CHAPTER 1

Cranes such as this one are required to lift extremely large loads. Their design is based on the basic principles of statics and dynamics, which form the subject matter of engineering mechanics.

GENERAL PRINCIPLES

CHAPTER OBJECTIVES

- To provide an introduction to the basic quantities and idealizations of mechanics.
- To state Newton's Laws of Motion.
- To review the principles for applying the SI system of units.
- To examine the standard procedures for performing numerical calculations.
- To present a general guide for solving problems.

1.1   MECHANICS

Mechanics can be defined as that branch of the physical sciences concerned with the state of rest or motion of bodies that are subjected to the action of forces. In this book we will study two important branches of mechanics, namely, statics and mechanics of materials. These subjects form a suitable basis for the design and analysis of many types of structural, mechanical, or electrical devices encountered in engineering.

Statics deals with the equilibrium of bodies, that is, it is used to determine the forces acting either external to the body or within it that are necessary to keep the body in equilibrium. *Mechanics of materials* studies the relationships between the external loads and the distribution of internal forces acting within the body. This subject is also concerned with finding the deformations of the body, and it provides a study of the body's stability.

In this book we will first study the principles of statics, since for the design and analysis of any structural or mechanical element it is *first* necessary to determine the forces acting both on and within its various members. Once these internal forces are determined, the size of the members, their deflection, and their stability can then be determined using the fundamentals of mechanics of materials, which will be covered later.

Historical Development.   The subject of statics developed very early in history because its principles can be formulated simply from measurements of geometry and force. For example, the writings of Archimedes (287–212 B.C.) deal with the principle of the lever. Studies of the pulley and inclined plane are also recorded in ancient writings—at times when the requirements for engineering were limited primarily to building construction.

The origin of mechanics of materials dates back to the beginning of the seventeenth century, when Galileo performed experiments to study the effects of loads on rods and beams made of various materials. However, at the beginning of the eighteenth century, experimental methods for testing materials were vastly improved, and at that time many experimental and theoretical studies in this subject were undertaken primarily in France, by such notables as Saint-Venant, Poisson, Lamé, and Navier.

Over the years, after many of the fundamental problems of mechanics of materials had been solved, it became necessary to use advanced mathematical and computer techniques to solve more complex problems. As a result, this subject has expanded into other areas of mechanics, such as the *theory of elasticity* and the *theory of plasticity*. Research in these fields is ongoing, in order to meet the demands for solving more advanced problems in engineering.

1.2   FUNDAMENTAL CONCEPTS

Before we begin our study, it is important to understand the definitions of certain fundamental concepts and principles.

Mass.   *Mass* is a measure of a *quantity of matter* that is used to compare the action of one body with that of another. This property provides a measure of the resistance of matter to a change in velocity.

Force.   In general, *force* is considered as a "push" or "pull" exerted by one body on another. This interaction can occur when there is direct contact between the bodies, such as a person pushing on a wall, or it can occur through a distance when the bodies are physically separated. Examples of the latter type include gravitational, electrical, and magnetic forces. In any case, a force is completely characterized by its magnitude, direction, and point of application.

Newton's Three Laws of Motion. Engineering mechanics is formulated on the basis of Newton's three laws of motion, the validity of which is based on experimental observation. These laws apply to the motion of a particle as measured from a *nonaccelerating* reference frame. They may be briefly stated as follows.

First Law. A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this equilibrium state provided the particle is *not* subjected to an unbalanced force, Fig. 1–1*a*.

