

Fractional indices

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the n th root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- $a^{-m} = \frac{1}{a^m}$
- Every positive number, a , has two square roots, positive \sqrt{a} and negative $-\sqrt{a}$. The two roots can be written using the \pm symbol, e.g. $\pm\sqrt{16} = \pm 4$.

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1
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Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$
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Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = (\sqrt[3]{27})^2$ $= 3^2$ $= 9$	<ol style="list-style-type: none"> 1 Use the rule $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ 2 Use $\sqrt[3]{27} = 3$
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Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$	1 Use the rule $a^{-m} = \frac{1}{a^m}$
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$$= \frac{1}{16}$$

$$2 \text{ Use } 4^2 = 16$$

Practice questions

1 Evaluate.

a $49^{\frac{1}{2}}$

b $64^{\frac{1}{3}}$

c $125^{\frac{1}{3}}$

d $16^{\frac{1}{4}}$

2 Evaluate.

a $25^{\frac{3}{2}}$

b $8^{\frac{5}{3}}$

c $49^{\frac{3}{2}}$

d $16^{\frac{3}{4}}$

3 Evaluate.

a 5^{-2}

b 4^{-3}

c 2^{-5}

d 6^{-2}

4 Evaluate.

a $4^{-\frac{1}{2}}$

b $27^{-\frac{2}{3}}$

c $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$

e $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

f $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

5 (a) Simplify fully $(3e)^0$

(b) Simplify $\left(\frac{64x^6}{25y^2}\right)^{\frac{1}{2}}$

Answers

1 a 7

b 4

c 5

d 2

2 a 125

b 32

c 343

d 8

3 a $\frac{1}{25}$

b $\frac{1}{64}$

c $\frac{1}{32}$

d $\frac{1}{36}$

4 a $\frac{1}{2}$

b $\frac{1}{9}$

c $\frac{8}{3}$

d $\frac{1}{4}$

e $\frac{4}{3}$

f $\frac{16}{9}$

5 a 1

b $\frac{5y}{8x^3}$