

Annotated Teacher's Edition

Calculus

AP[®] Edition

Second Edition

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*For Julie, Susan, Sally, Sue,
Katie, Jeremy, Elise, Mary, Claire, Katie, Chris, and Annie
whose support, patience, and encouragement made this book possible.*

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Preface

This book was specifically designed and written for teachers and students of Advanced Placement[®] Calculus. Our approach is based on many years of teaching calculus at diverse institutions using the best teaching practices we know. We collaborated with over 90 academic experts and classroom practitioners to develop the correct blend of rigorous content, accessible support, flexible resources, and engaging instruction to help students master the essential mathematical and communication skills they need to achieve their AP goals. For the second edition, we welcome T. Michael Brown of Prairie School (WI) and Michael Masterson of Benjamin Franklin High School (LA). Their many years of teaching AP calculus are reflected in this edition, particularly in the new pedagogical features.

Although the book was written with the most recent AP curriculum framework in mind, it also contains several topics that do not appear in the curriculum framework, but are often covered in the first two semesters of a standard college-level calculus sequence. For this reason, the book not only provides preparation for the AP Exam, but also serves as a foundation for additional mathematical course work at the college level.

Throughout this book, a concise and lively narrative motivates the ideas of calculus. All topics are introduced through concrete examples, applications, and analogies rather than through abstract arguments. We appeal to students' intuition and geometric instincts to make calculus natural and believable. Once this intuitive foundation is established, generalizations and abstractions follow.

New To This Edition

- The 2nd Edition is fully aligned to the 2016–2017 AP Calculus Curriculum Framework.
 - *AP Practice Questions* at the end of chapters have been updated to reflect the revised content and format of the AP Calculus Exams.
 - Content was updated in various sections to reflect content changes called for in the 2016–2017 Curriculum Framework.
 - The *AP Calculus Pacing Guide* (p. T-9) and *The Concept Outline* (p. T-1) have been fully updated to meet the new Curriculum Framework.
 - The addition of the Mathematical Practices for AP Calculus (MPACs) to the course description for AP Calculus AB and BC is arguably the biggest change in the new course curriculum from College Board. To help teachers address this important addition, we have included a *Guide to the Mathematical Practices for AP Calculus* in this text (p. T-5).
- The Suggested Assignments that appear prior to each set of section exercises in the Teacher's Edition now contain extension exercise recommendations (given in **blue boldface type**) to help teachers differentiate for more advanced students.

Also new to this edition:
The AP Calculus Learning Objectives (see p. T-1) and suggested pacing (see p. T-9) appear within each section of the book, right next to the section title.

LO 3.4A, 3.4C,
3.4E
Days: AB 4, BC 3

- A new feature called **AP[®] EXAM INSIGHTS** appears within the AP Practice Questions at the end of each chapter. The feature appears adjacent to the practice questions to which it relates and is designed to help students better prepare for the content and format of the AP Exams.
- The Pearson Education Test Prep Series for AP[®] Calculus Workbook has been revised to include more material to help students prepare for the AP Calculus Exams:
 - Introduction to the AB and BC Calculus Exams
 - Precalculus Review of Calculus Prerequisites
 - Review of Calculus AB and Calculus BC Topics
 - Practice Exams
 - Answers and Solutions
- The MyMathLab for School and MathXL for School online courses have been updated as follows:
 - Many more assignable algorithmic exercises have been added to provide greater flexibility in crafting assignments.
 - New Setup & Solve exercises require students to show how to set up a solution as well as their answer. This better mirrors what students do by hand and are required to do on tests.
 - More instructional videos are included to help students who need additional instruction or who miss a particular class.
 - A Guide to Video-Based Assignments enables teachers to create assignments that include video instruction followed by assessment exercises.

Pedagogical Features

Exercises

The exercises at the end of each section are one of the strongest features of the text. They are graded, varied, and original. In addition, they are labeled and carefully organized into groups.

- Each exercise set begins with *Review Questions* that check students' conceptual understanding of the essential ideas from the section.
- *Basic Skills* exercises are confidence-building problems that provide a solid foundation for the more challenging exercises to follow. Each example in the narrative is linked directly to a block of *Basic Skills* exercises via *Related Exercises* references at the end of the example solution.
- *Further Explorations* exercises expand on the *Basic Skills* exercises by challenging students to think creatively and to generalize newly acquired skills.
- *Applications* exercises connect skills developed in previous exercises to applications and modeling problems that demonstrate the power and utility of calculus.
- *Additional Exercises* are generally the most difficult and challenging problems; they include proofs of results cited in the narrative.
- *Technology Exercises* provide additional exercises that emphasize the use of graphing calculators or mathematical software.

Each chapter concludes with a comprehensive set of *Review Exercises* and a set of *AP Practice Questions*. Although these practice questions are not taken from actual AP Exams, they are designed to help students prepare for the AP Exam. Interspersed among

the AP practice questions is a new feature called **AP Exam Insights** that provides students with tips on exam taking and preparation.

Figures

Given the power of graphics software and the ease with which many students assimilate visual images, we devoted considerable time and deliberation to the figures in this book. Whenever possible, we let the figures communicate essential ideas using annotations reminiscent of a teacher's voice at the board. Readers will quickly find that the figures facilitate learning in new ways.

