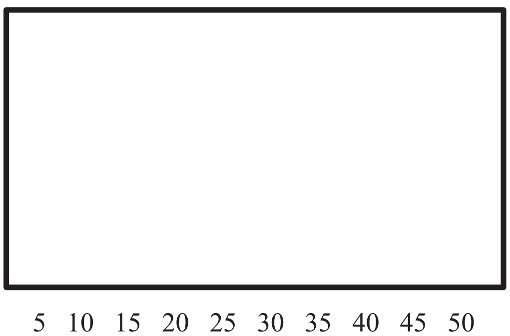
Understanding the Birthday Problem
A Numerical, Graphical, and Practical Investigation

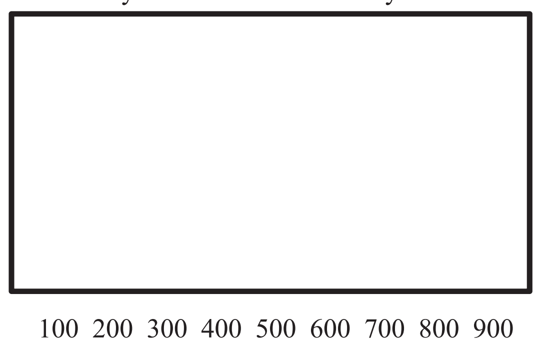
Before you begin: Review the section titled *Probability and Coincidence* in Unit 7E of your textbook and especially focus on Example 8: Birthday Coincidence to get an overview of the classic birthday problem. Try to anticipate the shape of the probability curves by sketching your best guess in the rectangles below. Let the *y*-axis represent the approximate probabilities ranging 0 to 1 (that is, impossible to certain). If you are working with a partner, discuss the possible shapes first before committing to paper.

Probability of Any Birthday in Common



Number of People in a Room

Probability of a Particular Birthday in Common



Number of People in a Room

Procedure: Now, with a blank spreadsheet, carefully follow the steps below:

<u>A Particular Birthday in Common</u>

1. Type in cell A1, "Particular Birthday in Common" to act as a header for this first scenario.
2. Now in cell A3 type "Persons" and in B3 type "Probabilities."
3. In cell A4 type 50 and A5 type 100.
4. With that pattern completed (increment of 50 each time), highlight cells A4 and A5 and then fill down the pattern to 1500 persons (point the arrow in the lower right corner and drag downward on the black cross hair).
5. To calculate the corresponding probabilities, refer to Example 8 part (a) and type the following formula in cell B4: `=1-(364/365)^A4`. This uses the at least once rule and cell A4 contains the number of persons in the room besides yourself.
6. Finish the probabilities by filling the formula in cell B4 down to the cell with 1500 persons.

<u>Any Birthday in Common</u>

1. Type in cell D1, "Any Birthday in Common" to act as your header for the second scenario.
2. Now in cell D3 type 'Persons' and in E3 type "Probabilities."

3. In cell D4 type 2 and D5 type 3.
4. With that pattern completed (increment of 1 person each time), highlight cells D4 and D5 and then fill down the pattern to 50 persons (use the black crosshair in the lower right corner) click and drag downward as before.
5. To calculate the corresponding probabilities, refer to Example 8 part (b) and type the following formula in cell E4: $=1-\text{PERMUT}(365, \text{D4})/365\,^\wedge\,\text{D4}$. As mentioned in the text, this formula still uses the *at least once* question but cell D4 contains the number of persons in the room <u>including</u> yourself. Notice how the function PERMUT accomplishes what many calculators cannot accomplish due to the large numbers. (Only calculators that have a capacity of 10^999 can handle these calculations. Examples include the TI-89 and TI-86.)

<u>Graphs of the Probabilities</u>

1. Review the Using Technology boxes of 5C to learn how to construct a line chart or scatterplot effectively.
2. Highlight the Particular Birthday probabilities from column B and create a line chart that displays the probabilities as they vary with the number of people. Now repeat the process for the probabilities in column E for the Any Birthday in Common scenario.
3. Print your line charts with adequate titles and labels if instructed by your teacher.

Results Summary/Analysis: Now with the two types of probabilities calculated on your spreadsheet, fill in the following tables and complete the questions below. Round your probabilities in the tables below to two decimal places.

A Particular Birthday in Common:

Persons	100	200	300	400	500	600	700	800	900
Probabilities									

Any Birthday in Common:

Persons	5	10	15	20	25	30	35	40	45
Probabilities									

1. How many students are in your class counting yourself and the teacher? __________
2. What is the probability that someone in the class has your very birthday? __________
3. What is the probability that there is a birthday match (that is, some birthday in common) in the room? ________
4. How confident are you that there will actually be a match for question #3 with your particular class of students?

5. ** With your teacher's help, the class will find out whether there is a birthday match in the room. After this practical survey, describe the outcome and your thoughts or insights:

6. According to your data from your spreadsheet, how many people are needed in a group in order to have approximately 0.50 probability that someone has a birthday in common with someone else in the group? __________

7. Now answer the same question from #6 but for the "Particular Birthday" scenario. That is, between _____ and _____ people would be needed in the room to have a 0.50 probability that two people share a particular birthday. What is the most accurate answer you can find? (Modify your spreadsheet slightly to find it.) ___________

Discussion:

1. How do your actual line charts compare to your first impressions you sketched out in the Before You Begin section above? Were you close in terms of shape or significantly off from your first guesses?
2. Disregarding Leap Day, how many people would be needed in the room to be 100% confident that at least two people share the same birthday? (This is known as the Pigeonhole Principle.) Justify your answer.
3. Now, looking at your results on the spreadsheet for the Particular Birthday scenario, discuss the theoretical number of people needed in a room to be absolutely certain someone has your birthday. Why is this different from your answer in question #2?
4. Describe how the two line charts (Particular Birthday vs. Any Birthday) differ in terms of the shape and the number of persons on the x-axis.
5. At approximately what probability does the curve from the Any Birthday line chart seem to bend? Or in other words, where does it change from increasing "faster and faster" to increasing "slower and slower"?
6. The story is told of a guest on the Johnny Carson's Tonight Show, back in the 1970s, who tried to impress everyone about the classic birthday problem. He told Cason that even though the studio audience consisted of only about 120 people (less than 1/3 of the number of days in the calendar year) there was an extremely high probability that someone would have Johnny Carson's birthday in the studio and the guest was obviously embarrassed. What would the probability have been for success given that there were about 120 people in the audience? Discuss how the guest was confused about the classic birthday problem and how it should have been stated on the show correctly.