Equilibrium

(a)
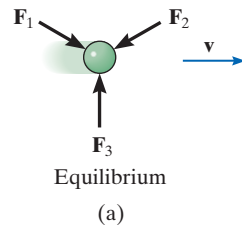
Second Law. A particle acted upon by an *unbalanced force* **F** experiences an acceleration **a** that has the same direction as the force and a magnitude that is directly proportional to the force, Fig. 1–1*b*.* If the particle has a mass m, this law may be expressed mathematically as

$$\mathbf{F} = m\mathbf{a} \tag{1-1}$$

Accelerated motion

(b)
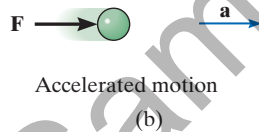
Third Law. The mutual forces of action and reaction between two particles are equal, opposite, and collinear, Fig. 1–1*c*.
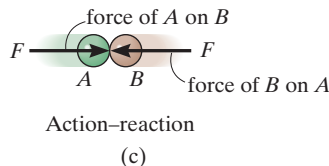

Action–reaction

(c)

Fig. 1–1

*Stated another way, the unbalanced force acting on the particle is proportional to the time rate of change of the particle's linear momentum.

Newton's Law of Gravitational Attraction.   Shortly after formulating his three laws of motion, Newton postulated a law governing the gravitational attraction between any two particles. Stated mathematically,

$$F = G\frac{m_1 m_2}{r^2} \tag{1-2}$$

where

F = force of gravitation between the two particles

G = universal constant of gravitation; according to experimental evidence, $G = 66.73\left(10^{-12}\right) \text{ m}^3/\left(\text{kg} \cdot \text{s}^2\right)$

m_1, m_2 = mass of each of the two particles

r = distance between the two particles

Weight.   According to Eq. 1–2, any two particles or bodies have a mutual attractive (gravitational) force acting between them. In the case of a particle located at or near the surface of the earth, however, the only gravitational force having any sizable magnitude is that between the earth, because of its very large mass, and the particle. Consequently, this force, called the *weight*, will be the only gravitational force we will consider.

From Eq. 1–2, if the particle has a mass $m_1 = m$, and we assume the earth is a nonrotating sphere of constant density and having a mass $m_2 = M_e$, then if r is the distance between the earth's center and the particle, the weight W of the particle becomes

$$W = G\frac{mM_e}{r^2}$$

If we let $g = GM_e/r^2$, we have

$$\boxed{W = mg} \tag{1-3}$$

The astronaut's weight is diminished since she is far removed from the gravitational field of the earth.

If we allow the particle to fall downward, then neglecting air resistance, the only force acting on the particle is its weight, and so Eq. 1–1 becomes $W = ma$. Comparing this result with Eq. 1–3, we see that $a = g$. In other words, g is the acceleration due to gravity. Since it depends on r, then the weight of the particle or body is *not* an absolute quantity. Instead, its magnitude depends upon the elevation where the measurement was made. For most engineering calculations, however, g is determined at sea level and at a latitude of 45°, which is considered the "standard location."

Idealizations. Models or idealizations are used in mechanics in order to simplify application of the theory. Here we will consider three important idealizations.

Particle. A *particle* has a mass, but a size that can be neglected. For example, the size of the earth is insignificant compared to the size of its orbit, and therefore the earth can be modeled as a particle when studying its orbital motion. When a body is idealized as a particle, the principles of mechanics reduce to a rather simplified form since the geometry of the body *will not be involved* in the analysis of the problem.

Rigid Body. A *rigid body* can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying a load. This model is important because the body's shape does not change when a load is applied, and so we do not have to consider the type of material from which the body is made. In most cases the actual deformations occurring in structures, machines, mechanisms, and the like are relatively small, and the rigid-body assumption is suitable for analysis.

Concentrated Force. A *concentrated force* represents the effect of a loading which is assumed to act at a point on a body. We can represent a load by a concentrated force, provided the area over which the load is applied is very small compared to the overall size of the body. An example would be the contact force between a wheel and the ground.

Three forces act on the ring. Since these forces all meet at a point, then for any force analysis, we can assume the ring to be represented as a particle.

Steel is a common engineering material that does not deform very much under load. Therefore, we can consider this railroad wheel to be a rigid body acted upon by the concentrated force of the rail.

Refer to the companion website for Lecture Summary and Quiz videos.