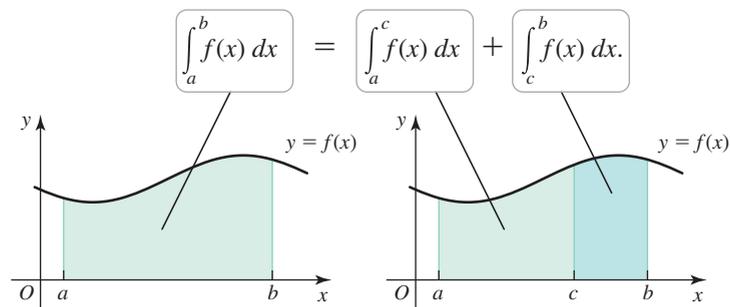


FIGURE 5.35

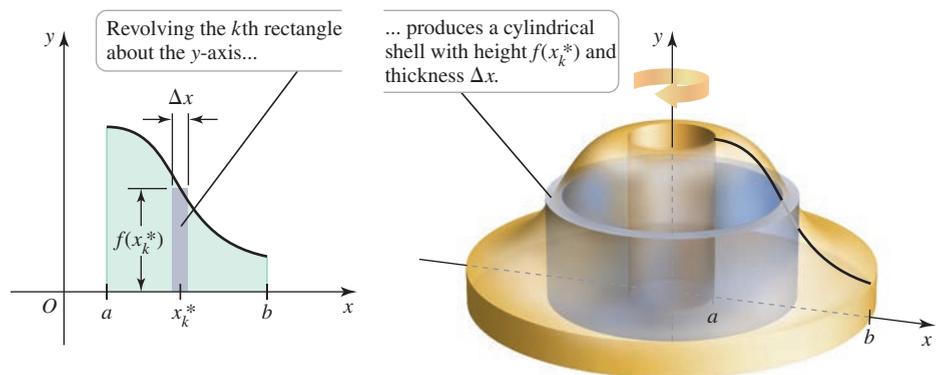


FIGURE 6.41

Quick Check and Margin Notes

The narrative is interspersed with *Quick Check* questions that encourage students to read with pencil in hand. These questions resemble the kinds of questions teachers pose in class. Answers to the *Quick Check* questions are found at the end of the section in which they occur. *Margin Notes* offer reminders, provide insight, and clarify technical points.

Guided Projects

The *Teacher's Resource Guide* contains 59 *Guided Projects*. These projects allow students to work in a directed, step-by-step fashion, with various objectives: to carry out extended calculations, to derive physical models, to explore related theoretical topics, or to investigate new applications of calculus. The *Guided Projects* vividly demonstrate the breadth of calculus and provide a wealth of mathematical excursions that go beyond the typical classroom experience. A list of suggested *Guided Projects* is included at the end of each chapter.

Technology

We believe that a calculus text should help students strengthen their analytical skills and demonstrate how technology can extend (not replace) those skills. The exercises and examples in this text emphasize this balance. Graphing calculators are additional tools in the kit, and students must learn when and when not to use them. Our goal is to support students as they prepare for the graphing calculator portion of the AP Exam.

Throughout the book, exercises marked with  indicate that a graphing calculator is required.

eBook with Interactive Figures (optional, available for purchase)

The textbook is supported by a groundbreaking and award-winning electronic book, created by Eric Schulz of Walla Walla Community College. This “live book” contains the complete text of the print book in addition to interactive versions of approximately 400 figures. Teachers can use these interactive figures in the classroom to illustrate the important ideas of calculus, and students can explore them while they are reading the textbook. Our experience confirms that the interactive figures help build students’ geometric intuition of calculus. The authors have written Interactive Figure Exercises that can be assigned via MyMathLab for School so that students can study the figures outside of class in a directed way. Available only within MyMathLab for School, the eBook provides teachers with powerful new teaching tools that expand and enrich the learning experience for students.

Supplements and Technology

Annotated Teacher’s Edition

- *Teacher Notes* on the page give an overview of the material being taught, helpful tips, and warnings, as well as suggested assignments.
- Answers are included on the same page as the problem for most exercises. All answers are included in the back of the book.

Pearson Education Test Prep Series for AP[®] Calculus (Workbook ISBN 0134705556)

- Introduction to the AB and BC Calculus Exams
- Precalculus Review of Calculus Prerequisites
- Review of Calculus AB and Calculus BC Topics
- Practice Exams
- Answers and Solutions

Graphing Calculator Manual (download only)

- An introduction to Texas Instruments graphing calculators, as they are used for calculus
- Discussion of the TI-84 Plus Silver Edition featuring MathPrint, the TI-83 Plus Silver Edition, and the TI-89 Titanium.

How To Access Electronic Teacher Materials

Many of the teacher supplements are available electronically, at no charge, to qualified adopters. These supplements may be obtained (a) within MyMathLab for School, or (b) in the Instructor Resource Center (IRC). To obtain access to the IRC, you must first register and set up a user name and password. To register, visit PearsonSchool.com/access_request and choose **Instructor Resource Center**. You will be required to complete a brief, one-time registration subject to verification of educator status. Upon verification, access information and instructions will be sent via email. After you receive your confirmation, go to pearsonhighered.com and enter **ISBN 013472576X** in the search box, then select the **Resources** tab to choose available online resources.

Teacher's Resource Guide (download only)

- Fifty-nine *Guided Projects*, correlated to specific chapters of the text, can be assigned to students for individual or group work. The *Guided Projects* vividly demonstrate the breadth of calculus and provide a wealth of mathematical excursions that go beyond the typical classroom experience.
- An annotated Table of Contents gives teacher's insight into overall content and organization of the text.
- *Learning Objectives Lists* and an *Index of Applications* are tools to help teachers gear the text to their course goals and students' interests.

Teacher's Solutions Manual (download only)

The *Teacher Solutions Manual* contains complete solutions to all the exercises in the text.

TestGen[®] (downloadable)

TestGen[®] enables teachers to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing teachers to create multiple but equivalent versions of the same question or test with the click of a button. Teachers can also modify test bank questions or add new questions.

PowerPoint[®] Lecture Slides (downloadable)

The PowerPoint slides contain key concepts, definitions, figures, and tables from the textbook.

Video Resources (online only)

The Video Lectures with optional captioning feature an engaging team of mathematics teachers who present comprehensive coverage of topics in the text. The lecturers' presentations include illustrative examples and exercises and support an approach that emphasizes visualization and problem solving. Available only through MyMathLab for School and MathXL for School.

MyMathLab[®] for School Online Course—www.mymathlabforschool.com (optional, access code required, for purchase only)

Built around Pearson's best-selling content, MyMathLab for School is an online homework, tutorial, and assessment program designed to work with this text to engage students and improve results. *Used by more than 37 million students worldwide, MyMathLab delivers consistent, measurable gains in student learning outcomes, retention, and subsequent course success.* Visit www.mymathlab.com/results to learn more.

PREPAREDNESS

One of the biggest challenges in calculus courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their course work. MyMathLab supports students with just-in-time remediation and key-concept review.

EXERCISES

Students are motivated to succeed when they're engaged in the learning experience and understand the relevance and power of mathematics. MyMathLab's online homework

offers students immediate feedback and tutoring, which means they retain more knowledge and are able to do more, which means they retain more knowledge and are able to do more, which means they retain more knowledge and are able to do more,

Should this be eBook? It should be consistent throughout the preface.

