

Factors and primes

The **factors** of a number are any numbers that divide into it exactly. A **prime number** has exactly two factors. The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19 and so on.

Prime factors

If a number is a factor of another number **and** it is a prime number then it is called a **prime factor**. You use a factor tree to find prime factors.

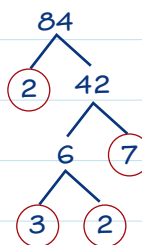
$$84 = 2 \times 2 \times 3 \times 7$$

$$= 2^2 \times 3 \times 7$$

Remember to put in the multiplication signs.

This is called a **product of prime factors**.

Remember to circle the prime factors as you go along. The order doesn't matter.



The highest common factor (HCF) of two numbers is the **highest number** that is a **factor** of both numbers.

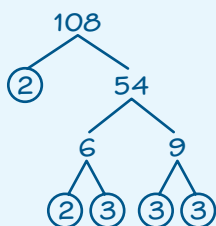
The lowest common multiple (LCM) of two numbers is the **lowest number** that is a **multiple** of both numbers.

Worked example

Target grade **4**

- (a) Express 108 as a product of powers of its prime factors.

(3 marks)



$$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$$

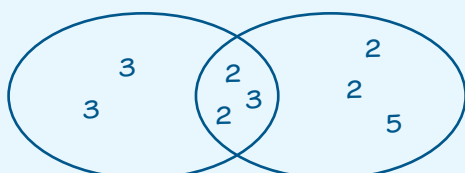
- (b) $240 = 2^4 \times 3 \times 5$

Find, as a product of powers of its prime factors,

- (i) the highest common factor (HCF) of 108 and 240 (1 mark)

Factors of 108

Factors of 240



$$\text{HCF} = 2 \times 2 \times 3 = 2^2 \times 3$$

- (ii) the lowest common multiple (LCM) of 108 and 240 (1 mark)

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$$

$$= 2^4 \times 3^3 \times 5$$

Examiners' report

If you have to write a number as a **product** of prime factors, make sure you use \times signs in your final answer. Don't use $+$, and don't just write a list of prime factors.

Real students have struggled with questions like this in recent exams – **be prepared!**



You can find the HCF and LCM by writing the products of prime factors in a **Venn diagram**. Use the powers to tell you how many times each prime factor occurs. Put the **common factors** in the intersection of the two ovals.

- HCF = product of all the prime factors in the intersection
- LCM = product of all the prime factors in the Venn diagram

There is more on Venn diagrams on page 125.

Now try this

Target grade **4**

- (a) Express 980 as a product of its prime factors. (3 marks)

(b) Find the highest common factor (HCF) of 980 and 56. (2 marks)
- $X = 2 \times 3^5 \times 7^2$ $Y = 3^2 \times 5 \times 7$

(a) Find the highest common factor (HCF) of X and Y . (2 marks)

(b) Find the lowest common multiple (LCM) of X and Y . (2 marks)

You can use numbers given in index form directly:
 HCF: Choose the **lowest** power of each prime.
 LCM: Choose the **highest** power of each prime.
 For example, $\text{HCF} = 2^0 \times 3^2 \times 5^0 \times 7^1 = 3^2 \times 7$