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TABLE 1–1 SI System of Units				
Name	Length	Time	Mass	Force
International System of Units	meter	second	kilogram	newton*
SI	m	s	kg	N $\left(\dfrac{\text{kg·m}}{\text{s}^2}\right)$

*Derived unit.

1.3   THE INTERNATIONAL SYSTEM OF UNITS

The four basic quantities—length, time, mass, and force—are not all independent from one another; in fact, they are *related* by Newton's second law of motion, $\mathbf{F} = m\mathbf{a}$. Because of this, the *units* used to measure these quantities cannot *all* be selected arbitrarily. The equality $\mathbf{F} = m\mathbf{a}$ is maintained only if three of the four units, called **base units**, are *defined* and the fourth unit is then *derived* from the equation.

For the International System of Units, abbreviated SI after the French "Système International d'Unités," length is in meters (m), time is in seconds (s), and mass is in kilograms (kg), Table 1–1. The unit of force, called a **newton** (N), is *derived* from $\mathbf{F} = m\mathbf{a}$. Thus, 1 newton is equal to a force required to give 1 kilogram of mass an acceleration of $1\,\text{m/s}^2 \left(\text{N} = \text{kg·m/s}^2\right)$.

If the weight of a body located at the "standard location" is to be determined in newtons, then Eq. 1–3 must be applied. Here measurements give $g = 9.806\,65\,\text{m/s}^2$; however, for calculations the value $g = 9.81\,\text{m/s}^2$ will be used. Thus,

$$W = mg \quad (g = 9.81\,\text{m/s}^2) \tag{1–4}$$

Therefore, a body of mass 1 kg has a weight of 9.81 N, a 2-kg body weighs 19.62 N, and so on, Fig. 1–1. Perhaps it is easier to remember that a small apple weighs one newton.
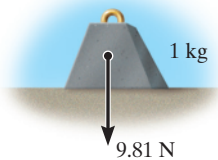
Prefixes.   When a numerical quantity is either very large or very small, the units used to define its size may be modified by using a prefix. Some of the prefixes used in the SI system are shown in Table 1–2. Each represents a multiple or submultiple of a unit which, if applied successively, moves

1 kg

9.81 N

Fig. 1–1

the decimal point of a numerical quantity to every third place.* For example, $4\,000\,000$ N $= 4\,000$ kN (kilo-newton) $= 4$ MN (mega-newton), or 0.005 m $= 5$ mm (milli-meter). Notice that the SI system does not include the multiple deca (10) or the submultiple centi (0.01), which form part of the metric system. Except for some volume and area measurements, the use of these prefixes is generally avoided in science and engineering.

TABLE 1–2   Prefixes

	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

Rules for Use.   Here are a few of the important rules that describe the proper use of the various SI symbols:

- Quantities defined by several units which are multiples of one another are separated by a *dot* to avoid confusion with prefix notation, as indicated by N $=$ kg$\cdot$m/s^2 $=$ kg$\cdot$m$\cdot$s^{-2}. Also, m$\cdot$s (meter-second), whereas ms (milli-second).

- The exponential power on a unit having a prefix refers to both the unit *and* its prefix. For example, μN^2 $=$ $(\mu$N$)^2$ $=$ μN$\cdot\mu$N. Likewise, mm^2 represents $($mm$)^2$ $=$ mm$\cdot$mm.

- With the exception of the base unit the kilogram, in general avoid the use of a prefix in the denominator of composite units. For example, do not write N/mm, but rather kN/m; also, m/mg should be written as Mm/kg.

- When performing calculations, represent the numbers in terms of their *base or derived units* by converting all prefixes to powers of 10. The final result should then be expressed using a *single prefix*. Also, after calculation, it is best to keep numerical values between 0.1 and 1000; otherwise, a suitable prefix should be chosen. For example,

$$(50\,\text{kN})(60\,\text{mm}) = \left[\,50(10^3)\ \text{N}\,\right]\left[\,60(10^{-9})\text{m}\,\right]$$
$$= 3000(10^{-6})\ \text{N}\cdot\text{m} = 3(10^{-3})\ \text{N}\cdot\text{m} = 3\ \text{mN}\cdot\text{m}$$

*The kilogram is the only base unit that is defined with a prefix.