- **Exercises with immediate feedback**—the hundreds of assignable exercises for this text regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyMathLab provides helpful feedback when students enter incorrect answers and includes optional learning aids including Help Me Solve This, View an Example, videos, and the eText.
- **Setup & Solve Exercises** require students to show how to set up a solution as well as the answer. This better mirrors what students do by hand and are required to do on tests. Here is an example:

4.4 Optimization Problems Close

Objective: Solve optimization pr... 21 of 42 (0 complete) 0 correct

4.4.27-Setup & Solve Question Help

A cone is constructed by cutting a sector from a circular sheet of metal with radius 14. The cut sheet is then folded up and welded (see figure). Find the radius and height of the cone with maximum volume that can be formed in this way.

Write the objective function relating V and h in a form that does not include r .

$V = \frac{196}{3}\pi h - \frac{1}{3}\pi h^3$

(Type an expression. Type an exact answer, using π as needed.)

The interval of interest of the objective function is $[0, 14]$.

(Simplify your answer. Type your answer in interval notation.)

The radius is $r = \square$ and the height is $h = \square$.

(Type exact answers, using radicals as needed.)

Enter your answer in the edit fields and then click Check Answer.

All parts showing Clear All Check Answer

- **Additional Conceptual Questions** focus on deeper, theoretical understanding of the key concepts in calculus. These questions are also assignable through Learning Catalytics. Here is an example:

When is the statement "Whether or not $\lim_{x \rightarrow a} f(x)$ exists, depends on how $f(a)$ is defined" true?

Choose the correct answer below.

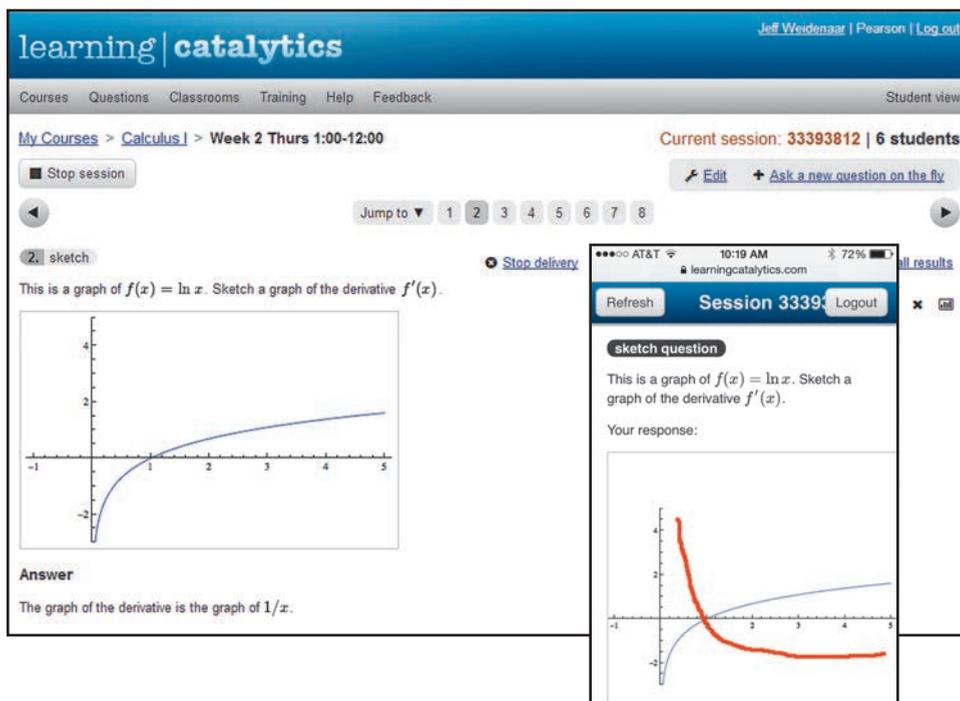
sometimes

always

never

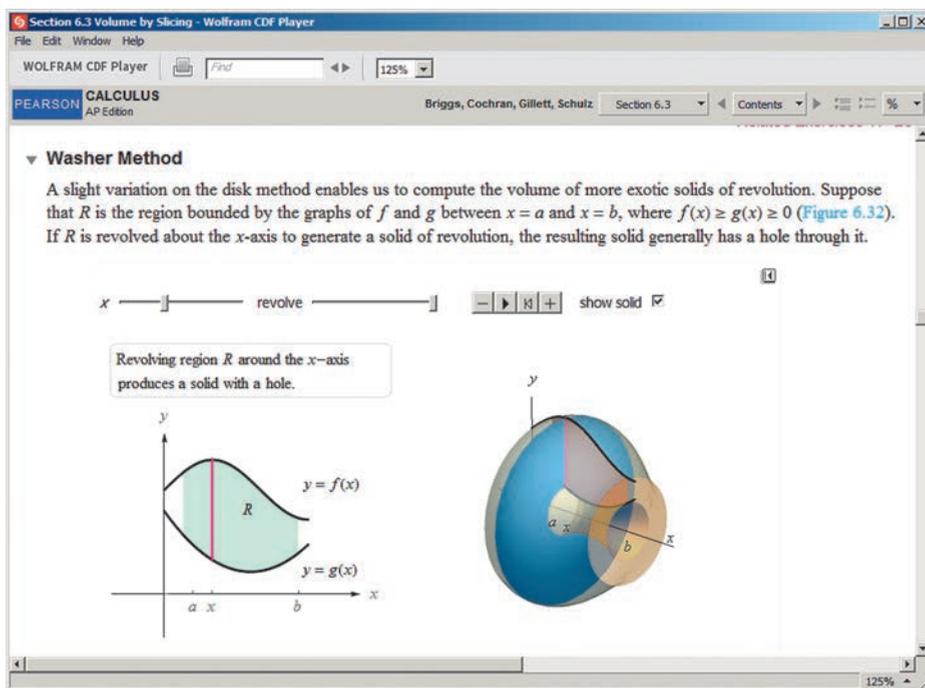
- **Learning Catalytics™** is a student response tool that uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking during lecture. Learning Catalytics fosters student engagement and peer-to-peer learning with

real-time analytics. Learning Catalytics is available to all MyMathLab users. Here is an example:

