and using variables, expressions and equations. The Australian Curriculum introduces ***linear and non-linear relationships*** as a sub-strand in Year 7, but ***functional thinking*** begins in the early years as students consider real-world situations where quantities covary, such as the relationship between someone's age and how tall they are. Students' algebraic thinking can be developed across all the content strands and is integral to mathematical reasoning, as can be seen in the descriptions of the proficiency strands at each year level of the Australian Curriculum.

BIG IDEAS

- Algebra is a useful tool for generalising arithmetic and representing patterns in our world. Explaining the regularities and consistencies across many problems gives students the chance to generalise.
- The methods we use to compute and the structures in our number system can and should be generalised. For example, the generalisation that $a + b = b + a$ tells us that $83 + 27 = 27 + 83$ without the need to compute the sums on each side of the equal sign.

▶ Learning objectives

To help readers know what they should expect to learn, each chapter begins with learning objectives.

▶ Big ideas

Much of the research and literature espousing a student-centred approach suggests that teachers plan their instruction around big ideas rather than isolated skills or concepts. At the beginning of each chapter in Section 2, you will find a list of the key mathematical ideas associated with the chapter. Teachers find these lists helpful to quickly envision the mathematics they are to teach.

▶ Activities

There are numerous activities in every chapter in Section 2. Some activity ideas are described directly in the text and in the illustrations. Others are presented in the numbered activity boxes. Every activity is a problem-based task (as described in Chapter 3), is aligned with the Australian Curriculum and is designed to engage students in doing mathematics.

ACTIVITY 15.2 ACMNA078

Who is winning?

STUDENTS with ADDITIONAL NEEDS

EAL/D STUDENTS

Use the Who is winning? activity page and give students paper strips or ask them to draw a number line. This activity can be done two ways (depending on your lesson goals). First, ask students to use reasoning to answer the question 'Who is winning?' Students can use reasoning strategies to compare and decide. Second, students can locate each person's position on a number line. Explain that the friends below are playing 'Red light, green light'. The fractions tell how much of the distance they have already moved. Can you place these friends on a line to show where they are between the start and finish? Second, rather than place them, ask students to use reasoning to answer the question, 'Who is winning?'

Aria: $\frac{3}{4}$ Harry: $\frac{1}{2}$ Luca: $\frac{5}{6}$

Hannah: $\frac{5}{8}$ Ahmed: $\frac{5}{9}$ Angela: $\frac{2}{3}$

This game can be differentiated by changing the value of the fractions or the number of friends (fractions). The game of 'Red light, green light' may not be familiar to EAL/D students. Modelling the game with people in the class and using estimation are good ways to build background and support all students, particularly those with additional learning needs.

▲ Adaptations for students with additional learning needs and EAL/D students

Chapter 6 provides detailed background and strategies for how to support students with additional learning needs and students for whom English is an additional language or dialect (EAL/D). Many adaptations, however, are specific to a particular activity or task. Therefore, Section 2 chapters offer adaptations and instructions within activities (look for the icons) that can meet the needs students with additional learning needs and EAL/D students.

ACTIVITY 19.20 ACMMG109

Fixed perimeters

Give students a loop of non-stretching string that is 24 centimetres in circumference and 1-centimetre grid paper or just use the grid paper alone. The task is to decide what different-sized rectangular gardens can be made with a perimeter of 24 centimetres. Each different rectangle can be recorded on the 1-centimetre grid paper with the area noted inside the sketch of the garden ($A = 20$ cm^2). Then record all of the results on the Fixed perimeter recording sheet.

ACTIVITY 19.21 ACMMG109

Fixed areas

Provide students with 1-centimetre grid paper. The task is to see how many rectangular gardens can be made with an area of 36 – that is, to make filled-in rectangles, not just borders. Each new rectangle should be recorded by sketching the garden and the dimensions on grid paper. For each rectangle, students should determine and record the perimeter inside the figure ($P = 24$ cm). Then record all the results on the Fixed area recording sheet. You might also use the Fixed areas expanded lesson for this activity.

◀ Blackline Masters, activity pages and teacher resource pages

More than 130 ready-to-use pages are provided to support the problems and activities in the book. These are highlighted in pink in the text and can be downloaded from the Pearson Australia website at www.pearson.com.au.

▶ Formative assessment notes

Assessment should be an integral part of teaching. Similarly, it makes sense to think about what to be listening for (assessing) as you read about different areas of content development. Throughout the content chapters, there are formative assessment notes with brief descriptions of ways to assess the topic in that section. Reading these assessment notes as you read the text can also help you understand how best to assist struggling students.