1.4   NUMERICAL CALCULATIONS

Numerical work in engineering practice is most often performed by using handheld calculators and computers. It is important, however, that the answers to any problem be reported with justifiable accuracy using appropriate significant figures. In this section we will discuss these topics together with some other important aspects involved in all engineering calculations.

Dimensional Homogeneity.   The terms of any equation used to describe a physical process must be *dimensionally homogeneous*; that is, each term must be expressed in the same units. Provided this is the case, all the terms of an equation can then be combined if numerical values are substituted for the variables. Consider, for example, the equation $s = vt + \frac{1}{2}at^2$, where, in SI units, s is the position in meters, m, t is time in seconds, s, v is velocity in m/s, and a is acceleration in m/s^2. Regardless of how this equation is evaluated, it maintains its dimensional homogeneity. In the form stated, each of the three terms is expressed in meters $\left[\text{m}, (\text{m}/s)s, (\text{m}/s^2)s^2 \right]$ or solving for a, $a = 2s/t^2 - 2v/t$, the terms are each expressed in units of $\text{m/s}^2 \left[\text{m/s}^2, \ \text{m/s}^2, \ (\text{m/s})/\text{s} \right]$.

Keep in mind that problems in mechanics always involve the solution of dimensionally homogeneous equations, and so this fact can then be used as a partial check for algebraic manipulations of an equation.

Significant Figures.   The number of significant figures contained in any number determines the accuracy of the number. For instance, the number 4981 contains four significant figures. However, if zeros occur at the end of a whole number, it may be unclear as to how many significant figures the number represents. For example, 23 400 might have three (234), four (2340), or five (23 400) significant figures. To avoid these ambiguities, we will use *engineering notation* to report a result. This requires that numbers be rounded off to the appropriate number of significant digits and then expressed in multiples of (10^3), such as (10^3), (10^6), or (10^{-9}). For instance, if 23 400 has five significant figures, it is written as $23.400(10^3)$, but if it has only three significant figures, it is written as $23.4(10^3)$.

If zeros occur at the beginning of a number that is less than one, then the zeros are not significant. For example, 0.008 21 has three significant figures. Using engineering notation, this number is expressed as $8.21(10^{-3})$. Likewise, 0.000 582 can be expressed as $0.582(10^{-3})$ or $582(10^{-6})$.

Rounding Off Numbers.   Rounding off a number is necessary so that the accuracy of the result will be the same as that of the problem data. As a general rule, any numerical figure ending in a number greater than five is rounded up and a number less than five is not rounded up. The rules for rounding off numbers are best illustrated by example. Suppose the number 3.5587 is to be rounded off to *three* significant figures. Because the fourth digit (8) is *greater than* 5, the third number is rounded up to 3.56. Likewise 0.5896 becomes 0.590 and 9.3866 becomes 9.39. If we round off 1.341 to three significant figures, because the fourth digit (1) is *less than* 5, then we get 1.34. Likewise 0.3762 becomes 0.376 and 9.871 becomes 9.87. There is a special case for any number that ends in a 5. As a general rule, if the digit preceding the 5 is an *even number*, then this digit is *not* rounded up. If the digit preceding the 5 is an *odd number*, then it is rounded up. For example, 75.25 rounded off to three significant digits becomes 75.2, 0.1275 becomes 0.128, and 0.2555 becomes 0.256.

Calculations.   When a sequence of calculations is performed, it is best to store the intermediate results in the calculator. In other words, do not round off calculations until expressing the final result. This procedure maintains precision throughout the series of steps to the final solution. In this book we will generally round off the answers to *three significant figures* since most of the data in engineering mechanics, such as geometry and loads, may be reliably measured to this accuracy.