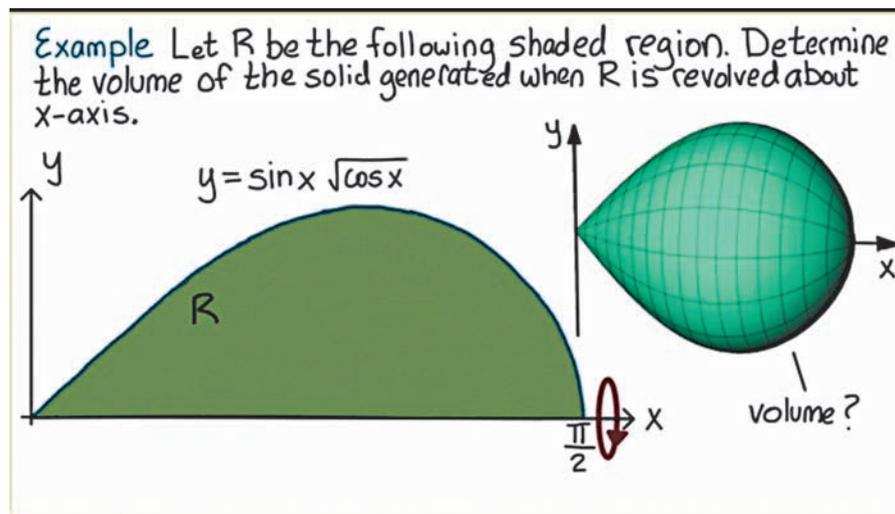


LEARNING AND TEACHING TOOLS

- The **eBook with Interactive Figures** features over 400 figures that can be manipulated to illuminate difficult-to-convey concepts. Teachers can use these interactive figures in the classroom, and students can manipulate the figures while they are using MyMathLab. The figures have proven to be a valued tool for helping to build geometric intuition of calculus.



- **Instructional videos**—hundreds of videos are available as learning aids within exercises and for self-study. The Guide to Video-Based Assignments makes it easy to assign videos for homework by showing which MyMathLab exercises correspond to each video.



- The **Graphing Calculator Manual** provides detailed guidance for integrating the Texas Instruments graphing calculators throughout the course, including syntax and commands.
- **PowerPoint Presentations** are available for download that cover each section of the book.
- **Accessibility** and achievement go hand in hand. MyMathLab is compatible with the JAWS screen reader, and enables multiple-choice and free-response problem types to be read and interacted with via keyboard controls and math notation input. MyMathLab also works with screen enlargers, including ZoomText, MAGic, and SuperNova. And, all MyMathLab videos have closed-captioning. More information is available at <http://mymathlab.com/accessibility>.
- **A comprehensive gradebook** with enhanced reporting functionality allows you to efficiently manage your course.
 - **The Reporting Dashboard** provides insight to view, analyze, and report learning outcomes. Student performance data is presented at the class, section, and program levels in an accessible, visual manner so you'll have the information you need to keep your students on track.
 - **Item Analysis** tracks class-wide understanding of particular exercises so you can refine your class lectures or adjust the course/department syllabus. Just-in-time teaching has never been easier!

Whether you are just getting started with MyMathLab, or have a question along the way, Pearson is here to help you learn about its technologies and how to incorporate them into your course. To learn more about how MyMathLab helps students succeed, visit www.mymathlabforschool.com or contact your Pearson School Account Executive.

MathXL for School—www.mathxlforschool.com (optional, for purchase only, access code required)

MathXL for School is a powerful online homework, tutorial, and assessment system that aligns to Pearson's textbooks in mathematics and statistics. MathXL for school is the homework and assessment engine within MyMathLab for School. It contains the online homework available within MyMathLab, but not other resources such as the eBook. For more information or to purchase student access codes after the first year, visit our website at www.mathxlforschool.com or contact your Pearson School Account Executive.

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The Concept Outline

This table lists the learning objectives in the concept outline from the new College Board *AP Calculus AB and AP Calculus BC Course and Exam Description* and matches them with the sections in the second edition of *Calculus: AP Edition*. Subject matter that is included only in the BC course is indicated with **blue shading**.

Big Idea 1: Limits

| Enduring Understandings (Students will understand that . . .) | Learning Objectives (Students will be able to . . .) | Essential Knowledge (Students will know that . . .) | <i>Calculus AP Edition 2e</i> |
|--|--|---|-------------------------------|
| EU 1.1: The concept of a limit can be used to understand the behavior of functions. | LO 1.1A(a) LO 1.1A(b) | EK 1.1A1 | Sect. 2.1–2.2 |
| | | EK 1.1A2 | Sect. 2.2, 2.4–2.5 |
| | | EK 1.1A3 | Sect. 2.2, 2.5 |
| | LO 1.1B | EK 1.1B1 | Sect. 2.1–2.2 |
| | LO 1.1C | EK 1.1C1 | Sect. 2.3 |
| | | EK 1.1C2 | Sect. 2.3 |
| | | EK 1.1C3 | Sect. 4.7 |
| | LO 1.1D | EK 1.1D1 | Sect. 2.5 |
| EK 1.1D2 | | Sect. 4.7 | |
| EU 1.2: Continuity is a key property of functions that is defined using limits. | LO 1.2A | EK 1.2A1 | Sect. 2.6 |
| | | EK 1.2A2 | Sect. 2.6 |
| | | EK 1.2A3 | Sect. 2.6 |
| | LO 1.2B | EK 1.2B1 | Sect. 2.6, 4.1, 4.6 |

Big Idea 2: Derivatives

| Enduring Understandings (Students will understand that . . .) | Learning Objectives (Students will be able to . . .) | Essential Knowledge (Students will know that . . .) | <i>Calculus AP Edition 2e</i> |
|---|--|---|-------------------------------|
| EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies. | LO 2.1A | EK 2.1A1 | Sect. 2.1, 3.1 |
| | | EK 2.1A2 | Sect. 3.1 |
| | | EK 2.1A3 | Sect. 3.1 |
| | | EK 2.1A4 | Sect. 3.1 |
| | | EK 2.1A5 | Sect. 3.1–3.2 |