FORMATIVE ASSESSMENT NOTES. Ask students to write a real-life story problem that involves an inequality. You can add expectations such as 'it must be multi-step' and 'you must illustrate the solution on a number line' (for more details see Whaley, 2012, for a full lesson, examples, rubric and discussion). Writing helps students connect representations and helps you see what misconceptions they might have. ■

TECHNOLOGY NOTE. Computer programs and graphing calculators can provide a variety of graphical displays. Use the time saved by technology to focus on the discussions about the information that each display provides. Students can make their own selections from among different graphs and justify their choice based on their own intended purposes. TinkerPlots (www.tinkerplots.com) and Geogebra (www.geogebra.org) are two examples of software that is available for students to graphically represent data. The graphing calculator puts data analysis technology in the hands of every student. The TI-73 calculator is designed for middle years students. It will produce eight different kinds of plots or graphs, including pie graphs, column graphs and picture graphs, and will compute and graph lines of best fit. The Internet also offers opportunities to explore different graphs. Create a Graph (NCES Kids Zone; https://nces.ed.gov/nceskids/) provides tools for creating five different graphical displays. NCTM's Illuminations Data grapher and Advanced data grapher (https://illuminations.nctm.org) allow students to enter data, select

AC Proficiency Strands
Reasoning

◄ Australian Curriculum proficiency strands margin notes

Connections to the four proficiency strands of the Australian Curriculum: Mathematics are highlighted in the margins. The location of the note indicates an example of the identified strand in the nearby text.

▲ Technology notes

Infusing technological tools is important in learning mathematics, as you will learn in Chapter 7. We have infused technology notes throughout Section 2. A technology icon is used to identify places within the text or activity where a technology idea or resource is discussed. Descriptions include open-source (free) software, applets and other online resources, as well as ideas for calculator use.

REFLECTIONS ON CHAPTER 14

WRITING TO LEARN

Assess your understanding and application of chapter content by answering the following questions.

1. Generalisation and symbolisation are essential aspects of algebraic thinking. Describe what you think each word means and give an example.
2. What misconceptions or limited conceptions do students have regarding the equal sign? What causes these misconceptions and how can teaching clear these up?
3. What misconceptions or limited conceptions do students have regarding variables? What causes these misconceptions and how can teaching clear these up?
4. Explain how to solve the equation $4x + 3 = x + 12$ on the pan balance.
5. What is a recursive relationship? A correspondence relationship (explicit rule)? Where in a table for a

RESOURCES FOR CHAPTER

LITERATURE CONNECTIONS

The following examples of books are excellent beginnings for patterns and building tables of values.

Anno's magic seeds *Anno (1994)*
Anno's magic seeds has several patterns. A wise man gives Jack two magic seeds, one to eat and one to plant. The planted seed will produce two new seeds by the following year. Several years later, Jack decides to plant both seeds. Then he has a family and starts to sell seeds. At each stage of the story, there is an opportunity to develop a table of values and extend the current pattern into the future. Austin and Thompson (1997) describe how they used the story to develop patterns and tables of values with Year 6/7 students.

Bats on parade *Appelt and Sweet (1999)*
This story includes the pattern of bats walking 1 by 1, then 2 by 2, and so on. One activity from this enjoyable book is determining the growing pattern of the number of bats given the array length (e.g. 3 for the 3 × 3 array). There is also one mouse, so this can be included in a second investigation. Activity pages for these two ideas and two others can be found in Roy and Beckmann (2007).

◄ End-of-chapter resources

The end of each chapter includes two major subsections: *Reflections*, which includes 'Writing to learn' and 'For discussion and exploration', and *Resources*, which includes 'Literature connections' (found in all Section 2 chapters) and 'Recommended readings'.

Writing to learn Questions are provided that help you reflect on the important pedagogical ideas related to the content in the chapter. Writing out the answers to these questions in your own words, or talking about them with peers, is one of the best ways for you to develop your understanding of each chapter's main ideas. Answers and feedback for each question are provided.

For discussion and exploration These questions ask you to explore an issue related to that chapter's content, applying what you have learned. For example, questions may ask you to reflect on classroom observations, analyse curriculum materials or take a position on controversial issues. We hope that these questions will stimulate thought and cause spirited conversations.

Literature connections Section 2 chapters contain great children's literature suggestions for launching into the mathematics concepts in the chapter just read. For each title suggested, there is a brief description of how the mathematics concepts in the chapter can be connected to the story. These literature-based mathematics activities will help you engage students in interesting contexts for doing mathematics.

Recommended readings In this section, you will find an annotated list of articles and books to augment the information found in the chapter. These recommendations include professional resources designed for the classroom teacher. (In addition to the recommended readings, there is a References list at the end of the book for all sources cited within the chapters.)