When solving problems, do the work as neatly as possible. Being neat will stimulate clear and orderly thinking, and vice versa.

1.5   GENERAL PROCEDURE FOR ANALYSIS

Attending a lecture, reading this book, and studying the example problems helps, but **the most effective way of learning the principles of engineering mechanics is to *solve problems***. To be successful at this, it is important to always present the work in a *logical* and *orderly manner*, as suggested by the following sequence of steps:

- Read the problem carefully and try to correlate the actual physical situation with the theory studied.
- Tabulate the problem data and *draw to a large scale* any necessary diagrams.
- Apply the relevant principles, generally in mathematical form. When writing any equations, be sure they are dimensionally homogeneous.
- Solve the necessary equations, and report the answer with no more than three significant figures.
- Study the answer with technical judgment and common sense to determine whether or not it seems reasonable.

IMPORTANT POINTS

- Statics is the study of bodies that are at rest or move with constant velocity.
- A particle has a mass but a size that can be neglected, and a rigid body does not deform under load.
- A force is considered as a "push" or "pull" of one body on another.
- Concentrated forces are assumed to act at a point on a body.
- Newton's three laws of motion should be memorized.
- Mass is measure of a quantity of matter that does not change from one location to another. Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located.
- In the SI system the unit of force, the newton, is a derived unit. The meter, second, and kilogram are base units.
- Prefixes G, M, k, m, μ, and n are used to represent large and small numerical quantities. Their exponential size should be known, along with the rules for using the SI units.
- Perform numerical calculations with several significant figures, and then report the final answer to three significant figures.
- Algebraic manipulations of an equation can be checked in part by verifying that the equation remains dimensionally homogeneous.
- Know the rules for rounding off numbers.

Lecture videos that cover the material in this and most of the other sections of the book can be obtained at www.pearsonglobaleditions.com.

EXAMPLE   1.1

Convert 100 km/h to m/s and 24 m/s to km/h.

SOLUTION

Since 1 km = 1000 m and 1 h = 3600 s, the factors of conversion are arranged in the following order, so that a cancellation of the units can be applied:

$$100\,\text{km/h} = \frac{100\,\cancel{\text{km}}}{\cancel{\text{h}}}\left(\frac{1000\,\text{m}}{\cancel{\text{km}}}\right)\left(\frac{1\,\cancel{\text{h}}}{3600\,\text{s}}\right)$$

$$= \frac{100(10^3)\,\text{m}}{3600\,\text{s}} = 27.8\,\text{m/s} \qquad \textit{Ans.}$$

$$24\,\text{m/s} = \left(\frac{24\,\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\frac{1\,\text{km}}{1000\,\cancel{\text{m}}}\right)\left(\frac{3600\,\cancel{\text{s}}}{1\,\text{h}}\right)$$

$$= \frac{86.4(10^3)\,\text{km}}{1000\,\text{h}} = 86.4\,\text{km/h} \qquad \textit{Ans.}$$

NOTE: Remember to round off the final answer to three significant figures.

EXAMPLE   1.2

Convert the density of steel $7.85\,\text{g/cm}^3$ to kg/m^3.

SOLUTION

Using 1 kg = 1000 g and 1 m = 100 cm, and arrange the conversion factor in such a way that g and cm^3 can be canceled out.

$$7.85\,\text{g/cm}^3 = \left(\frac{7.85\,\text{g}}{\text{cm}^3}\right)\left(\frac{1\,\text{kg}}{1000\,\text{g}}\right)\left(\frac{100\,\text{cm}}{1\,\text{m}}\right)^3$$

$$= \left(\frac{7.85\,\cancel{\text{g}}}{\cancel{\text{cm}^3}}\right)\left(\frac{1\,\text{kg}}{1000\,\cancel{\text{g}}}\right)\left(\frac{100^3\,\cancel{\text{cm}^3}}{1\,\text{m}^3}\right)$$

$$= 7.85(10^3)\,\text{kg/m}^3 \qquad \textit{Ans.}$$