| Enduring Understandings (Students will understand that . . .) | Learning Objectives (Students will be able to . . .) | Essential Knowledge (Students will know that . . .) | <i>Calculus</i> <i>AP Edition 2e</i> |
|---|--|---|---|
| EU 2.1: <i>The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.</i> | LO 2.1B | EK 2.1B1 | Sect. 3.1 |
| | LO 2.1C | EK 2.1C1 | Sect. 3.3, 3.5, 3.9 |
| | | EK 2.1C2 | Sect. 3.3, 3.5, 3.9–3.10 |
| | | EK 2.1C3 | Sect. 3.3–3.4 |
| | | EK 2.1C4 | Sect. 3.7 |
| | | EK 2.1C5 | Sect. 3.8 |
| | | EK 2.1C6 | Sect. 3.10 |
| | | EK 2.1C7 (BC) | Sect. 11.6 |
| | LO 2.1D | EK 2.1D1 | Sect. 3.3 |
| | | EK 2.1D2 | Sect. 3.3 |
| EU 2.2: A function’s derivative, which is itself a function, can be used to understand the behavior of the function. | LO 2.2A | EK 2.2A1 | Sect. 4.1–4.3 |
| | | EK 2.2A2 | Sect. 4.1–4.3 |
| | | EK 2.2A3 | Sect. 4.1–4.3 |
| | | EK 2.2A4 (BC) | Sect. 11.4 |
| | LO 2.2B | EK 2.2B1 | Sect. 3.2 |
| EK 2.2B2 | | Sect. 3.2 | |
| EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rate of change. | LO 2.3A | EK 2.3A1 | Sect. 3.6 |
| | | EK 2.3A2 | Sect. 3.6 |
| | LO 2.3B | EK 2.3B1 | Sect. 3.1 |
| | | EK 2.3B2 | Sect. 3.1, 4.5 |
| | LO 2.3C | EK 2.3C1 | Sect. 3.6 |
| | | EK 2.3C2 | Sect. 3.11 |
| | | EK 2.3C3 | Sect. 4.4 |
| | | EK 2.3C4 (BC) | Sect. 11.7 |
| | LO 2.3D | EK 2.3D1 | Sect. 3.6, 3.11 |
| | LO 2.3E | EK 2.3E1 | Sect. 8.1 |
| | | EK 2.3E2 | Sect. 8.1–8.2 |
| | LO 2.3F | EK 2.3F1 | Sect. 8.1–8.2 |
| | | EK 2.3F2 (BC) | Sect. 8.2 |
| EU 2.4: The Mean Value Theorem connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval. | LO 2.4A | EK 2.4A1 | Sect. 4.6 |

Big Idea 3: Integrals and the Fundamental Theorem of Calculus

| Enduring Understandings (Students will understand that . . .) | Learning Objectives (Students will be able to . . .) | Essential Knowledge (Students will know that . . .) | <i>Calculus AP Edition 2e</i> |
|---|--|---|-------------------------------|
| EU 3.1: Antidifferentiation is the inverse process of differentiation. | LO 3.1A | EK 3.1A1 | Sect. 5.1 |
| | | EK 3.1A2 | Sect. 5.1 |
| EU 3.2: The definite integral of a function over an interval is the limit of a Riemann Sum over that interval and can be calculated using a variety of strategies. | LO 3.2A(a) LO 3.2A(b) | EK 3.2A1 | Sect. 5.2 |
| | | EK 3.2A2 | Sect. 5.2–5.3 |
| | | EK 3.2A3 | Sect. 5.2–5.3 |
| | LO 3.2B | EK 3.2B1 | Sect. 5.2–5.3 |
| | | EK 3.2B2 | Sect. 5.2–5.3 |
| | LO 3.2C | EK 3.2C1 | Sect. 5.3 |
| | | EK 3.2C2 | Sect. 5.3 |
| | | EK 3.2C3 | Sect. 5.3 |
| | LO 3.2D (BC) | EK 3.2D1 (BC) | Sect. 7.4 |
| | | EK 3.2D2 (BC) | Sect. 7.4 |
| EU 3.3: The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration. | LO 3.3A | EK 3.3A1 | Sect. 5.4 |
| | | EK 3.3A2 | Sect. 5.4 |
| | | EK 3.3A3 | Sect. 5.4 |
| | LO 3.3B(a) LO 3.3B(b) | EK 3.3B1 | Sect. 5.4 |
| | | EK 3.3B2 | Sect. 5.4 |
| | | EK 3.3B3 | Sect. 5.1, 5.4 |
| | | EK 3.3B4 | Sect. 5.7 |
| | | EK 3.3B5 | Sect. 5.6, 7.1 |
| EK 3.3B5 (BC) | Sect. 7.1–7.3 | | |
| EU 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation. | LO 3.4A | EK 3.4A1 | Sect. 6.1, 6.6 |
| | | EK 3.4A2 | Sect. 6.1, 6.6 |
| | | EK 3.4A3 | Sect. 5.3 |
| | LO 3.4B | EK 3.4B1 | Sect. 5.5 |
| | LO 3.4C | EK 3.4C1 | Sect. 6.1 |
| | | EK 3.4C2 (BC) | Sect. 11.7 |
| | LO 3.4D | EK 3.4D1 | Sect. 5.5, 6.2 |
| | | EK 3.4D1 (BC) | Sect. 11.4 |
| | | EK 3.4D2 | Sect. 6.3 |
| | | EK 3.4D3 (BC) | Sect. 6.5, 11.2, 11.4 |
| LO 3.4E | EK 3.4E1 | Sect. 6.1, 6.6 | |

Shade these two cells blue.

| Enduring Understandings (Students will understand that . . .) | Learning Objectives (Students will be able to . . .) | Essential Knowledge (Students will know that . . .) | <i>Calculus AP Edition 2e</i> |
|---|--|---|-------------------------------|
| EU 3.5: Antidifferentiation is an underlying concept in solving separable differential equations. Solving separable differentiable equations involves determining a function or relation given its rate of change. | LO 3.5A | EK 3.5A1 | Sect. 8.1, 8.4 |
| | | EK 3.5A2 | Sect. 8.3 |
| | | EK 3.5A3 | Sect. 8.3–8.4 |
| | | EK 3.5A4 | Sect. 8.3 |
| | LO 3.5B | EK 3.5B1 | Sect. 8.4 |
| | | EK 3.5B2 (BC) | Sect. 8.3 |

Big Idea 4: Series (BC)

| Enduring Understandings (Students will understand that . . .) | Learning Objectives (Students will be able to . . .) | Essential Knowledge (Students will know that . . .) | <i>Calculus AP Edition 2e</i> |
|--|--|---|-------------------------------|
| EU 4.1: The sum of an infinite number of real numbers may converge. | LO 4.1A | EK 4.1A1 | Sect. 9.1–9.2 |
| | | EK 4.1A2 | Sect. 9.1–9.2 |
| | | EK 4.1A3 | Sect. 9.3–9.4 |
| | | EK 4.1A4 | Sect. 9.6 |
| | | EK 4.1A5 | Sect. 9.6 |
| | | EK 4.1A6 | Sect. 9.4–9.5 |
| | LO 4.1B | EK 4.1B1 | Sect. 9.3 |
| | | EK 4.1B2 | Sect. 9.3 |
| | | EK 4.1B3 | Sect. 9.6 |
| EU 4.2: A function can be represented by an associated power series over its interval of convergence. | LO 4.2A | EK 4.2A1 | Sect. 10.1 |
| | | EK 4.2A2 | Sect. 10.1 |
| | | EK 4.2A3 | Sect. 10.2–10.3 |
| | | EK 4.2A4 | Sect. 10.1 |
| | | EK 4.2A5 | Sect. 9.6 |
| | LO 4.2B | EK 4.2B1 | Sect. 10.1 |
| | | EK 4.2B2 | Sect. 10.3–10.4 |
| | | EK 4.2B3 | Sect. 10.3 |
| | | EK 4.2B4 | Sect. 10.3 |
| | | EK 4.2B5 | Sect. 10.2 |
| | LO 4.2C | EK 4.2C1 | Sect. 10.2 |
| | | EK 4.2C2 | Sect. 10.2 |
| | | EK 4.2C3 | Sect. 10.3 |
| | | EK 4.2C4 | Sect. 10.2 |

Guide to the Mathematical Practices for AP Calculus (MPACs)

The addition of the Mathematical Practices for AP Calculus (MPACs) to the course description for AP Calculus AB and BC is arguably the biggest change in the new course curriculum from the College Board. In the past, the curriculum has consisted of a list of calculus concepts, goals, and information that students needed to understand, accomplish, and know. This has been replaced by the new Curriculum Framework, consisting of six MPACs and a Concept Outline, which presents the subject matter of the courses in a table format (p. T-1).

The MPACs are meant to suffuse both the way AP Calculus courses are taught and the way students solve calculus problems, whether on homework problem sets, quizzes, tests, or exams. Therefore, one cannot point to a single section or chapter in this textbook that covers or contains a particular MPAC. Each of the MPACs recurs many times during the year as the students are learning calculus. The following outline gives specific examples of each MPAC and indicates how each MPAC is woven throughout the second edition of *Calculus: AP Edition*.

MPAC 1 Reasoning with Definitions and Theorems

Students can:

- a. use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;
- b. confirm that hypotheses have been satisfied in order to apply the conclusion of a theorem;
- c. apply definitions and theorems in the process of solving a problem;
- d. interpret quantifiers in definitions and theorems (e.g., “for all,” “there exists”);
- e. develop conjectures based on exploration with technology; and
- f. produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures

Definitions and theorems occur throughout the textbook; however, there are several specific opportunities for developing the skills comprised in this MPAC using theorems and definitions that are covered on the AP Exam.

- Definition of Limits, p. 68
- The Squeeze Theorem, p. 83
- Definition of Continuity, pp. 107, 111
- The Intermediate Value Theorem, p. 115
- The Derivative Function, p. 141
- Rolle’s Theorem, p. 302, and the Mean Value Theorem, p. 303

In addition to providing opportunities to apply theorems, exercise types such as “Explain why or why not” and “Pitfalls using technology” develop the ability to reason and conjecture.

Note that the Precise Definition of Limits (Section 2.7) may also be used, though this topic is not included on the AP Exam.

MPAC 2 Connecting Concepts

Students can:

- a. relate the concept of a limit to all aspects of calculus;
- b. use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;
- c. connect concepts to their visual representations with and without technology; and
- d. identify a common underlying structure in problems involving different contextual situations.

Limits underlie all of calculus and can be used as an overarching connector when teaching. The following main topics are based on limits:

- Definition of Continuity, pp. 107, 111
- The Derivative Function, p. 141
- Derivations of Derivative Rules, Chapter 3
- L'Hôpital's Rule, Section 4.7
- The Definite Integral, Section 5.3
- Infinite Series, Section 9.1

The Fundamental Theorem of Calculus (Section 5.4) further connects these topics.

Procedure boxes and Guidelines boxes throughout the textbook help students identify common structures and problem-solving processes in problems. The following sections have suggested procedures or guidelines appearing in boxes:

- Related Rates, Section 3.11, p. 233
- Optimization, Section 4.4, p. 283
- Linear Approximation, Section 4.5, p. 296
- Velocity-Displacement vs. General Net Change, Section 6.1, pp. 433, 437
- Partial Fractions, Section 7.3, p. 519
- Slope Fields and Euler's Method, Section 8.2, p. 564
- Separable Differential Equations, Section 8.3, p. 573
- Choosing a Series Test, Section 9.5, p. 649

Many sections of the textbook also use the exercises to connect concepts. These exercises are typically found in the "Additional Exercises" section. For example, in Section 5.4 (Fundamental Theorem of Calculus), the following exercises connect with topics from other chapters.

- Exercise 109. Exercise 112. Sine integral connects with differential equations.
- Exercise 110. Max/min of area functions connects with optimization.
- Exercise 115. Discrete version of the Fundamental Theorem connects with the definition of derivative.

MPAC 3 Implementing Algebraic/Computational Processes

Students can:

- a. select appropriate mathematical strategies;
- b. sequence algebraic/computational procedures logically;
- c. complete algebraic/computational processes correctly;
- d. apply technology strategically to solve problems;
- e. attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
- f. connect the results of algebraic/computational processes to the question asked.

Selecting strategies and sequencing steps in a procedure are treated throughout the textbook through extensive Examples and in the Procedure boxes and Guidelines boxes related to MPAC 2 (see above). Examples are keyed to specific exercises so students can practice a procedure immediately after working the Example. Exercises using technology (such as a graphing calculator) are marked with a  throughout the book, allowing one to focus on the technology portion of the AP Exam as needed.

Students working exercises online in MyMathLab (see p. xiii) have access to extra guidance in sequencing steps by using the Help Me Solve This and View an Example features. Additionally, MyMathLab now includes Setup & Solve exercises, which require students to show how to set up a problem as well as solve it.

A variety of topics are developed using numerical or graphic investigations that include considerations of precision, including:

- The Idea of Limits, Section 2.1
- A Limit with Strange Behavior, p. 71
- Graphing Functions, Section 4.3
- Linear Approximations, Example 2, p. 294
- Newton's Method (not covered on the AP Exam), Section 4.8
- Euler's Method, Section 8.2

MPAC 4 Connecting Multiple Representations

Students can:

- a. associate tables, graphs, and symbolic representations of functions;
- b. develop concepts using graphical, symbolical, verbal, or numerical representations with and without technology;
- c. identify how mathematical characteristics of functions are related in different representations;
- d. extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);
- e. construct one representational form from another (e.g., a table from a graph or a graph from given information); and
- f. consider multiple representations (graphical, numerical, analytical, and verbal) of a function to select or construct a useful representation for solving a problem.

The treatment of functions and calculus topics using tables, graphs, and formulas is maintained throughout the textbook. The need for analytic graphing (without technology) is clearly developed in Section 4.3, Graphing Functions, while concomitantly cultivating skills with technology, as in the technology exercises of that section, p. 281.

The online eBook (see p. xv) features hundreds of Interactive Figures that dramatically leverage understanding of graphical representations and concepts. For example, one can move a slider to see a secant becoming a tangent on a diagram defining the derivative at a point.

MPAC 5 Building Notational Fluency

Students can:

- a. know and use a variety of notations (e.g., $f'(x)$, y' , $\frac{dy}{dx}$);
- b. connect notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum);
- c. connect notation to different representations (graphical, numerical, analytical and verbal);
- d. assign meaning to notation, accurately interpreting the notation in a given problem and across different context

Important new notations are introduced carefully and completely, starting with one representation and then developing variety. For example, after $f'(x)$ notation is explained (p. 140) and exemplified, alternative notations are explained and used (p. 142). Other examples include different versions of the Chain Rule, p. 192, and the connection between definite integral and summation notations in the side bar of p. 368.

MPAC 6 Communicating

Students can:

- a. clearly present methods, reasoning, justifications, and conclusions;
- b. use accurate and precise language and notation;
- c. explain the meaning of expressions, notation, and results in terms of a context (including units);
- d. explain the connections among concepts;
- e. critically interpret and accurately report information provided by technology; and
- f. analyze, evaluate, and compare the reasoning of others.

The textbook provides many tools and features to facilitate teaching that encourages learning these communication skills. Every set of exercises begins with “Review Questions” that feature questions that require students to explain their thinking (i.e., questions such as, “Explain what net area means.”). Exercise sets also include more involved “Explain why or why not” questions requiring proof or counterexample. These exercises are perfect for discussion, Socratic method, or think/pair/share.

The Teacher’s Resource Guide (see details on p. xiii) includes fifty-nine Guided Projects that provide excellent templates for facilitating group and individual practice of all these skills in the context of complex problem solving.

AP Calculus Pacing Guide

Numbers in **bold** indicate topics covered on the AP Exam.

| | | AB | BC |
|----------|--|----------|----------|
| | | 7 days | 0 day |
| 1 | Functions | | |
| 1.1 | Review of Functions | 2 | 0 |
| 1.2 | Representing Functions | 1 | 0 |
| 1.3 | Inverse, Exponential, and Logarithmic Functions | 1 | 0 |
| 1.4 | Trigonometric Functions and Their Inverses. | 1 | 0 |
| | <i>Review Exercises & AP Practice Questions</i> | 2 | 0 |
| 2 | Limits | 18 days | 10 days |
| 2.1 | The Idea of Limits | 2 | 1 |
| 2.2 | Definitions of Limits | 3 | 2 |
| 2.3 | Techniques for Computing Limits | 3 | 1 |
| 2.4 | Infinite Limits | 1 | 1 |
| 2.5 | Limits at Infinity | 2 | 1 |
| 2.6 | Continuity | 2 | 1 |
| 2.7 | Precise Definitions of Limits | 2 | 1 |
| | <i>Review Exercises & AP Practice Questions</i> | 3 | 2 |
| 3 | Derivatives | 37 days | 25 days |
| 3.1 | Introducing the Derivative | 2 | 1 |
| 3.2 | Working with Derivatives | 3 | 2 |
| 3.3 | Rules of Differentiation | 3 | 2 |
| 3.4 | The Product and Quotient Rules | 3 | 2 |
| 3.5 | Derivatives of Trigonometric Functions | 3 | 2 |
| 3.6 | Derivatives as Rates of Change | 3 | 2 |
| 3.7 | The Chain Rule | 3 | 2 |
| 3.8 | Implicit Differentiation | 3 | 2 |
| 3.9 | Derivatives of Logarithmic and Exponential Functions | 3 | 2 |
| 3.10 | Derivatives of Inverse Trigonometric Functions | 3 | 2 |
| 3.11 | Related Rates | 5 | 4 |
| | <i>Review Exercises & AP Practice Questions</i> | 3 | 2 |

| | | AB | BC |
|--|--|---------|---------|
| 4 Applications of the Derivative | | 25 days | 18 days |
| 4.1 | Maxima and Minima | 2 | 2 |
| 4.2 | What Derivatives Tell Us | 3 | 2 |
| 4.3 | Graphing Functions | 3 | 2 |
| 4.4 | Optimization Problems | 5 | 4 |
| 4.5 | Linear Approximation and Differentials | 3 | 2 |
| 4.6 | Mean Value Theorem. | 3 | 2 |
| 4.7 | L'Hôpital's Rule | 2 | 1 |
| 4.8 | Newton's Method | 1 | 1 |
| <i>Review Exercises & AP Practice Questions.</i> | | 3 | 2 |

| | | 26 days | 18 days |
|--|---|---------|---------|
| 5 Integration | | 26 days | 18 days |
| 5.1 | Antiderivatives | 4 | 2 |
| 5.2 | Approximating Areas under Curves | 3 | 2 |
| 5.3 | Definite Integrals | 3 | 2 |
| 5.4 | Fundamental Theorem of Calculus | 4 | 3 |
| 5.5 | Properties of Integrals and Average Value | 3 | 2 |
| 5.6 | Substitution Rule. | 4 | 3 |
| 5.7 | Numerical Integration | 2 | 2 |
| <i>Review Exercises & AP Practice Questions.</i> | | 3 | 2 |

| | | 16 days | 13 days |
|--|----------------------------------|---------|---------|
| 6 Applications of Integration | | 16 days | 13 days |
| 6.1 | Velocity and Net Change. | 4 | 3 |
| 6.2 | Regions between Curves | 3 | 1 |
| 6.3 | Volume by Slicing | 3 | 2 |
| 6.4 | Volume by Shells. | 3 | 2 |
| 6.5 | Length of Curves. | 0 | 1 |
| 6.6 | Physical Applications | 0 | 2 |
| <i>Review Exercises & AP Practice Questions.</i> | | 3 | 2 |

| | | 0 day | 7 days |
|--|---------------------------------------|-------|--------|
| 7 Integration Techniques | | 0 day | 7 days |
| 7.1 | Basic Approaches | 0 | 1 |
| 7.2 | Integration by Parts | 0 | 1 |
| 7.3 | Partial Fractions | 0 | 1 |
| 7.4 | Improper Integrals | 0 | 1 |
| 7.5 | Trigonometric Substitutions | 0 | 1 |
| <i>Review Exercises & AP Practice Questions.</i> | | 0 | 2 |

8 Differential Equations

| | AB | BC |
|--|---------|---------|
| | 16 days | 11 days |
| 8.1 Basic Ideas | 4 | 2 |
| 8.2 Slope Fields and Euler's Method. | 2 | 2 |
| 8.3 Separable Differential Equations. | 4 | 3 |
| 8.4 Exponential Models | 3 | 2 |
| <i>Review Exercises & AP Practice Questions.</i> | 3 | 2 |

9 Sequences and Infinite Series

| | 0 day | 15 days |
|--|-------|---------|
| 9.1 An Overview | 0 | 2 |
| 9.2 Sequences | 0 | 2 |
| 9.3 Infinite Series. | 0 | 2 |
| 9.4 The Divergence and Integral Tests | 0 | 2 |
| 9.5 The Ratio, Root, and Comparison Tests | 0 | 2 |
| 9.6 Alternating Series | 0 | 3 |
| <i>Review Exercises & AP Practice Questions.</i> | 0 | 2 |

10 Power Series

| | 0 day | 18 days |
|--|-------|---------|
| 10.1 Approximating Functions with Polynomials. | 0 | 4 |
| 10.2 Properties of Power Series | 0 | 4 |
| 10.3 Taylor Series | 0 | 4 |
| 10.4 Working with Taylor Series | 0 | 4 |
| <i>Review Exercises & AP Practice Questions.</i> | 0 | 2 |

11 Polar, Parametric, and Vector Curves

| | 0 day | 11 days |
|---|-------|---------|
| 11.1 Parametric Equations | 0 | 1 |
| 11.2 Calculus with Parametric Equations | 0 | 1 |
| 11.3 Polar Coordinates | 0 | 1 |
| 11.4 Calculus in Polar Coordinates | 0 | 2 |
| 11.5 Vectors in The Plane | 0 | 1 |
| 11.6 Calculus of Vector-Valued Functions | 0 | 2 |
| 11.7 Two-Dimensional Motion | 0 | 1 |
| <i>Review Exercises & AP Practice Questions</i> | 0 | 2 |

**Total: 146 Teaching Days
(including non-AP Exam topics)**

Note: This timeline is based on a school year starting after Labor Day, with approximately 160 teaching days before the Advanced Placement[®] exam. This timeline gives approximately 14 days to review the course before the exam. This review time is essential, and 3 weeks is a minimum time to set aside for this purpose.

Credits

Cover Image, “Handmade glass”, Gekon/Fotolia.

Chapter 1

Page 1, “Team of three alpinists climbing a mountain,” Mikadun/Shutterstock. **Page 26**, Tucker, V. A. (2000). The Deep Fovea, Sideways Vision and Spiral Flight Paths in Raptors. *The Journal of Experimental Biology*, 203, 3745–3754. **Page 26**, Collings, B. J. (2007, January). Tennis (and Volleyball) Without Geometric Series. *The College Mathematics Journal*, 38(1). **Page 26**, U.S. Fish and Wildlife Service. **Page 51**, Isaksen, D. C. (1996, September). How to Kick a Field Goal. *The College Mathematics Journal*, 27(4).

Chapter 2

Page 60, “African flamingos,” Anna Om/Fotolia. **Page 119**, Arthur Koestler, The Act of Creation.

Chapter 3

Page 136, “Surfer on Amazing Blue Wave in the Barrel, Epic Tube,” EpicStockMedia/Shutterstock. **Page 138**, Facebook SEC filing. **Page 145**, U.S. Bureau of Census. **Page 214**, Hook, E. G. & Lindsjo, A. (1978, January). Down syndrome in live births by single year maternal age interval in a Swedish study: comparison with results from a New York State study. *American Journal of Human Genetics*, 30(1).

Chapter 4

Page 246, “Meteor tumbling towards earth,” Storm/Fotolia. **Page 246**, Courtesy NASA/JPL-Caltech. **Page 248**, THOMAS, GEORGE B.; WEIR, MAURICE D.; HASS, JOEL; GIORDANO, FRANK R., THOMAS’ CALCULUS, EARLY TRANSCENDENTALS, MEDIA UPGRADE, 11th, ©2008. Printed and Electronically reproduced by permission of Pearson Education, Inc., Upper Saddle River, New Jersey. **Page 249**, THOMAS’ CALCULUS, EARLY TRANSCENDENTALS, MEDIA UPGRADE, 11th edition, by George B. Thomas, Maurice D. Weir, Joel Hass, and Frank R. Giordano. Copyright © 2008 Pearson Education, Inc. Printed and Electronically reproduced by permission of Pearson Education, Inc., Upper Saddle River, New Jersey. **Page 250**, THOMAS’ CALCULUS, EARLY TRANSCENDENTALS, MEDIA UPGRADE, 11th edition, by George B. Thomas, Maurice D. Weir, Joel Hass, and Frank R. Giordano. Copyright © 2008 Pearson Education, Inc. Printed and Electronically reproduced by permission of Pearson Education, Inc., Upper Saddle River, New Jersey. **Page 262**, THOMAS’ CALCULUS, EARLY TRANSCENDENTALS, MEDIA UPGRADE, 11th edition, by George B. Thomas, Maurice D. Weir, Joel Hass, and Frank R. Giordano. Copyright © 2008 Pearson Education, Inc. Printed and Electronically reproduced by permission of Pearson Education, Inc., Upper Saddle River, New Jersey. **Page 266**, THOMAS’ CALCULUS, EARLY TRANSCENDENTALS, MEDIA UPGRADE, 11th edition, by George B. Thomas, Maurice D. Weir, Joel Hass, and Frank R. Giordano. Copyright © 2008 Pearson Education, Inc. Printed and Electronically reproduced by permission of Pearson Education, Inc., Upper Saddle River, New Jersey.

COMP: replace wording for pgs 249, 250, 262, 266 Thomas credits with exact wording as shown for pg 248

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Chapter 5

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